On the Validation of Chrono::Granular

Luning Fang, Ruochun Zhang, Jason Zhou, Dan Negrut
Abstract

This report summarizes a set of validation tests for Chrono::Granular, a module that is part of the Chrono dynamics simulation engine [1, 2]. These tests range from simple one-sphere simulations, to complex scenarios in which many spheres interact with external objects represented as meshes. The Chrono::Granular results are compared against analytical results or data published in the literature. No non-physical behavior is observed; the results match the expected outcomes.

Keywords: Chrono, granular dynamics, friction force, normal contact force, cosimulation, mesh-granular material interaction
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1 Overview

Chrono::Granular, a software leveraging commodity Graphics Processing Unit (GPU), has been developed to run large-scale dynamics problem [3] using the Discrete Element Method (DEM) framework [4]. The software utilizes mixed-precision data types, shared memory and the CUB library, thus demonstrating linear scaling for 3D scenarios with up to 710 million frictionless monodisperse particles and up to 210 million frictional ones. The software offers an interface for cosimulation with third-party dynamics engines by defining external geometries with triangle meshes. The list of time integrators available in Chrono::Granular includes first-order Explicit Euler, extended Taylor, centered differences, and Chung [5]. Chrono::Granular uses a Hertzian contact force in normal direction; a history-based tangential friction model [6]; and a constant torque rolling friction model [7]. Currently, Chrono::Granular only supports monodisperse systems; i.e., the granular material is made up of spheres of identical radii.

This report focuses on validating the code for tests for which validation data is readily available. Some tests include a single sphere since in this case an analytical solution is likely available. Several other tests include more spheres; in this cases, validation data might come from the literature. Specifically, the small-scale tests performed are settling (Sec. 2.1), rolling with and without rolling friction (Sec. 2.2 and Sec. 2.3), and stacking (Sec. 2.5). Large-scale tests carried out are spheres settling in a container (Sec. 2.6), angle of repose (Sec. 2.7), crater/ball-drop test (Sec. 2.8) and Goldenberg (Sec. 2.9).

2 Numerical Experiments

2.1 Single sphere settling on ground

This experiment measures the penetration length of a sphere setting on a plane, under gravity only. Although the contact force model adopted in this version of Chrono::Granular calls for user-defined parameters, \( k_n, \gamma_n, k_t \) and \( \gamma_t \), a material-based one can still be implemented with minor adjustments. For this test, a nonlinear Hertzian contact model [8] was used to validate the elastic component of the normal force, expressed as,

\[
F_N = K\delta^2, \quad (1)
\]

where \( K \) is the generalized stiffness and \( \delta \) is the normal penetration. The generalized stiffness \( K \) is dependent on material properties and the the radius of particles in contact, \( r_i \) and \( r_j \),

\[
K = \frac{4}{3(\sigma_i + \sigma_j)} \sqrt{\frac{r_i r_j}{r_i + r_j}}, \quad (2)
\]

in which parameters \( \sigma_i \) and \( \sigma_j \) are associated with Young’s modulus and Poisson ratio of each particle, \( E_i, E_j, \nu_i \) and \( \nu_j \),

\[
\sigma_i = \frac{1 - \nu_i^2}{E_i}, \quad \sigma_j = \frac{1 - \nu_j^2}{E_j}. \quad (3)
\]
For a sphere of radius $r$ resting on the ground scenario, assuming the same material property of the sphere and the ground, Eq.(2) can be reduced to

$$K = \frac{2\sqrt{r}}{3\sigma}, \quad \sigma = \frac{1 - \nu^2}{E}. \quad (4)$$

In Chrono::Granular, the user can define normal stiffness parameter as,

$$k_n = K\sqrt{r}, \quad (5)$$

to achieve the same normal penetration as a material-based Hertzian contact force model represented by Eq.(1). To demonstrate this, spheres with various Young’s moduli, $E = 10^{11}, 10^{10}, 10^{9}, 10^{8}$ and $10^{7}\text{ g} \cdot \text{cm}^{-1} \cdot \text{s}^{-2}$, and radius $r = 0.5, 0.7, 0.9, 1.1$ and $1.3\text{ cm}$, were settled on the ground and the penetration in each case was compared with the analytical solution. All tests used the same density, $\rho = 7.8\text{ g} \cdot \text{cm}^{-3}$, and Poisson ratio, $\nu = 0.3$. Note that the damping coefficient $\gamma_n$ is irrelevant in this test, considering only the settled stage was compared. The ground is modeled both as a numerical boundary condition and a one-facet mesh to compare the result. Figure 1 shows the screenshots of the mesh. In Fig. 1 (b) it is clear that the sphere penetrates the mesh (ground), in line with our penalty-based model.

![Figure 1: Screenshots of the sphere settling test, with a mesh](image)

The penetration lengths that Chrono::Granular outputs with $r$, $E$, and ground modeled as numerical boundary $\delta_B$ and mesh $\delta_M$ are given in Table 1, along with the corresponding analytical penetration lengths ($\delta_E$) for reference.

There is no notable difference between that of theory and the Chrono::Granular outputs. This experiment validates the normal force model concerning the interaction between spheres and meshes.

### 2.2 Single sphere rolling on ground without rolling resistance

This section introduces a Chrono::Granular validation test in which a sphere is rolling on the frictional ground; the quantities of interest are its translational and angular velocities.
Table 1: Penetration lengths that Chrono::Granular outputs ($\delta_M$ for mesh and $\delta_B$ for numerical boundary) and that of analytical ($\delta_E$), w.r.t. sphere radii ($r$), Young’s modulus ($E$) as well as the corresponding normal stiffness ($K$ in Eq.(4) and $k_n$ in Eq.(5)) in the contact force model

\begin{center}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$E$ (g·cm$^{-1}$·s$^{-2}$) & $K$ (g·cm$^{-\frac{3}{2}}$·s$^{-2}$) & $k_n$ (g·s$^{-2}$) & $\delta_B$ (cm) & $\delta_M$ (cm) & $\delta_E$ (cm) \\
\hline
1e11 & 5.18e10 & 3.66e10 & 1.84e-5 & 1.8e-5 & 1.81e-5 \\
1e10 & 5.18e9 & 3.66e9 & 8.42e-5 & 8.4e-5 & 8.42e-5 \\
1e9 & 5.18e8 & 3.66e8 & 3.9e-4 & 3.9e-4 & 3.91e-4 \\
1e8 & 5.18e7 & 3.66e7 & 1.8e-3 & 1.81e-3 & 1.81e-3 \\
1e7 & 5.18e6 & 3.66e6 & 8.42e-5 & 8.42e-3 & 8.42e-3 \\
\hline
\end{tabular}
\end{center}

\begin{center}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$E$ (g·cm$^{-1}$·s$^{-2}$) & $K$ (g·cm$^{-\frac{3}{2}}$·s$^{-2}$) & $k_n$ (g·s$^{-2}$) & $\delta_B$ (cm) & $\delta_M$ (cm) & $\delta_E$ (cm) \\
\hline
1e11 & 6.13e10 & 5.13e10 & 3.2e-5 & 3.18e-5 & 3.18e-5 \\
1e10 & 6.13e9 & 5.13e9 & 1.47e-5 & 1.47e-5 & 1.48e-5 \\
1e9 & 6.13e8 & 5.13e8 & 6.84e-4 & 6.85e-4 & 6.85e-4 \\
1e8 & 6.13e7 & 5.13e7 & 3.18e-3 & 3.18e-3 & 3.18e-3 \\
1e7 & 6.13e6 & 5.13e6 & 1.48e-2 & 1.48e-2 & 1.48e-2 \\
\hline
\end{tabular}
\end{center}

2.2.1 Ground represented by boundary

We first use Chrono::Granular plane boundary type to represent the ground. The sphere has the radius of 0.5cm and density of 7.8g·cm$^{-3}$, and is given an initial velocity of 1cm·s$^{-1}$. The sphere–wall normal and tangential stiffness used in input files are 1e11g·s$^{-2}$ and 1e7g·s$^{-2}$, respectively. The static friction coefficient between the wall and the sphere is set to be 0.25. The translational and angular velocities obtained in simulation are reported in Fig. 2. Within 0.1s, the sphere establishes an angular velocity of around 1.42s$^{-1}$, and its translational velocity reduces to around 0.71cm·s$^{-1}$. This indicates entering a pure rolling state. The sphere then rolls indefinitely due to the lack of rolling resistance.

2.2.2 Ground represented by mesh

While essentially repeating the test above, the flat, “one body” ground is replaced by a mesh this time. A one-facet mesh is used to model the ground. The sphere has the radius of 0.5cm and density of 7.8g·cm$^{-3}$, and is given an initial velocity of 1cm·s$^{-1}$. The sphere–mesh normal and tangential stiffness used in input files are 1e11g·s$^{-2}$ and 1e7g·s$^{-2}$, respectively. The static friction coefficient between the mesh and the sphere is set to be 0.25. We plot the translational and angular velocities of this sphere in Fig. 3. The results are seen to be identical to the ones obtained for the case when the ground was represented as one body, see Fig. 2.
Figure 2: Velocity and angular velocity of a sphere rolling on a frictional Chrono::Granular boundary wall, with a prescribed initial velocity.

Figure 3: Velocity and angular velocity of a sphere rolling on a frictional mesh, with a prescribed initial velocity.
2.3 Single sphere rolling on ground with rolling resistance

This section focuses on a Chrono::Granular validation test in which a sphere was rolled on the ground with an initial velocity; both the sliding friction $F_r$ and rolling friction $M_r$ were at work. The rolling friction model adopted in Chrono::Granular is a constant torque one, with its magnitude proportional to the rolling friction coefficient $\mu_r$, the normal contact force magnitude $F_n$, and effective radius $R_r$. Its orientation is opposite of the relative angular velocity, $\omega_{rel} = \omega_i - \omega_j$:

$$M_r = -\frac{\omega_{rel}}{|\omega_{rel}|} \mu_r R_r F_n.$$  \hfill (6)

Note that this model leads to a “zero divided by zero” scenario associated with $\omega_{rel}$, and the current solution is to set rolling friction $M_r$ to zero once $v_{rot} = |\omega_{rel}| R_{eff}$ is less than a threshold value, 0.005. This implementation causes numerical artifacts when the sphere is supposed to come to a stop. When $M_r$ is hard-coded to zero with nonzero angular velocity, the non-zero friction force $F_r$ can still increase angular velocity, pushing $v_{rot}$ above the threshold value, resulting a non-zero $M_r$ again. The non-zero rolling resistance $M_r$ then decreases angular velocity until it reaches the threshold value again. As a result, the sphere would experience small oscillation when it settles, see [9,10] for more details.

2.3.1 Ground represented by boundary

We first used Chrono::Granular plane boundary type to represent the ground. The sphere had the radius of 0.5 cm, density of 7.8 g · cm$^{-3}$, and was given an initial velocity of 1 cm · s$^{-1}$. The sphere–wall normal and tangential stiffness used, $k_n$ and $k_t$, were $1e11$ g · s$^{-2}$ and $1e7$ g · s$^{-2}$, respectively. The static friction coefficient between the wall and the sphere was set to 0.25. The rolling friction coefficient, $\mu_r$, between the wall and the sphere was set to 0.01. The translational and angular velocities of this sphere were plotted in Fig. 4.

![Figure 4: Velocity and angular velocity of a sphere rolling on a frictional Chrono::Granular boundary wall that has rolling resistance, with a prescribed initial velocity](image-url)
Within 0.2s, the sphere stopped moving altogether. It is notable that between 0.1s and 0.15s, there was a time window where the introduction of rolling friction caused the sphere to move backwards. When this effect died down, the sphere comes to a halt.

2.3.2 Ground represented by mesh

![Figure 5: Velocity and angular velocity of a sphere rolling on a frictional mesh that has rolling resistance, with a prescribed initial velocity](image)

While essentially repeating the test above, we are once more using a mesh to account for the presence of the ground. A one-facet mesh was used to model the ground. Similarly, the sphere had the radius of 0.5 cm and density of 7.8 g · cm⁻³, and was given an initial velocity of 1 cm · s⁻¹. The sphere–mesh normal and tangential stiffness used, $k_n$ and $k_t$, were $1e11$ g · s⁻² and $1e7$ g · s⁻², respectively. The static friction coefficient between the mesh and the sphere was set to 0.25. The rolling friction coefficient, $\mu_r$, between the wall and the sphere was set to be 0.01. The velocity and angular velocity of this sphere were plotted in Fig. 5. We can draw the same conclusion as for the sphere-wall case.

2.4 Sphere–mesh interaction: corner cases

This section focuses on two corner cases in Chrono::Granular sphere–mesh interaction and suggests measures to remedy potential problems.

2.4.1 Potential mesh penetration

While using the penalty-based force model, Chrono::Granular does not employ an explicit safety measure for extremely large penetrations. The question that arises is whether a “punch-through”, or tunneling, scenario happens, where a sphere seizes to be in contact with a plane (or a mesh facet, for that matter) when the penetration becomes larger than the sphere diameter.
The test in this section demonstrates that the aforementioned scenario can happen only under extreme and unrealistic conditions. The test is letting particles with certain densities hit a mesh facet with a range of velocities, and inspect whether they go through the mesh (tunneling). The experiment setup is illustrated in Fig. 6. The spheres used in this test have radius of 0.5cm. The sphere–mesh normal stiffness input in Chrono::Granular input is $1\text{e}11\text{g}\cdot\text{s}^{-2}$, mimicking a material similar to steel.

Figure 6: The experiment is letting a single sphere hit a mesh facet, and see it can “brute force” through it

The experimental outcome is shown in Table 2. If the sphere has the normal steel density of $7.8\text{g}\cdot\text{cm}^{-3}$, the mesh breach/tunneling happens only at velocity $2000\text{m/s}$. If we are to let it happen at a more realistic velocity such as $10\text{m/s}$ (still quite high), the sphere needs to have 1000 times the steel density. The conclusion is therefore that such mesh breach will not happen in simulations where the governing physics is mild, justifying the current “no-treatment” policy.

It is worth noting however, this is only a single-sphere test, and if the sphere is under the influence of a large external force (such as due to compression) then the result could be different.

Table 2: Experimental results on whether a sphere with certain velocity and density can (T) or cannot (F) penetrate through a mesh facet

<table>
<thead>
<tr>
<th>Density (g/cm$^3$)</th>
<th>1e3cm/s</th>
<th>1e4cm/s</th>
<th>1e5cm/s</th>
<th>2e5cm/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7.8\text{g}\cdot\text{cm}^{-3}$</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>$78\text{g}\cdot\text{cm}^{-3}$</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>$780\text{g}\cdot\text{cm}^{-3}$</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>$7800\text{g}\cdot\text{cm}^{-3}$</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>
2.4.2 Mesh facet sub-domain assignment

In this section we document an issue we encountered in determining the sub-domain (SD) assignment for a mesh facet, as well as the corresponding treatment. The related code is found in file `ChGranularGPU_SM_C_trimesh.cuh`.

Figure 7 gives a demonstration. In rare cases, a mesh facet (red triangle) can lie right in the plane between 2 SDs (blue and yellow cubes), as seen in (a). Due to the floating point accuracy, this could result in that this triangle being assigned to no SD, rendering it inactive in this simulation; or being assigned to a SD, but in the contact detection phase, the contact point is considered to be in the other SD, invalidating this contact event.

This rare scenario is handled as follows. In the SD assignment phase, the bounding boxes of mesh facets are used to do a coarse SD assignment sweep. We artificially enlarge this bounding box (by 0.1% of the SD size, with current implementation) as seen in (b), the red box. We can thus ensure that a mesh facet that lies in such an unfavorable position is picked up by both SDs, and in this way no sphere–mesh contact will be missed.

Not that this hardly invokes any computational overhead, because such a small inflation in bounding box size does not influence the SD assignment for the vast majority of mesh facets, see (c).

![Diagram](image)

(a) A mesh facet lies right between 2 SDs could have ambiguity in SD assignment.
(b) Artificially enlarging the bounding box ensures both SDs compute this mesh facet.
(c) For most mesh facets this treatment works as if we use only “true” bounding boxes, not affecting efficiency.

Figure 7: Demonstrating the challenge in assigning a mesh facet to a SD. There is ambiguity which SD a facet (red triangle) belongs to, if it lies right between 2 SDs (blue and yellow cubes). We enlarge its bounding box (red box) to ensure proper assignment, while not hindering the performance.
2.5 Spheres stacking in a column

Nine simulations have been conducted in the stacking unit test. The test consisted of 2 to 10 spheres being dropped on top of each other. Initially, the gap in the z-direction between any two neighboring spheres was 1 cm, which is also equal to the radius of the spheres. The quantities monitored in this test are the total kinetic energy of each system and the normal contact force between the spheres. The visualization of the simulation can be found in Fig. 8. The parameters used for the simulation are listed in Table 3.

![Figure 8: Visualization for the stacking unit test with n=2,3,4,5,6,7,8,9 from left to right](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle Radius</td>
<td>1 [cm]</td>
</tr>
<tr>
<td>Particle Density</td>
<td>2.5 [g \cdot cm^{-3}]</td>
</tr>
<tr>
<td>Simulation Step Size</td>
<td>5e-5 [s]</td>
</tr>
<tr>
<td>Normal Force Stiffness</td>
<td>1e7 [g \cdot s^{-2}]</td>
</tr>
<tr>
<td>Normal Force Damping Coefficient</td>
<td>2e4 [s^{-1}]</td>
</tr>
<tr>
<td>Tangential Force Stiffness</td>
<td>1e6 [g \cdot s^{-1}]</td>
</tr>
<tr>
<td>Tangential Force Damping Coefficient</td>
<td>50 [s^{-1}]</td>
</tr>
<tr>
<td>Friction Coefficient</td>
<td>0.5 [-]</td>
</tr>
</tbody>
</table>

The kinetic energy plot shown in Fig. 9 compares the kinetic energy of the granular columns containing 2 to 10 particles. The initial kinetic energy surge is due to the free fall of the particles as potential energy gets converted into kinetic energy. Each collision
between two spheres causes energy loss owing to the dissipative damping component in normal contact force. Thus, each sudden total kinetic energy decrease indicates an impact between a particle to another particle. Based on Fig 9, in all nine simulations, the impact time between a specific pair of particles is the same, which is expected as a specific particle was dropped from a fixed height in all nine simulations. After all particles finish stacking, the total kinetic energy decreases and returns to zero as the system settles.

Finally, Fig. 10 shows the magnitude of the normal contact force between a certain pair of particles in the 10-particle simulation. Particles were numbered from 0 to 9 and the contact force between each pair of particles was recorded. The contact force peaks when the pair of particles collides, and decrease to \((9 - i) \times \text{particle-weight}\), \(i\) is the number of the particle, after the system settles.

### 2.6 Spheres settling in a container

In this test, a cube of spheres is dropped and settles inside a container. The container is defined by 6 boundaries - bottom and upper boundaries in the Z direction, front and back boundaries in the Y direction, and the left and right boundaries in the X direction. The boundaries form a cube volume with a side length of 100 cm. The mass of one sphere is 4.18879 g; the total mass of the 40581 spheres in the experiment is 169.98530 kg.

This setup was run twice in Chrono::Granular - with zero friction, and non-zero friction. The net forces applied on different boundaries by the particles coming in contact with the box walls were extracted from the simulation and subsequently analyzed. A visualization of the box walls along with their IDs is provided in Fig. 11: boundaries/walls 0 and 1 are facing the negative and positive x direction outward; boundaries are 2 and 3 facing the
Figure 10: Normal contact force between various pairs of particles

negative and positive y direction outward; and boundaries are 4 and 5 facing the negative and positive z direction outward. The direction of arrows also indicates the direction of the force applied on the boundary by the particles. The parameters used in the simulation can be found in Table 4. In Fig. 12, the images show the visualization of the system at time t = 0s, the initial stage when the spheres are being dropped; t = 0.14s, when the bottom layers of spheres started to hit and settled in the confined volume; t = 0.32s, when spheres start bouncing back into the air and hitting the upper boundary; and t = 10s, when all spheres are settled in the container.

Table 4: Simulation Parameters for the Sphere Settling in Container test

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle Radius</td>
<td>1 [cm]</td>
</tr>
<tr>
<td>Particle Density</td>
<td>1 [g · cm⁻³]</td>
</tr>
<tr>
<td>Simulation Step Size</td>
<td>5e⁻⁵ [s]</td>
</tr>
<tr>
<td>Normal Force Stiffness</td>
<td>1e⁸ [g · s⁻²]</td>
</tr>
<tr>
<td>Normal Force Damping Coefficient</td>
<td>2e³ [s⁻¹]</td>
</tr>
<tr>
<td>Tangential Force Stiffness</td>
<td>1e⁷ [g · s⁻¹]</td>
</tr>
<tr>
<td>Tangential Force Damping Coefficient</td>
<td>0 [s⁻¹]</td>
</tr>
<tr>
<td>Friction Coefficient</td>
<td>0.5 [-]</td>
</tr>
</tbody>
</table>
Figure 11: Visualization of box surface numbering
(a) At $t=0s$, a cube of spheres were created, all spheres are subject to gravity and have initial velocity of 0

(b) At $t=0.14s$, spheres on the bottom layers reach the bottom boundary of the container, some of the spheres start bouncing back up

(c) At $t=0.32s$, most spheres were settled inside the container, while some spheres bounced back up and hit the upper boundary

(d) At $t=10s$, the end of the unit test simulation, all spheres were settled inside the container

Figure 12: Visualization for 'Spheres settling in a container' at timestamps t=0s, t=0.14s, t=0.32s, and t=10s
The system-wide total kinetic energy in two different running modes was calculated as

\[ TotalKE = \sum_{i=1}^{NumSphere} \left[ \frac{1}{2} m_i (v_{ix}^2 + v_{iy}^2 + v_{iz}^2) + \frac{1}{2} I_i (\omega_{ix}^2 + \omega_{iy}^2 + \omega_{iz}^2) \right] \]  

(7)

where NumSphere is the total number of particles, which is 40581 in the simulation, \( m_i \), \( I_i \), \( v_{ix} \), \( \omega_{ix} \), \( v_{iy} \), \( \omega_{iy} \) and \( v_{iz} \), \( \omega_{iz} \) are the mass, inertia and translational and angular velocity in x-, y-, z- direction of \( i^{th} \) particle, respectively. Figure 13 compares the total kinetic energy in the system in the frictionless and frictional cases. Compared to the frictionless scenarios, the simulation with friction has smaller peak kinetic energy, and it settles down faster after the cube of particles were dropped.

![Total Kinetic Energy in the System](image)

Figure 13: Total Kinetic Energy of all particles in the system

Figure 14 shows the magnitudes of forces applied on the boundary 0 - 1 in the first 1-second period after the simulation starts. The pattern of forces applied on boundaries 0 to 3, the side walls of the box, are extremely similar, due to the symmetric geometry of the simulation. The force applied on boundary 4, theoretically, should be the total weight of all particles after the system settles down. Therefore the final static force applied to the bottom plane in the Z direction should be 1665.85594 N, theoretically. The force applied on boundary 5 shows particles bounced back to the top surface after it hit the bottom stack of settled particles.
Figure 14: Force applied on every boundary from t=0s to t=0.5s
2.7 Angle of Repose

In this simulation, a column of particles was created and then dropped upon one layer of particles that were fixed at (glued to) the bottom of the container. The sliding and rolling friction models used were as introduced in Sec. 2.3. The simulation parameters are provided in Table 5; snapshots of the simulation are shown in Fig.15.

Table 5: Simulation parameters for the Angle of Repose test

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle Radius</td>
<td>0.2 [cm]</td>
</tr>
<tr>
<td>Particle Density</td>
<td>1.53 [g · cm³]</td>
</tr>
<tr>
<td>Simulation Step Size</td>
<td>1e-4 [s]</td>
</tr>
<tr>
<td>Normal Force Stiffness</td>
<td>1e7 [g · s⁻²]</td>
</tr>
<tr>
<td>Normal Force Damping Coefficient</td>
<td>4e4 [s⁻¹]</td>
</tr>
<tr>
<td>Sphere-Sphere Tangential Force Stiffness</td>
<td>2e6 [g · s⁻¹]</td>
</tr>
<tr>
<td>Sphere-Wall Tangential Force Stiffness</td>
<td>1e6 [g · s⁻¹]</td>
</tr>
<tr>
<td>Tangential Force Damping Coefficient</td>
<td>50 [s⁻¹]</td>
</tr>
</tbody>
</table>

The results of five simulations are shown in Fig. 16. Three simulations analyze the effect of static friction coefficient on the repose angle by varying static friction coefficient among 0.1, 0.25, and 0.5, with a fixed rolling friction coefficient of 0.5. Another three simulations analyze the effect of rolling friction coefficient on the repose angle by varying rolling friction coefficient among 0.1, 0.25, 0.5, with a fixed static friction coefficient of 0.5. The repose angles were measured by the Matlab Image Angle Measurement Tool. The results show that the repose angle increases as either the static friction coefficient increases or the rolling friction coefficient increases. The static friction coefficient might have a larger influence on the repose angle.

An important artifact caused by the current rolling friction model expressed through Eq. 6 was noticed in the simulation results. Specifically, the system settled at a relatively high kinetic energy, and the repose peak failed to hold after a long simulation duration due to the oscillation artifacts discussed in Sec. 2.3. A similar phenomenon was also observed in [10], where the kinetic energy of the pile kept decreasing slowly and so did the pile height, even after 50 seconds.
(a) At $t=0\text{s}$, a cylindrical-shaped particle cluster is created, along with a layer of fixed particle on the ground to simulate frictional effect.

(b) At $t=0.08\text{s}$, particles starts free falling.

(c) At $t=0.15\text{s}$, particles starts scattering to form a repose angle with the simulated frictional effect.

(d) At $t=1.2\text{s}$, the system settles as the kinetic energy of the system stabilized at $4e^{-2}$ Joule.

Figure 15: Visualization for Repose Angle Test at $t=0\text{s}$, $t=0.08\text{s}$, $t=0.15\text{s}$, and $t=1.2\text{s}$
(a) 0.1 static friction coefficient and 0.5 rolling friction coefficient, the repose angle is 12.81 degrees

(b) 0.25 static friction coefficient and 0.5 rolling friction coefficient, the repose angle is 16.78 degrees

(c) 0.5 static friction coefficient and 0.5 rolling friction coefficient, the repose angle is 24.04 degrees

(d) 0.5 static friction coefficient and 0.25 rolling friction coefficient, the repose angle is 23.06 degrees

(e) 0.5 static friction coefficient and 0.1 rolling friction coefficient, the repose angle is 22.66 degrees

Figure 16: Angle of Repose Test with Various Rolling Friction and the Various Static Friction
2.8 Ball drop

A low-impact crater test was carried out using a sphere of diameter $D_b$ and density $\rho_b$. The sphere was dropped from different heights onto a bed of loosely packed, noncohesive granular material. The penetration depth, $d$, of the sphere was measured in simulation and compared against an empirically established relation [11]

$$d = \frac{0.14}{\mu} \left( \frac{D_b}{\rho_g} \right)^{1/2} D_b^{2/3} H^{1/3},$$  \hspace{1cm} (8)

where $\rho_g$ is the bulk density of the granular material, and $H$ is the total free fall height, as shown in Fig. 17. Nine tests were performed, with different combinations of the density, $\rho_b = 0.28, 0.7, 2.2 \text{ g/cm}^3$, and drop height $H = 5, 10, 20 \text{ cm}$. All simulations used the same granular medium composed of particles with radius 0.1 cm, density 2.5 g/cm$^3$ and friction coefficient $\mu = 0.3$. It is shown in [12] experimentally that the penetration of the big ball is independent of the grain size.

The simulation was composed of two steps. Initially, the particles were settled in a 9 cm by 9 cm container with a filling height of 10 cm. The average kinetic energy of a particle reached $2.18 \times 10^{-12} \text{ J}$ after 3 sec. The contact force parameters are listed in Table 6, and the same value were used for sphere-sphere, sphere-wall and sphere-mesh contact.

Table 6: Contact force model parameters of ball drop test (units: stiffness - [g · s$^{-2}$], damping coefficient - [s$^{-1}$])

<table>
<thead>
<tr>
<th>contact model parameter</th>
<th>notation</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal force stiffness</td>
<td>$k_n$</td>
<td>$10^9$</td>
</tr>
<tr>
<td>normal force damping coefficient</td>
<td>$\gamma_n$</td>
<td>1000</td>
</tr>
<tr>
<td>tangential force stiffness</td>
<td>$k_t$</td>
<td>$10^8$</td>
</tr>
<tr>
<td>tangential force damping coefficient</td>
<td>$\gamma_t$</td>
<td>2000</td>
</tr>
</tbody>
</table>

Once settled, the granular medium reached a packing fraction of 0.58 and bulk density of $\rho_g = 1.4564 \text{ g/cm}^3$. The position of all the particles were recorded for the next step. Subsequently, the big ball, modeled as a mesh, was dropped onto the granular media initialized...
from the settled position recorded. The penetration of the ball $d$ was measured when the vertical position of the spherical object reached its minimum, rather than when the ball was settled, due to the fact that the ball bounced back slightly after the impact. All simulations used a step size of $5 \times 10^{-6}$ sec and a centered difference time integrator.

The relation between depth $d$ and the scaled total dropped height $(\rho_b/\rho_g)^{1/2}D_b^{2/3}H^{1/3}$ is illustrated in Fig. 18. Here, each simulation result is denoted by a marker, where the color represents various density $\rho_b$ and the shape for drop height $H$. The green line is the linear fitting result from the markers, with a slope of $0.1222/\mu$, compared with the empirical one [11], $0.14/\mu$, plotted as the blue line. A similar result was also achieved in [13] where a non-smooth contact model was implemented for the granular system [14,15].

![Figure 18: Penetration depth $d$ vs scaled total drop height $H$ from various simulations.](image)

**2.9 The Goldenberg test**

To validate the contact force at a microscopic level, a system of particles arranged on a triangular lattice was modeled with a downward external force, $F_{ext}$, applied at the top. The system consisted of 15 horizontal layers of spheres, with each layer alternating between 60 and 61 spheres. For example, the bottom (top) layer had 61 particles, while the layer above (underneath, that is) had 60 particles, as seen in Fig. 19. The experiment originated from the work of Goldenberg et al. [16,17], however, note that their experimental and numerical results were based on 2D disks, while the simulation herein modeled 3D spheres. Both 2D and 3D tests demonstrated that static friction enhances the elasticity in granular solids.

The test had three steps. First, an initial configuration of non-touching particles were settled with gravity only into a box of width $122R$, where $R$ is the particle radius. Once
settled, the vertical contact forces applied to the bottom container was recorded as $F_0$; note that this $F_0$ value changed from sphere to sphere and it’s distribution is expected to be symmetric relative to the center of the box. The process took 2 sec. Next, an external force $F_{ext}$ was exerted gradually on the top center particle over a time period of 2 sec. Eventually, with the constant external force, the system was allowed to settle, and the contact force between a sphere and the bottom was collected as $F_y$; again, the $F_y$ value changed from sphere to sphere. The monodisperse particles had radius 5 cm and density of 7.8 g/cm$^3$. The parameters used in the contact force calculation are listed in Table 7. The simulation used a forward Euler time integrator with a step size of $10^{-5}$ sec.

Table 7: Contact force model parameters of Goldenberg test (units: stiffness - $[g \cdot s^{-2}]$, damping coefficient - $[s^{-1}]$)

<table>
<thead>
<tr>
<th>contact model parameter</th>
<th>notation</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>sphere-sphere normal force stiffness</td>
<td>$k_n$</td>
<td>30000 mg/R</td>
</tr>
<tr>
<td>sphere-sphere normal force damping coefficient</td>
<td>$\gamma_n$</td>
<td>50000</td>
</tr>
<tr>
<td>sphere-wall normal force stiffness</td>
<td>$k_{wall}^n$</td>
<td>$2k_n$</td>
</tr>
<tr>
<td>sphere-wall normal force damping coefficient</td>
<td>$\gamma_{wall}^n$</td>
<td>$\gamma_n$</td>
</tr>
<tr>
<td>sphere-sphere tangential force stiffness</td>
<td>$k_t$</td>
<td>0.8$k_n$</td>
</tr>
<tr>
<td>sphere-sphere tangential force damping coefficient</td>
<td>$\gamma_t$</td>
<td>0.8$\gamma_n$</td>
</tr>
<tr>
<td>sphere-wall tangential force stiffness</td>
<td>$k_{wall}^t$</td>
<td>0.8$k_n$</td>
</tr>
<tr>
<td>sphere-wall tangential force damping coefficient</td>
<td>$\gamma_{wall}^t$</td>
<td>0.8$\gamma_n$</td>
</tr>
</tbody>
</table>

A series of simulations were carried out using various values of $F_{ext}$ and sphere-sphere friction coefficient $\mu$. The normalized reaction force, $\Delta F/F_{ext}$, where $\Delta F = F_y - F_0$, are plotted over the normalized particle position in Fig. 20. The force distribution exhibits two profile types. One type has only one peak, and the maximum force occurs directly below where the force is applied (pos = 0). In contrast, the other type can have two peaks. The first type indicates an isotropic elastic response, while the latter, a hyperbolic one. Figure 21 plots the normalized contact force increment at the bottom center, $\Delta F_y(0)/F_{ext}$, over normalized external force, $F_{ext}/mg$. For each granular system, the elasticity disappeared with increasing external force $F_{ext}$, and the introduction of friction extended the elastic range (flat part of the curve).
Figure 20: Force profile on the bottom in response to different values of applied force, $F_{ext}$, at the top of the grid at pos = 0, the effect of gravity is excluded.
Figure 21: Vertical force at bottom center with various $F_{ext}$


3 Code information

All the models used in the validation tests discussed in this document, except for the Goldenberg one, can be found in [18]. The Goldenberg test used a forked Chrono::Granular version by Luning Fang.

4 Conclusions and directions of future work

This document reports on a set of validations tests carried out to gauge the quality of results produced by the new Chrono::Granular module. The results reported compare well with data obtained from analytical solutions or reports. In terms of future work, an upcoming 2021 technical report will consider several other validation tests. It also remains to compare the Chrono::Granular implementation with a Chrono::FSI implementation that looks at the dynamics of granular material via a continuum representation. Details for the continuum representation are provided in [19], where the discussion focuses on the mobility of an eight wheel rover operating on granular terrain.

References


