Autonomous Driving via Model Predictive Control for Path Tracking and PID Speed Control

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Abstract

Path tracking is an essential task for autonomous vehicles. In this report, we demonstrate a Model Predictive Control (MPC)-based approach to path tracking combined with a PID strategy for speed control to ensure the vehicle follows a desired path and speed profile. To implement this controller, we first considered the situation in which the speed is constant and designed the model predictive path tracking controller to follow an 8-shaped curve path. Then, we added the PID speed control in the model predictive control (MPC) scheme to make the vehicle track the desired longitudinal velocity. We present Matlab® simulation results for these two scenarios. Compared with the reference objectives, the tracking errors of the heading angle \( \psi \), lateral displacement of vehicle center of mass (C.M.) \( Y \) and the speed from both controllers are small. This work uses the MPC fundamentals discussed in [1,2] to build on preliminary results discussed in [3] using vehicle models presented in [4].

Keywords: Model Predictive Control, Vehicle Dynamics, Path Tracking, Speed Control
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1 Introduction

Model predictive control has its roots in optimal control. The basic concept of MPC is to use a dynamic model to forecast system behavior, and optimize the control move at the current time to produce the best performance in the future [5]. MPC method, which combines predictive models, a receding horizon optimization and a feedback correction, has a very good ability to solve the control problem with multiple constraints. The constraints are used to build the objective function of an MPC controller to obtain an optimal control output [6]. Recent work has shown that MPC can be used to rigorously handling the multiple vehicle dynamics and safety constraints and the stability of this algorithm is also well studied [7]. The lateral dynamics control and the longitudinal dynamics control are essential for autonomous vehicles. In the literature, lateral and longitudinal control have been studied in a decoupled way. Actually, numerous studies implemented the path tracking controller based on the assumption that the longitudinal velocity is constant for the lateral control. However, the longitudinal and lateral dynamics of an automotive vehicle are highly nonlinear and coupled and the vehicle speed may be varied along the path [8]. In actual situation, a vehicle should drive or brake according to the road conditions, for instance, when a vehicle runs into a large curvature bend, slowing down in advance is necessary. After a vehicle go through a bend and enters a straight road, the necessary acceleration should be applied. Therefore, a time-varying velocity condition should be considered [9].

In this report, the model predictive path tracking and the speed controller are designed and implemented in Matlab®. For the lateral control, we used model predictive control and took the 8-DOF model as the prediction model, the 14-DOF model as the plant. The tracking path is a straight line connected with the top of an 8-shaped curve path shown in Figure 1. When implementing the path following controller, we first consider the desired vehicle speed is constant (Case A) and then consider the vehicle speed is time-varied (Case B). For the longitudinal control, we used PID controller to track the desired speed. The tire model here is simplified as a linear tire model with assumption of small slip ratio and slip angle. The lateral control and longitudinal control are combined in the MPC controller to generate the optimal control output: the front wheel steering angle and the total driving or braking wheel torque which were passed to the plant vehicle simultaneously so as to track the reference trajectory and speed.

The remainder of this paper is organized as follows. In Section 2 we described the tire model, 8-DOF vehicle model, 14-DOF vehicle model and wheel rotational dynamics equations, which were used in the MPC controller and the plant. In Section 3 we specified the design of model predictive control for path tracking and the PID controller for the speed tracking; In Section 4 we implemented the proposed controller with two cases and provided the simulation results compared with the references. In Section 5 we presented the conclusion based on the simulation results and the potential future work for this study.
2 Vehicle Modeling

2.1 Tire model

The tire model for calculating the lateral and longitudinal forces is complex and assumed to depend on normal force, slip angle, surface friction, and longitudinal slip ratio. However, when the slip ratio and slip angle are limited within small values, the tire model can be simplified and generate linearized lateral force and longitudinal force \[10\]. Herein, the tire model used in the controller and the plant is linear tire model assuming that the slip ratio and slip angle are small.

Under this assumption, the tire lateral force can be given as \( F_c = C_\alpha(\mu, F_z)\alpha \), where \( C_\alpha \) is tire cornering stiffness related to tire normal force and road-tire friction coefficient; The tire longitudinal force can be given as \( F_l = C_x(\mu, F_z)s_{f,r} \), where \( s_{f,r} \) is the slip ratio of front tire or rear tire, \( C_x \) is the tire longitudinal stiffness which also related to the tire normal force and road-tire friction coefficient.

2.2 8-DOF vehicle model

An 8-DOF full vehicle model is often used as a simplified lower order model for studying vehicle handling in scenarios which do not involve significant longitudinal accelerations. In this section, a formulation for the 8-DOF model, adapted from various references \[11\] \[12\], that can match the 14-DOF model reasonably accurately is presented.

The schematic of the 8-DOF full vehicle model is shown in Figure 2. The model has four degrees of freedom for the chassis and one degree of freedom at each of the four wheels.
representing the wheel spin dynamics. The chassis includes the longitudinal velocity, $u$, the lateral velocity, $v$, the roll angular velocity, $\omega_x$, and the yaw angular velocity, $\omega_z$. The pitch and heave motions are not modeled and the front and rear suspensions are represented simply by their respective equivalent roll stiffness ($k_{\phi f}/k_{\phi r}$) and roll damping coefficients ($b_{\phi f}/b_{\phi r}$) [11].

Figure 2: Schematic of 8-DOF full vehicle model [11]

$$m_t(\dot{u} - \omega_z v) = \sum F_{xgij} + (m_{uf} a - m_{ur} b)\omega_z^2 - 2h_r c m \omega_z \omega_x$$
$$m_t(\dot{v} + \omega_z u) = \sum F_{ygi j} + (m_{ur} b - m_{uf} a)\dot{\omega}_z + h_r c m \dot{\omega}_x$$
$$J_z \ddot{\omega}_z + J_{xz} \ddot{\omega}_x = (F_{yglf} + F_{ygrf}) a - (F_{yglr} + F_{ygrr}) b + \frac{(F_{xgrf} - F_{xgfr}) c_f}{2}$$
$$+ \frac{(F_{xgrr} - F_{xgfr}) c_r}{2} + (m_{ur} b - m_{uf} a)(\dot{v} + \omega_z u)$$
$$(J_x + m h_r^2) \ddot{\omega}_x + J_{xz} \ddot{\omega}_z = m g h_r \phi - (k_{\phi f} + k_{\phi r}) \phi - (b_{\phi f} + b_{\phi r}) \dot{\phi} + h_r c m (\dot{v} + \omega_z u),$$
where

\[ h_{rc} = \frac{h_{rcf} b + h_{rcr} a}{a + b}. \] (5)

In these equations, the forces \( F_{xgij} \) and \( F_{ygij} \) are the longitudinal and lateral forces at the four tire contact patches and the subscript ‘ij’ denotes \( lf, \ rf, \ lr, \) and \( rr. \) As before, \( h_{rcf} \) and \( h_{rcr} \) are the vertical distances of the front and rear roll centers below the sprung mass C.M., and thus \( h_{rc} \) is the vertical distance from the sprung mass C.M. to the vehicle roll center. It should be noted that as Eq.(4) for the roll degree of freedom is written by considering moments acting about the vehicle roll center rather than the sprung mass C.M., the roll inertia of the sprung mass about the vehicle roll center \( (J_x + m h_{rc}^2) \) is considered in Eq.(4) [11].

The equations for the wheel dynamics and the longitudinal and lateral tire forces are the same as those used in the 14-DOF vehicle model. The longitudinal and lateral velocities at, for example, the right front tire contact patch required in these equations are given as [11]

\[ u_{grf} = u + \frac{\omega_z c_f}{2} \] (6)
\[ v_{grf} = v + \omega_z a. \] (7)

The definition and calculation method of the lateral sideslip angle and the longitudinal slip ratio for each wheel are the same as those used in 14-DOF vehicle model.

The normal forces at the four tires are determined as [11]

\[
F_{zgij} = \frac{mg b}{2(a+b)} \left( \frac{u}{2} + \frac{m a h_{uf}}{c_f} + \frac{m b (h_{cg} - h_{rcf})}{c_f (a+b)} \right) (v + \omega_z u) - \frac{\left( k_{\phi_f} + b_{\phi_f} \right)}{c_f} \] (8)

\[
F_{zgrf} = \frac{mg b}{2(a+b)} \left( \frac{u}{2} + \frac{m a h_{uf}}{c_f} + \frac{m b (h_{cg} - h_{rcf})}{c_f (a+b)} \right) (v + \omega_z u) + \frac{\left( k_{\phi_f} + b_{\phi_f} \right)}{c_f} \] (9)

\[
F_{zgrtr} = \frac{mg a}{2(a+b)} \left( \frac{u}{2} + \frac{m a h_{ur}}{c_r} + \frac{m b (h_{cg} - h_{rcr})}{c_r (a+b)} \right) (v + \omega_z u) - \frac{\left( k_{\phi_r} + b_{\phi_r} \right)}{c_r} \] (10)

\[
F_{zgrr} = \frac{mg a}{2(a+b)} \left( \frac{u}{2} + \frac{m a h_{ur}}{c_r} + \frac{m b (h_{cg} - h_{rcr})}{c_r (a+b)} \right) (v + \omega_z u) + \frac{\left( k_{\phi_r} + b_{\phi_r} \right)}{c_r} \] (11)
These equations are fairly simple and linearized. It is possible to include several additional terms in the equations for the chassis velocities as well as the tire forces. However, in our experience, these terms have very little effect on the vehicle responses and can be ignored. The 8-DOF model cannot simulate vehicle behavior beyond wheel lift-off. Nevertheless, the model is valid for applications which do not involve wheel lift-off such as active steering and active throttle/brake systems for yaw control [11].

2.3 14-DOF vehicle model

In order to better represent the vehicle lateral and yaw dynamics as well as coupling of yaw-roll motion due to the transient lateral load transfer during extreme maneuvers, higher order model such as 8-DOF and 14-DOF are also used in rollover studies. A 14-DOF vehicle model, which considers the suspension at each corner, has the same benefits of an 8-DOF vehicle model, with the additional capabilities of predicting vehicle pitch and heave motions. It also offers the flexibility of modeling nonlinear springs and dampers and can simulate the vehicle responses to normal force inputs in the case of an active suspension system. Moreover, the 14-DOF model, unlike the 8-DOF model, can predict vehicle behavior even after wheel lift-off and thus can be used in rollover prediction/prevention strategies [11].

The schematic shown in Figure 3 exhibits the two axle, 14-DOF, vehicle model used to investigate vehicle roll response to steering and torque inputs. This schematic includes 6 DOF at the vehicle lumped mass C.M. and 2 DOF at each of the four wheels, including vertical suspension travel and wheel spin. The body is modeled as being rigid, with body-fixed coordinates, \((xyz)\), attached at the center of mass and aligned in principal directions (coordinate frame 1). \(u, v, w\) indicate forward, lateral, and vertical velocities, respectively, of the sprung mass [11].

The force and velocity components in the right front corner of a vehicle is depicted in Figure 4. The velocity \(u_{sr f}, v_{sr f},\) and \(w_{sr f}\) are the velocities of the right front strut mounting point in the longitudinal, lateral, and vertical directions, respectively, in the body-fixed coordinate frame, which is attached to the sprung mass C.M. (coordinate frame 1). These velocities can be obtained by transforming the C.M. velocities as [11]

\[
\begin{pmatrix}
u_{sr f} \\
v_{sr f} \\
w_{sr f}
\end{pmatrix} = \begin{pmatrix}
0 & 0 & \frac{c_f}{2} \\
0 & 0 & a \\
-\frac{c_f}{2} & -a & 0
\end{pmatrix} \begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{pmatrix} + \begin{pmatrix}
u \\
v \\
w
\end{pmatrix}.
\] (12)

The forces \(F_{xsrf}, F_{ysrf}\) and \(F_{zsrf}\) are the forces transmitted to the sprung mass along the longitudinal, lateral, and vertical directions, respectively, of coordinate frame 1. The forces \(F_{xgrf}, F_{ygrf},\) and \(F_{zgrf}\) are the forces acting at the tire ground contact patch in the same coordinate frame 1. These forces can be written in terms of the tire forces \(F_{xgrf}, F_{ygrf},\) and
$F_{zgrf}$ by projecting its components along coordinate frame 2 as,

$$
\begin{pmatrix}
F_{xgsrf} \\
F_{ygsrf} \\
F_{zgsrf}
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos\phi & \sin\phi \\
0 & -\sin\phi & \cos\phi
\end{pmatrix}
\begin{pmatrix}
\cos\theta & 0 & -\sin\theta \\
0 & 1 & 0 \\
\sin\theta & 0 & \cos\theta
\end{pmatrix}
\begin{pmatrix}
F_{xgrf} \\
F_{ygrf} \\
F_{zgrf}
\end{pmatrix}.
$$  (13)

The forces $F_{xgrf}$ and $F_{ygrf}$ are obtained by resolving the longitudinal($F_{xtrf}$) and cornering($F_{ytrf}$) forces at the tire contact patch as,

$$
F_{xgrf} = F_{xtrf}\cos\delta - F_{ytrf}\sin\delta 
$$  (14)

$$
F_{ygrf} = F_{ytrf}\cos\delta + F_{xtrf}\sin\delta,
$$  (15)

where the $\delta$ is the steering angle at the road wheels.

Here, the linear tire model has been used in the development of the tire forces $F_{xtrf}$ and $F_{ytrf}$. The lateral tire slip angle used in the tire model was calculated as follows:

$$
\alpha_{r.f} = \tan^{-1}\left(\frac{v_{grf}}{u_{grf}}\right) - \delta.
$$  (16)

So the lateral tire force in small angle region can be modeled as,

$$
F_{ytrf} = C_{\alpha f}\alpha_{r.f},
$$  (17)

where $C_{\alpha f}$ is the cornering stiffness of the front tire.
The longitudinal slip is defined as the difference between the actual longitudinal velocity at the axle of wheel $V_x$ and the equivalent rotational velocity $r_{eff}\omega$ of the tire, i.e. $r_{eff}\omega - V_x$. And, the longitudinal slip ratio is defined as

$$\sigma_x = \frac{r_{eff}\omega - V_x}{V_x}$$

(18) during braking

$$\sigma_x = \frac{r_{eff}\omega - V_x}{r_{eff}\omega}$$

(19) during acceleration,

where the $V_x$ for front and rear wheel are given as

$$V_{xrf} = u_{grf}cos\delta + v_{grf}sin\delta$$

(20)

$$V_{xrr} = u_{grr}$$

(21)

So, the longitudinal tire force in small-slip region can be modeled as

$$F_{xtrf} = C_{xf}\sigma_x,$$

(22)

where $C_{xf}$ is the longitudinal tire stiffness of the front tire.

The $u_{grf}$ and $v_{grf}$ at the right front can be determined as

$$u_{grf} = cos\theta(u_{urf} - \omega_y \cdot r_{rf}) + sin\theta(w_{urf}cos\phi + sin\phi(\omega_x \cdot r_{rf} + v_{urf}))$$

(23)

$$v_{grf} = cos\phi(v_{urf} + \omega_x \cdot r_{rf}) - w_{urf}sin\phi.$$
The longitudinal \((u_{urf})\) and lateral \((v_{urf})\) velocities of the unsprung mass in the body coordinate frame used in the earlier equations are simply written as

\[ u_{urf} = u_{srf} - l_{srf} \omega_y, \]  
\[ v_{urf} = v_{srf} + l_{srf} \omega_x, \]

where \(l_{srf}\) is the instantaneous length of the strut.

The derivative of \(u_{urf}\) and \(v_{urf}\) is given as

\[ \dot{u}_{urf} = \dot{u}_{srf} - (\omega_{srf} - \omega_{urf}) \omega_y + l_{srf} \dot{\omega}_y, \]  
\[ \dot{v}_{urf} = \dot{v}_{srf} + (\omega_{srf} - \omega_{urf}) \omega_y + l_{srf} \dot{\omega}_y. \]

The unsprung mass vertical velocity \(w_{urf}\) represents the degree of freedom corresponding to the suspension deflection and can be expressed by applying Newton’s law for the vertical motion of the unsprung mass as

\[ m_u \ddot{w}_{urf} = \cos \phi \left( \cos \theta \left( F_{zgrf} - m_u g \right) + \sin \theta F_{ygrf} - F_{dzrf} \right) - x_{srf} \cdot k_{sf} - \dot{x}_{srf} \cdot b_{sf} - m_u \left( v_{urf} \cdot \omega_x - u_{urf} \cdot \omega_y \right), \]

where \(k_{sf}\) is the suspension stiffness, \(b_{sf}\) the suspension damping coefficient, and \(x_{srf}\) the instantaneous compression of the right front suspension spring. The force, \(F_{dzrf}\), represents the additional load transfer that occurs at the wheels through the suspension links because of the reaction force to the force transmitted to the sprung mass through the roll center.

The instantaneous suspension spring deflection \(x_{srf}\) is given as

\[ \dot{x}_{srf} = -w_{srf} + u_{urf}. \]

The vertical force \(F_{zgrf}\) acting at the tire ground contact patch in coordinate frame 2 can be written in terms of the tire stiffness \((k_{tf})\) and the instantaneous tire deflection \((x_{trf})\) as

\[ F_{zgrf} = F_{ztrf} = x_{trf} k_{trf}. \]

The instantaneous tire deflection \(x_{trf}\) in equation (31) is given as

\[ \dot{x}_{trf} = w_{grf} - w_{uirf} = w_{grf} - \left( \cos \theta \left( w_{urf} \cos \phi + v_{urf} \sin \phi \right) - u_{urf} \sin \theta \right), \]

where \(w_{uirf}\) is the vertical velocity of the wheel center in the inertial coordinate frame. For the simulations in this article, it is assumed that the vertical velocity \(w_{grf}\) at the tire contact patch is zero (smooth road).

The instantaneous tire radius is then determined as

\[ r_{rf} = \frac{r_0 - x_{trf}}{\cos \theta \cos \phi}. \]
To account for the wheel lift-off, when the tire radial compression becomes less than zero, the tire normal force $F_{zgrf}$ is set equal to zero. In addition, the instantaneous tire radius is considered equal to the nominal tire radius until the tire returns to the road surface [11].

If $x_{trf} < 0$ then $F_{zgrf} = 0$ and $r_{rf} = r_0$. (34)

As the tire normal force becomes zero, no lateral ($F_{ygrf}$) and longitudinal ($F_{xgrf}$) tire forces are developed at that contact patch. Thus, the only forces acting at that suspension corner are the weight and inertia forces of the unsprung mass. As the cardan angles and appropriate coordinate transformations between the body-fixed coordinate frame 1 and the coordinate frame 2 at the tire-ground contact patch are considered for all the forces and velocities in the system, the model is able to simulate vehicle behavior after wheel lift-off and during the rollover event with just the modification mentioned in [34] [11].

The instantaneous length of the strut $l_{srf}$ is given as [11]

$$l_{srf} = l_{sif} - (x_{srf} - x_{sif}),$$

(35)

where $l_{sif}$ is the initial length of strut and $x_{sif}$ is the initial suspension spring deflection. The initial length of the strut $l_{sif}$ is taken such that [11]

$$l_{sif} = h - (r_0 - x_{tif})$$

(36)

where $x_{tif}$ is the initial tire compression.

The initial spring compression $x_{sif}$ and the initial tire compression $x_{tif}$ are determined from the static conditions as [11]

$$x_{sif} = \frac{mb}{2(a + b)k_{sf}},$$

(37)

$$x_{tif} = \frac{(mg/2(a + b) + m_{uf}g)}{k_{tf}}.$$  

(38)

The forces $F_{xgrf}$ and $F_{ygrf}$ transmitted to the sprung mass along the $u-$ and $v-$axes of the body-fixed coordinate frame are obtained after subtracting the components of the unsprung mass weight and inertia forces from the corresponding forces $F_{xgrsf}$ and $F_{ygrsf}$ acting at the tire contact patch as [11]

$$F_{xgrf} = F_{xgrsf} + m_ug\sin\theta - m_u\dot{u}_{urf} + m_u\omega_z v_{urf} - m_u\omega_y w_{urf}$$

(39)

$$F_{ygrf} = F_{ygrsf} - m_ug\sin\phi\cos\theta - m_u\dot{v}_{urf} + m_u\omega_x w_{urf} - m_u\omega_z u_{urf}.$$  

(40)

The vertical force $F_{zgrf}$ transmitted to the sprung mass through the strut is given as [11]

$$F_{zgrf} = x_{grf}k_{sr} + \dot{x}_{grf}b_{sr}.$$  

(41)

Figure 5 shows the forces and velocities in the roll plane of, for example, the front suspension. Generally, the roll center height is defined with reference to the ground. However,
for the development of this model, the front and rear roll centers are assumed to be fixed at distances $h_{rcf}$ and $h_{rcr}$, respectively, below the sprung mass C.M. along the negative $w$-axis of the body-fixed coordinate frame 1. Moreover, the roll center is simply considered to be a point of application of the forces transmitted to the sprung mass through the suspension links and not as a kinematic constraint [11].

![Figure 5: Forces and velocities in the front suspension roll plane](image)

In the figure, $F_{zstf}$ and $F_{zsr}$ are the forces transmitted to the sprung mass through the struts. $F_{ysf}$ and $F_{yrf}$ represet the lateral forces transmitted to the sprung mass through the suspension links. In the absence of a roll center, i.e., when the roll center is assumed to be in the ground plane, the total roll moment transmitted to the sprung mass at, for example, the right front corner along the $\omega_x$ direction is given as [11]

\[
M_{xrf} = F_{ysrf}(l_{sr} + r_{rf}) - (m_u g \sin \phi \cos \theta + m_u \dot{u}_{urf} - m_u \omega_x w_{urf} + m_u \omega_z u_{urf}) \cdot l_{sr} \tag{42}
\]

\[
= F_{ysrf} \cdot r_{rf} + F_{ysr} \cdot l_{sr} \tag{43}
\]

When a roll center is modeled as shown in Figure 5, the roll moment $M_{xrf}$ transmitted to the sprung mass by the right front corner suspension is given as [11]

\[
M_{xrf} = F_{ysrf} h_{rcf}. \tag{44}
\]

Thus, the inclusion of a roll center reduces the total roll moment transferred to the sprung mass by the front suspension. The difference between the roll moments in the absence of the roll center and when the roll center is considered acts directly on the unsprung mass and is
responsible for the link load transfer forces (jacking forces), \( F_{dzrf} \) and \( F_{dzlf} \). These forces can be estimated as \[11\]

\[ F_{dzrf} = -F_{dzlf} = F_{ygrf}r_{rf} + F_{ysrflrf} + F_{ysrflsf} - (F_{ygrf} + F_{ysrfl})h_{ruf}. \] (45)

The moments \( M_{yrf} \) and \( M_{zrf} \) transmitted to the sprung mass at, for example, the right front corner by the suspension along the \( \omega_y \) and \( \omega_z \) directions can be given as \[11\]

\[ M_{yrf} = -\left( F_{xsgf}(l_{sr} + r_{rf}) - (m_u g \sin \theta + m_u \dot{u}_{ur} - m_u \omega_z v_{ur} + m_u \omega_y w_{ur}) \cdot l_{sr} \right) \] (46)
\[ = (F_{xsgf} \cdot r_{rf} + F_{xsrff} \cdot l_{sr}) \] (47)
\[ M_{zrf} = 0. \] (48)

The equation of motion for the 6 DOF of the sprung mass model can now be derived from the direct application of Newton’s laws for the system as \[11\]

\[ m(\dot{u} + \omega_y w - \omega_z v) = \sum (F_{xsi}) + mg \sin \theta \] (49)
\[ m(\dot{v} + \omega_z u - \omega_x w) = \sum (F_{ysi}) - mg \sin \phi \cos \theta \] (50)
\[ m(\dot{w} + \omega_x v - \omega_y u) = \sum (F_{zsi} + F_{dzi}) - mg \cos \phi \cos \theta \] (51)
\[ J_x \dot{\omega}_x = \sum (M_{xij}) + \frac{(F_{zslf} - F_{zsr})c_f + (F_{zslr} - F_{zsr})c_r}{2} \] (52)
\[ J_y \dot{\omega}_y = \sum (M_{yij}) + (F_{zslf} + F_{zsr})b - (F_{zslf} + F_{zsr})a \] (53)
\[ J_z \dot{\omega}_z = \sum (M_{zij}) + (F_{yglf} + F_{ygrf})a - (F_{yglf} + F_{ygrf})b \] (54)
\[ + \frac{(-F_{zslf} + F_{zsr})c_f + (-F_{zslf} + F_{zsr})c_r}{2}, \]

where \( m \) is the sprung mass and the subscript ‘ij’ denotes left front (lf), right front (rf), left rear (lr), and right rear (rr).

The cardan angles \( \theta, \psi, \phi \) needed in the aforementioned equations are obtained by performing the integration of the following equations \[11\]

\[ \dot{\theta} = \omega_y \cos \phi - \omega_z \sin \phi \] (55)
\[ \dot{\psi} = \frac{\omega_y \sin \phi}{\cos \theta} + \frac{\omega_z \cos \phi}{\cos \theta} \] (56)
\[ \dot{\phi} = \omega_x + \omega_y \sin \phi \tan \theta + \omega_z \cos \phi \tan \theta. \] (57)

### 2.4 Wheel rotational dynamics

In the case that the vehicle is front wheel driven, the rotational dynamics for the right front and right rear wheels can be given as \[12\]

\[ J_{wrf} \dot{\omega}_{rf} = T_{arf} - T_{brf} - F_{xtrf} R \] (58)
\[ J_{wrr} \dot{\omega}_{rr} = -T_{brr} - F_{xtrr} R, \] (59)
where \( T_{arf}, T_{brf} \) is the acceleration torque and braking torque applied to the front wheel. \( T_{brr} \) is the braking torque applied to the rear wheel.

Then, we reconsider the longitudinal vehicle dynamics equation in the plant, 14-DOF vehicle model. The longitudinal model considered here is based on one wheel vehicle model. The sum of the longitudinal forces acting on the vehicle C.M. is given by [8][9]:

\[
m(\dot{u} + \omega_y w - \omega_z v) = F_p - F_r + mgsin\theta, \tag{60}
\]

where \( v \) is the vehicle speed, \( F_p \) is the propelling force and \( F_r \) is the sum of resisting forces. The propelling force \( F_p \) is the controlled input resulting from brake and throttle actions. The sum of the resisting force \( F_r \) is given by [8][9]:

\[
F_r = F_a + F_g + F_{rr}, \tag{61}
\]

where \( F_a \) is the aerodynamic force, \( F_g \) is the gravitational force and \( F_{rr} \) is the rolling resistance force. The form of the wheel dynamics has been slightly modified to distinguish the total brake torque \( T_b \) and the total traction torque \( T_a \) as follows [8][9]:

\[
I_w\dot{\omega} = -F_l R + T_a - T_b. \tag{62}
\]

For longitudinal controller synthesis, the simplifying assumption of a non-slip rolling is considered, then the following relationship hold [8][9]:

\[
u = R\omega \tag{63}
\]

\[
F_p = F_l = \frac{1}{R}(T_a - T_b - I_u\dot{\omega}) = \frac{T_a - T_b}{R} - \frac{I_u}{R^2}\dot{\omega}. \tag{64}
\]

For simplicity, the sum of the resisting force in the longitudinal dynamics is ignored, i.e., \( F_r = 0 \). The driving torque is divided equally and applied to the front left and right wheels, the braking torque is divided equally and applied to the front and rear wheels, (i.e., \( T_{alf} = T_{arf} = 0.5T_a, T_{afl} = T_{brf} = T_{brl} = T_{brl} = 0.25T_b \)), so the vehicle longitudinal dynamics equation (60) for the plant becomes:

\[
\dot{u} = \frac{1}{m + \frac{J_w}{r_{lf}^2} + \frac{J_w}{r_{rf}^2} + \frac{J_w}{r_{lr}^2} + \frac{J_w}{r_{rr}^2}} \left( \frac{(0.5T_a - 0.25T_b)}{r_{lf}} + \frac{(0.5T_a - 0.25T_b)}{r_{rf}} + \frac{(-0.25T_b)}{r_{lr}} 
+ \frac{(-0.25T_b)}{r_{rr}} - m\omega_y w + m\omega_z v + mgsin\theta \right). \tag{65}
\]

### 3 Lateral and Longitudinal Control

#### 3.1 Lateral control: model predictive control for path tracking

The basic concept of MPC is to use a dynamic model to forecast system behavior, and optimize the control move at the current time to produce the best performance in the future.
Such a prediction is accomplished by employing an internal model over a fixed finite time horizon, called the prediction horizon, from the current system state. At each sampling time, the controller generates an optimal control sequence, called control horizon, by solving an optimization problem and the first element of this sequence is applied to the plant. The repetition of this process over time by using the updated measurements creates a feedback loop which continually controls the system, pushing it towards an optimal path \[5\] \[14\].

### 3.1.1 Linearization of the vehicle model

Here, we take the 8-DOF vehicle model as the internal prediction model and set the control input \( u = \delta \) and the state variable \( \chi = \begin{bmatrix} \dot{x}, \dot{y}, \phi, \psi, \dot{\phi}, \dot{\psi}, Y, X \end{bmatrix}^T \), then the general form of the system can be given as:

\[
\dot{\chi} = f(\chi, u). \tag{66}
\]

The general form around the operating point is

\[
\dot{\chi}_o = f(\chi_o, u_o). \tag{67}
\]

Using the Taylor series expansion at the operating point and ignoring higher order terms, we can obtain \[6\]

\[
\dot{\chi} = f(\chi_o, u_o) + \frac{\partial f(\chi, u)}{\partial \chi} \bigg|_{\chi=\chi_o, u=u_o} (\chi - \chi_o) + \frac{\partial f(\chi, u)}{\partial u} \bigg|_{\chi=\chi_o, u=u_o} (u - u_o). \tag{68}
\]

subtracting Eq.(67) from Eq.(68) results in

\[
\dot{\hat{\chi}} = A \hat{\chi} + B \hat{u}, \tag{69}
\]

where

\[
A = \frac{\partial f(\chi, u)}{\partial \chi} \bigg|_{\chi=\chi_o, u=u_o}, \quad B = \frac{\partial f(\chi, u)}{\partial u} \bigg|_{\chi=\chi_o, u=u_o}, \quad \hat{\chi} = \chi - \chi_o, \quad \hat{\chi}_o = \chi_o - \chi_o, \quad \text{and} \quad \hat{u} = u - u_o.
\]

Eq.(69) is the linear error model \[6\].

In order to apply the model to the design of the MPC controller, it is discretized in the form of state-space representation \[6\]

\[
\hat{\chi}(k+1) = A_d \hat{\chi}(k) + B_d \hat{u}(k), \tag{70}
\]

where \( A_d = I + TA, B_d = TB \) and \( T \) is the sampling time.

The Eq.(70) can be also given in the following form for the MPC controller design

\[
\chi(k+1) = A_d \chi(k) + B_d u(k) + d_k(k), \tag{71}
\]

where

\[
d_k(k) = f(\chi_o(k), u_o(k)) - (A_d \chi_o(k) + B_d u_o(k)).
\]
3.1.2 State prediction

We define \( \xi(k) = \begin{bmatrix} \chi(k) \\ u(k-1) \end{bmatrix} \) as the new state variable, \( \eta(k) \) as the output state variable, and \( \Delta u(k) = u(k) - u(k-1) \) as the control input increment. Then, the discrete state-space controller model can be translated into a new form as follows [6] [15]

\[
\begin{align*}
\xi(k+1) &= \tilde{A}_d \xi(k) + \tilde{B}_d \Delta u(k) + \tilde{d}_k(k) \\
\eta(k) &= \tilde{C}_d \xi(k),
\end{align*}
\]

(72a)

(72b)

where \( \tilde{A}_d = \begin{bmatrix} A_d & B_d \\ 0_{m \times n} & I_m \end{bmatrix}, \tilde{B}_d = \begin{bmatrix} B_d \end{bmatrix}, \tilde{d}_k(k) = \begin{bmatrix} d_k(k) \end{bmatrix}, \tilde{C}_d = \begin{bmatrix} C_d & 0_{p \times m} \end{bmatrix} \) (where \( m \) denotes the dimension of control input, \( n \) denotes the dimension of state variable, and \( p \) denotes the dimension of output) [6].

We denote the sequence of future control input computed at time \( k \) as \( \Delta U_m \), that is \( \Delta U_m = [\Delta u(k), \ldots, \Delta u(k+m), \ldots, \Delta u(k+N_c-1)]^T \). The control input varies for \( N_c \) time steps (i.e., the control horizon) and then is held constant up to the preview horizon. We define the predicted output for the prediction state-space model as \( \eta_m(k) = [\eta(k+1), \ldots, \eta(k+N_p)]^T \). In this situation, it is straightforward to derive the prediction model of performance outputs over the prediction horizon \( N_p \) in a compact matrix form as [6]

\[
\eta_m(k) = \Theta_m \xi(k) + \Gamma_m \Delta U_m + \Psi_m D_k,
\]

(73)

where

\[
\Theta_m = \begin{bmatrix} \tilde{C}_d \tilde{A}_d & \tilde{C}_d \tilde{A}_d^2 & \ldots & \tilde{C}_d \tilde{A}_d^{N_c} & \ldots & \tilde{C}_d \tilde{A}_d^{N_p} \end{bmatrix}^T
\]

(74)

\[
\eta_m(k) = [\eta(k+1) \ldots \eta(k+N_p)]^T
\]

(75)

\[
\Delta U_m = [\Delta u(k) \ldots \Delta u(k+m) \ldots \Delta u(k+N_c-1)]^T
\]

(76)

\[
D_k = [\tilde{d}_k(k) \tilde{d}_k(k+1) \ldots \tilde{d}_k(k+N_p-1)]^T
\]

(77)

\[
\Gamma_m = \begin{bmatrix}
\tilde{C}_d \tilde{B}_d & 0 & \ldots & 0 \\
\tilde{C}_d \tilde{A}_d \tilde{B}_d & \tilde{C}_d \tilde{B}_d & \ldots & 0 \\
\vdots & \vdots & \ldots & \vdots \\
\tilde{C}_d \tilde{A}_d^{N_c-1} \tilde{B}_d & \tilde{C}_d \tilde{A}_d^{N_c-2} \tilde{B}_d & \ldots & \tilde{C}_d \tilde{B}_d \\
\vdots & \vdots & \ldots & \vdots \\
\tilde{C}_d \tilde{A}_d^{N_p-1} \tilde{B}_d & \tilde{C}_d \tilde{A}_d^{N_p-2} \tilde{B}_d & \ldots & \tilde{C}_d \tilde{A}_d^{N_p-N_c} \tilde{B}_d \\
\end{bmatrix}
\]

(78)

\[
\Psi_m = \begin{bmatrix}
\tilde{C}_d & 0 & 0 & \ldots & 0 \\
\tilde{C}_d \tilde{A}_d & \tilde{C}_d & 0 & \ldots & 0 \\
\tilde{C}_d \tilde{A}_d^2 & \tilde{C}_d \tilde{A}_d & \tilde{C}_d & \ldots & 0 \\
\vdots & \vdots & \vdots & \ldots & \vdots \\
\tilde{C}_d \tilde{A}_d^{N_p-1} & \tilde{C}_d \tilde{A}_d^{N_p-2} & \tilde{C}_d \tilde{A}_d^{N_p-3} & \ldots & \tilde{C}_d \\
\end{bmatrix}
\]

(79)
3.1.3 Cost function definition

The objective function of the path tracking controller can be given as [6]

\[ J(k) = \sum_{j=1}^{N_p} \tilde{\chi}^T(k+j)Q\tilde{\chi}(k+j) + \tilde{u}^T(k+j-1)R\tilde{u}(k+j-1), \]  

(80)

where Q and R represent weight matrices, where the \( \tilde{\chi} = \chi - \chi_o \) and \( \tilde{u} = u - u_o \). The first term in Eq.(80) reflects the capability of tracking performance, while the second reflects the constraint on the change of the control output.

Considering the soft constraints concept, we can get an alternative form of the objective function as follows [6]:

\[ J(k) = \sum_{i=1}^{N_p} \|\eta(k+i|t) - \eta_{ref}(k+i|t)\|^2_Q + \sum_{i=1}^{N_c} \|\Delta U(k+i|t)\|^2_R + \rho\varepsilon^2. \]  

(81)

Considering Eq.(73), the objective function can be given as

\[ J(k) = (\Theta_m\xi(k) + \Psi_m\tilde{d}_k - \eta_{ref})^T Q(\Theta_m\xi(k) + \Psi_m\tilde{d}_k - \eta_{ref}) + \Delta U_m^T R\Delta U_m + \rho\varepsilon^2. \]  

(82)

To solve the following optimization problem, the objective function is converted into a standard quadratic form [6].

\[ J(\xi(t), u(t-1), \Delta U(t)) = [\Delta U(t)^T, \varepsilon]^T H_t[\Delta U(t)^T, \varepsilon] + G_t[\Delta U(t)^T, \varepsilon], \]  

(83)

where \( H_t = \begin{bmatrix} \Gamma_m^T Q \Gamma_m + R & 0 \\ 0 & \rho \end{bmatrix} \), \( G_t = \begin{bmatrix} 2e_t^T Q \Gamma_m \\ 0 \end{bmatrix} \), \( e_t = (\Theta_m\xi(k) + \Psi_m\tilde{d}_k - \eta_{ref}) \) and \( e_t \) is the tracking error in the prediction horizon [6].

After obtaining the solution to Eq.(83) in each control cycle, a series of control input increments in the control horizon can be calculated as [6]

\[ \Delta U_t^* = [\Delta u^*_t, \Delta u^*_{t+1}, \ldots, \Delta u^*_{k+N_c-1}]^T. \]  

(84)

The first element of the control sequences is taken as the actual control input increment of the controller [6].

\[ u(t) = u(t-1) + \Delta u^*_t. \]  

(85)

3.1.4 Constraint analysis

There are constraints imposed on the control increment and the outputs. Specifically, the vehicle dynamics model, the constraints imposed on steering angle for the path-tracking problem are specified as [6]
1. The constraints for control can be given as
   \[ u_{\text{min}}(t + k) \leq u(t + k) \leq u_{\text{max}}(t + k), \quad k = 0, 1, \cdots, N_c - 1 \quad (86) \]

2. The constraints for control increments can be given as
   \[ \Delta u_{\text{min}}(t + k) \leq \Delta u(t + k) \leq \Delta u_{\text{max}}(t + k), \quad k = 0, 1, \cdots, N_c - 1 \quad (87) \]

3. The output constraints can be given as
   \[ y_{\text{min}}(t + k) \leq y(t + k) \leq y_{\text{max}}(t + k), \quad k = 0, 1, \cdots, N_c - 1 \quad (88) \]

   According to the tire modeling, the relationship between the tire slip angle and cornering force is linear when the tire slip angle is small. With small angle assumption for the linear tire models, the front wheel tire slip angle is limited in \(-2.5^\circ \leq \alpha \leq 2.5^\circ\), and the control increment is limited in \(-0.85^\circ \leq \Delta \alpha \leq 0.85^\circ\).

   In the cost function, the variables to be solved are control increments in the control horizon. The constraint conditions can only be expressed as a form of the control increment or a form of the control increment multiplied by the transformation matrix. Thus, Eq.(86) through (88) need to be converted for obtaining the corresponding transformation matrix. The relationship is defined as \[6\]

\[ u(t + k) = u(t + k - 1) + \Delta u(t + k), \quad (89) \]

assuming that

\[ U_t = 1_{N_c} \otimes u(k - 1) \quad (90) \]

\[ A = \begin{pmatrix}
1 & 0 & \cdots & \cdots & 0 \\
1 & 1 & 0 & \cdots & 0 \\
1 & 1 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & 0 \\
1 & 1 & \cdots & 1 & 1
\end{pmatrix} \otimes I_m \quad (91) \]

where \(1_{N_c}\) is the column vector of \(N_c\) ones, \(I_m\) is the identity matrix with a dimension of \(m\), \(\otimes\) is Kronecker product, and \(u(k - 1)\) is the previous control input \[6\].

Combining Eq.(89) through (91) can be converted into \[6\]

\[ U_{\text{min}} \leq A U_t + U_t \leq U_{\text{max}}, \quad (92) \]

where \(U_{\text{min}}\) and \(U_{\text{max}}\) are the lower and upper bound of the control input, respectively. This concludes the setting up of the constraint optimization problem induced by the model prediction control algorithm \[6\].
3.2 Longitudinal Control: PID speed control

3.2.1 Longitudinal dynamics equation

The wheel rotational dynamics of front right and rear right wheel in the plant are given as,

\[ J_w \dot{\omega}_{rf} = 0.5T_a - 0.25T_b - r_{rf} F_{xtrf} \]  \hspace{1cm} (93) \\
\[ J_w \dot{\omega}_{rr} = -0.25T_b - r_{rr} F_{xtrr}, \]  \hspace{1cm} (94)

where the \( T_a \) is total driving torque applied to the vehicle, \( T_b \) is the total braking torque applied to the vehicle. With these wheel rotational equations and simplification in Section 2.4, the longitudinal equation for the plant is given as

\[ \dot{u} = \frac{1}{m + \frac{J_w}{r_{lf}^2} + \frac{J_w}{r_{rf}^2} + \frac{J_w}{r_{lr}^2} + \frac{J_w}{r_{rr}^2}} \left( (0.5T_a - 0.25T_b) \frac{1}{r_{lf}} + (0.5T_a - 0.25T_b) \frac{1}{r_{rf}} + (-0.25T_b) \frac{1}{r_{lr}} \right) \\
+ \frac{(-0.25T_b)}{r_{rr}} - m\omega_y w + m\omega_z v + mgsin\theta. \]  \hspace{1cm} (95)

The longitudinal velocity and acceleration of the plant vehicle will be obtained from the above equation, which will be used in the PID speed controller.

3.2.2 PID controller

More than half of the industrial controllers in use today are PID controllers or modified PID controllers. The usefulness of PID controls lies in their general applicability to most control systems. Figure 6 shows a PID control of a plant \[ 16 \].

The speed tracking error is defined as

\[ e_u = u_d - u \]  \hspace{1cm} (96) \\
\[ e_\dot{u} = \dot{u}_d - \dot{u} \]  \hspace{1cm} (97) \\
\[ e_{ss} = \int_{t_1}^{t_2} e_u dt = e_u T_s, \]  \hspace{1cm} (98)

where \( u_d \) refers to the desired speed, \( u \) refers to the speed of the plant; \( \dot{u}_d \) refers to the desired longitudinal acceleration, \( \dot{u} \) refers to the longitudinal acceleration of the plant. \( T_s \) is the sample time.
Here, when the speed of the plant is smaller than the desired speed, then $T_b = 0$, $T_a = K_P e_u + K_I e_{s_x} + K_D e_{\dot{u}}$; when the speed of the plant is greater than the desired speed, then $T_a = 0$, $T_b = K_P e_u + K_I e_{s_x} + K_D e_{\dot{u}}$; when the speed of the plant equals to the desired speed, then $T_a = 0$, $T_b = 0$.

4 Simulation and analysis

4.1 Model parameters

The parameters for these vehicle models are listed in the table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>Sprung mass</td>
<td>1400(kg)</td>
</tr>
<tr>
<td>$J_x$</td>
<td>Sprung mass roll moment of inertia</td>
<td>900(kgm$^2$)</td>
</tr>
<tr>
<td>$J_y$</td>
<td>Sprung mass yaw moment of inertia</td>
<td>2000(kgm$^2$)</td>
</tr>
<tr>
<td>$J_z$</td>
<td>Sprung mass pitch moment of inertia</td>
<td>2420(kgm$^2$)</td>
</tr>
<tr>
<td>$a$</td>
<td>Distance of sprung mass C.M. from front axle</td>
<td>1.14(m)</td>
</tr>
<tr>
<td>$b$</td>
<td>Distance of sprung mass C.M. from rear axle</td>
<td>1.4(m)</td>
</tr>
<tr>
<td>$h$</td>
<td>Sprung mass C.M. height</td>
<td>0.75(m)</td>
</tr>
<tr>
<td>$c_f$</td>
<td>Front track width</td>
<td>1.5(m)</td>
</tr>
<tr>
<td>$c_r$</td>
<td>Rear track width</td>
<td>1.5(m)</td>
</tr>
<tr>
<td>$k_{sf}$</td>
<td>Front suspension stiffness</td>
<td>35000 (N/m)</td>
</tr>
<tr>
<td>$k_{sr}$</td>
<td>Rear suspension stiffness</td>
<td>30000 (N/m)</td>
</tr>
<tr>
<td>$b_{sf}$</td>
<td>Front suspension damping coefficient</td>
<td>2500 (Ns/m)</td>
</tr>
<tr>
<td>$b_{sr}$</td>
<td>Rear suspension damping coefficient</td>
<td>2000 (Ns/m)</td>
</tr>
<tr>
<td>$m_{uf}$</td>
<td>Front unsprung mass</td>
<td>80 (kg)</td>
</tr>
<tr>
<td>$m_{ur}$</td>
<td>Rear unsprung mass</td>
<td>80 (kg)</td>
</tr>
<tr>
<td>$k_{tf}$</td>
<td>Front tire stiffness</td>
<td>200000 (N/m)</td>
</tr>
<tr>
<td>$k_{tr}$</td>
<td>Rear tire stiffness</td>
<td>200000 (N/m)</td>
</tr>
<tr>
<td>$C_{\alpha f}$</td>
<td>Front right tire cornering stiffness</td>
<td>44000 (N/rad)</td>
</tr>
<tr>
<td>$C_{\alpha r}$</td>
<td>Rear right tire cornering stiffness</td>
<td>47000 (N/rad)</td>
</tr>
<tr>
<td>$C_{zf}$</td>
<td>Front right tire longitudinal stiffness</td>
<td>5000 (N)</td>
</tr>
<tr>
<td>$C_{xr}$</td>
<td>Rear right tire longitudinal stiffness</td>
<td>5000 (N)</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Nominal tire radius</td>
<td>0.285 (m)</td>
</tr>
<tr>
<td>$J_w$</td>
<td>Tire/wheel roll inertia</td>
<td>1 (kgm$^2$)</td>
</tr>
<tr>
<td>$h_{rcf}$</td>
<td>Front roll center distance below sprung mass C.M.</td>
<td>0.65 (m)</td>
</tr>
<tr>
<td>$h_{rcr}$</td>
<td>Rear roll center distance below sprung mass C.M.</td>
<td>0.6 (m)</td>
</tr>
</tbody>
</table>
4.2 Tracking objective definition

We implemented the controller for path tracking under the conditions of an 8-shaped curve path. In Case A, we consider the speed of the vehicle is constant at $10m/s$. In Case B, we consider the speed is increased from $0.5m/s$ to $10.4m/s$ in the straight line and then kept constant after entering the curve. In the path tracking controller, we chose the heading angle ‘$\psi$’ and the lateral displacement of vehicle C.M. in global coordinates ‘$Y$’ as the objectives; In the speed controller, we chose the speed of the vehicle C.M. as the objective.

In every simulation step, the optimal steering angle control input calculated from the MPC controller and the total driving or braking torque calculated from the PID controller are simultaneously transmitted to the plant to implement the path tracking and speed control.

The reference path, the reference lateral displacement of vehicle C.M. ‘$Y$’ in global coordinates, the reference heading angle ‘$\psi$’ and the speed reference are given as follows:

![Reference trajectory](image)

Figure 7: Tracking path: 8-shaped curve
Figure 8: Speed tracking objective (Case A)

Figure 9: Speed tracking objective (Case B)
Figure 10: path tracking objective: desired lateral displacement

Figure 11: path tracking objective: desired heading angle
4.3 Path tracking and speed control simulation results

4.3.1 Case A: Path tracking with constant speed (10m/s)

Figure 12: Comparison of Trajectory

Figure 13: Comparison of lateral displacement
Figure 14: Comparison of heading angle

Figure 15: Comparison of yaw rate
4.3.2 Case B: Path tracking with speed control

The speed reference in case B is firstly increased in the straight line and kept constant in the 8-shaped curve, the tracking performance of the simulation results are presented as below:
Figure 18: Comparison of speed

Figure 19: Comparison of lateral displacement
Figure 20: Comparison of heading angle

Figure 21: Comparison of longitudinal displacement
Figure 22: Comparison of yaw rate

Figure 23: Steering angle input
5 Conclusions and Future Work

In this study, the model predictive path tracking with PID speed control was proposed and implemented to ensure that the vehicle follows the given 8-shaped curve path and the desired speed. With respect to the lateral control, we used the 8-DOF vehicle model as the prediction model in the MPC controller and used the 14-DOF vehicle model as the plant. With respect to the longitudinal control, we used the PID controller to track the desired speed. Here, two cases for the reference speed were considered, case A is path tracking with constant speed, case B is path tracking and speed control with varied speed. Based on the simulation results, both controllers tracked the desired path well and compared with the reference heading angle ‘ψ’, the yaw rate ‘ψ̇’, the lateral displacement of vehicle C.M. ‘Y’ and the desired speed, the tracking errors are small.

In the future work, designing a speed profile along the given path will be considered for the vehicle in order to track the path as fast as possible.

Acknowledgments

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