ME 440
Intermediate Vibrations

Th, March 5, 2009

Single DOF Harmonic Excitation

Before we get started…

- Last Time:
  - Examples
  - Beating phenomena
  - Support excitation

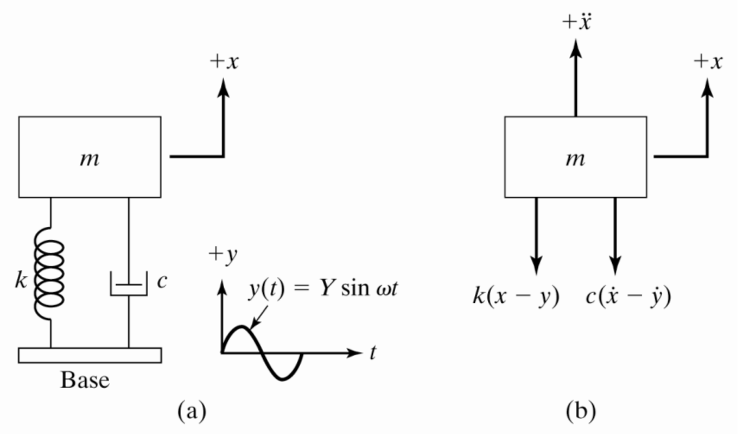
- Today:
  - HW Assigned (due March 12): 3.40, 3.51
  - Material Covered:
    - Examples
    - Flow Induced Vibration

Next alternative:

The "Relative Motion" Alternative

- Notation used:
  - x(t) captures the motion of the machine
  - y(t) captures the motion of the base
  - z(t) captures the relative motion:

$$z(t) = x(t) - y(t)$$



- EOM: Apply N2L in on body of mass m: $\sum F_x = m\ddot{x}$

$$\Downarrow$$

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

For symmetry, it misses a term in the y acceleration… Better subtract it from both sides of the equation

$$\ddot{y} = -\omega^2 Y \sin \omega t$$

- Finally,

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} = m\omega^2 Y \sin \omega t$$

The "Relative Motion" Alternative (Cont)

- EOM that needs to be solved:

$$m\ddot{z} + c\dot{z} + kz = m\omega^2 Y \sin \omega t$$

- Steady-state solution given by

$$z(t) = \frac{m\omega^2 Y \sin(\omega t - \phi_1)}{[(k - m\omega^2)^2 + (c\omega)^2]^{\frac{1}{2}}} = Z \sin(\omega t - \phi_1)$$

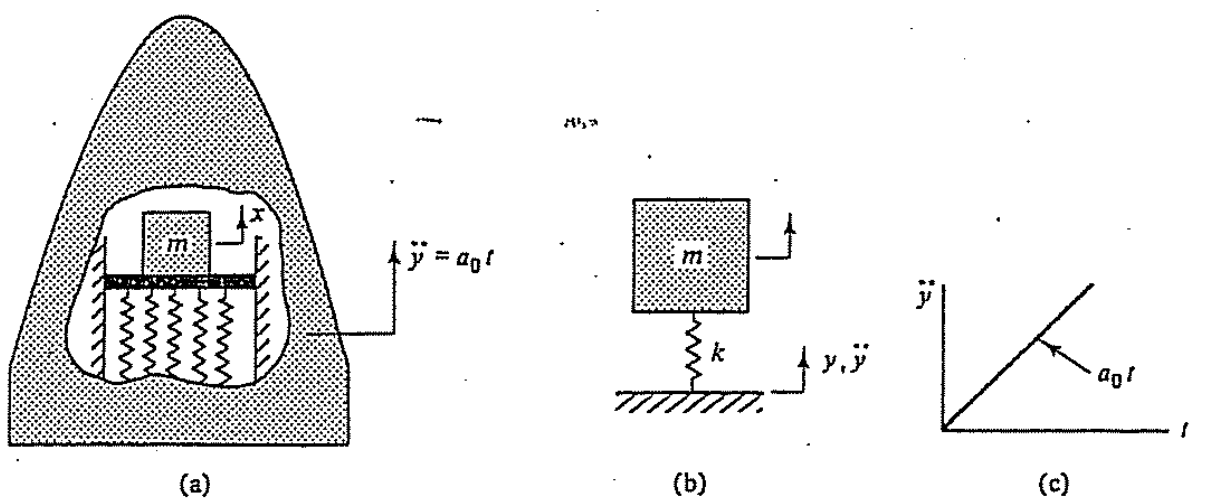
- Amplitude and phase of response computed as

$$Z = \frac{m\omega^2 Y}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} = Y \frac{r^2}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

$$\tan \phi_1 = \frac{c\omega}{k - m\omega^2} = \frac{2\zeta r}{1 - r^2}$$

Example: Instrument Package [AO2]

An instrument package of mass m is mounted on a support structure of stiffness k in the nose of a rocket as shown in part (a) of the figure. The support structure, which could involve springs, beams, plates, and so forth, is represented schematically by the spring-and-mass system shown in part (b) of the figure. Assuming that the nose cone to which the support structure is rigidly fastened experiences the acceleration shown in part (c) of the figure, and neglecting damping, determine (1) an expression for the relative displacement z of the instrument package with respect to the rocket, and (2) the absolute acceleration R of the instrument package as a function of time. If z is to be kept small, should k/m be large or small?



New Topic:

Flow Induced Vibration

- What is it?
  - Vibration caused by fluid flowing around or through objects
- What is causing it?
  - A multitude of causes, a common one is “vortex shedding” phenomenon

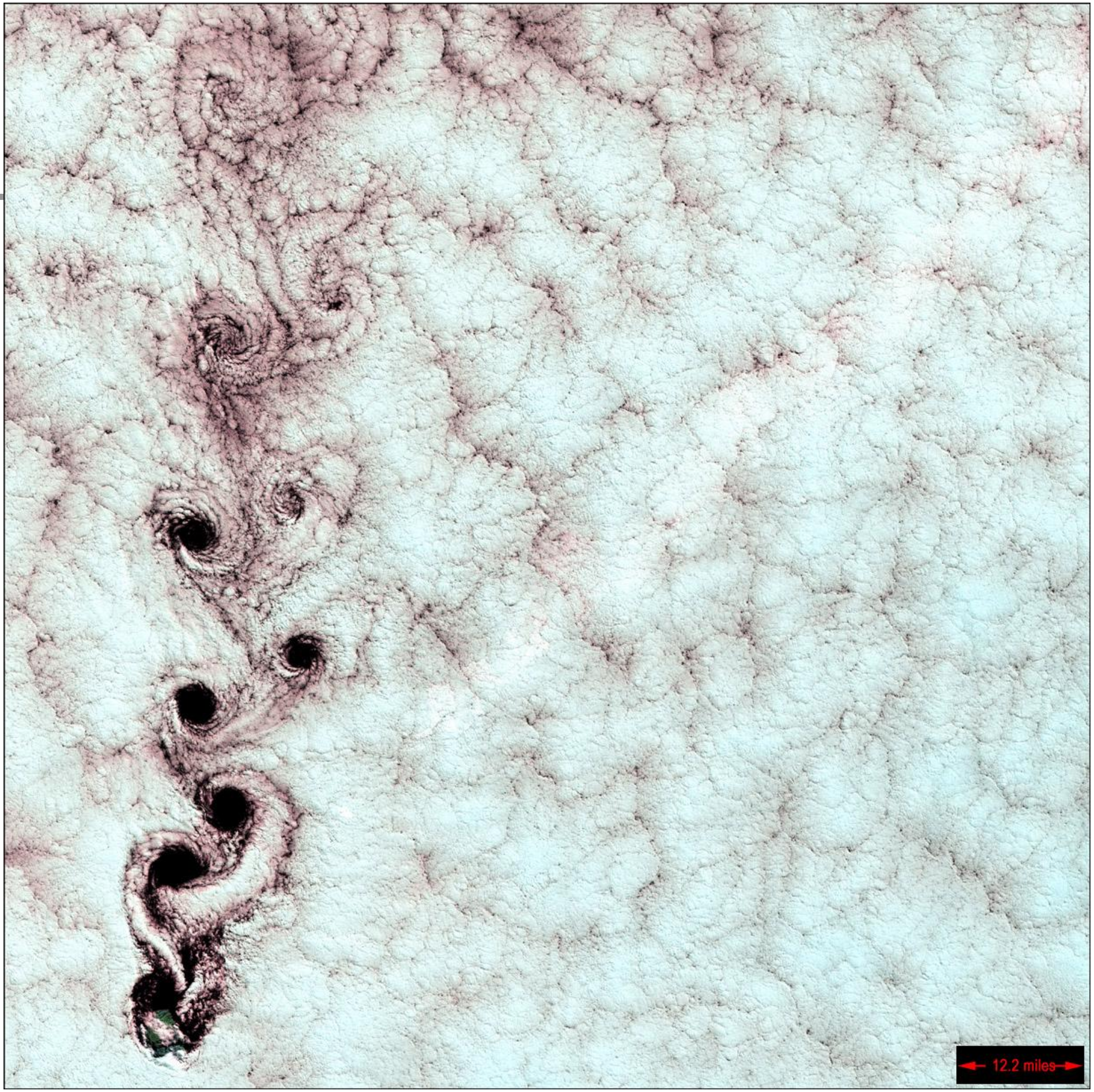
- Examples, “around”:
  - Electric transmission lines
  - Nuclear fuel rods
  - Tall air chimneys
  - Cross-flow in heat-exchangers
  - Submarine periscopes
  - Early bi-plane wires

- Examples, “through”
  - Water & Oil pipes
  - Fire hoses
  - Tubes in air compressors

Vortex Shedding

- Phenomenon appears when fluid flows past a smooth cylinder
  - Under certain conditions, a regular pattern of alternating vortices formed downstream.
  - Called Karman vortices, in honor of Theodor Von Karman.
    - Von Karman theoretically predicted these in 1911.
- The vortices shed alternately clockwise and counter clockwise from opposite sides of the cylinder with frequency f .
- This causes an alternating pressure on each side of the cylinder, which causes a harmonically varying force on the cylinder perpendicular to the velocity of the fluid.

Picture of Selkirk Island (Pacific Ocean off the cost of Chile) taken in September 1999 by the Landsat7 satellite. The white stuff are clouds.



Vortex Shedding (Cntd)

$$F = \frac{C_D \rho v^2 A}{2} \sin(2\pi f t)$$

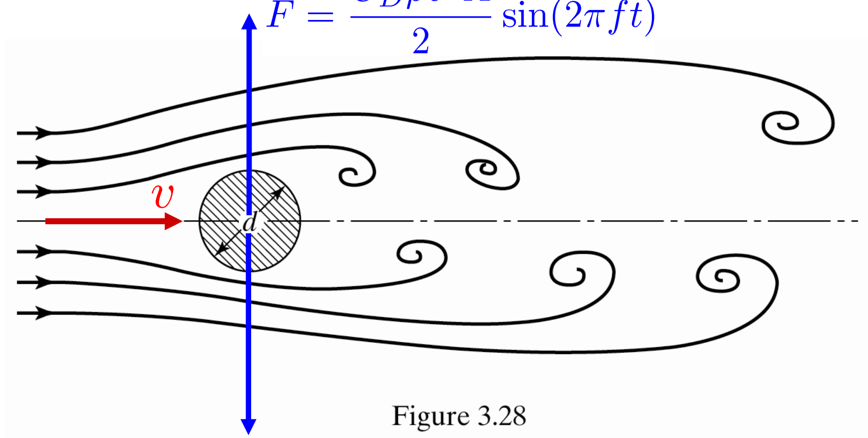
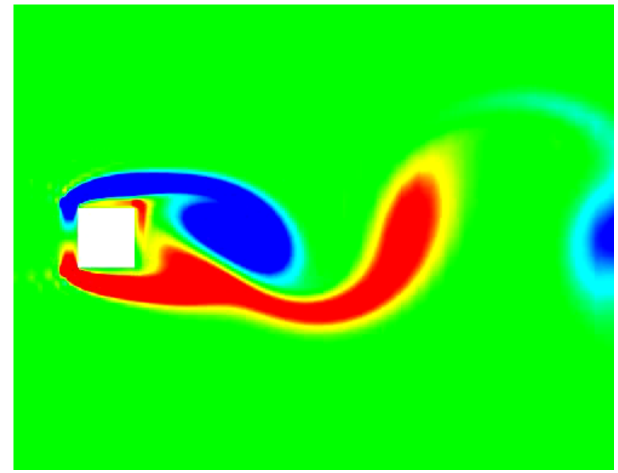


Figure 3.28
Fluid flow past a cylinder.



- Experimental data shows that regular vortex shedding occurs strongly in the range of Reynolds numbers (Re) from about 60 to 5000

$$\mathrm{Re} = \frac{v d \rho}{\mu} \quad \text{where} \quad \begin{cases} \rho & - \quad \text{density of fluid} \\ v & - \quad \text{velocity of fluid} \\ d & - \quad \text{diameter of cylinder} \\ \mu & - \quad \text{absolute viscosity of fluid} \end{cases}$$

dimensionless...

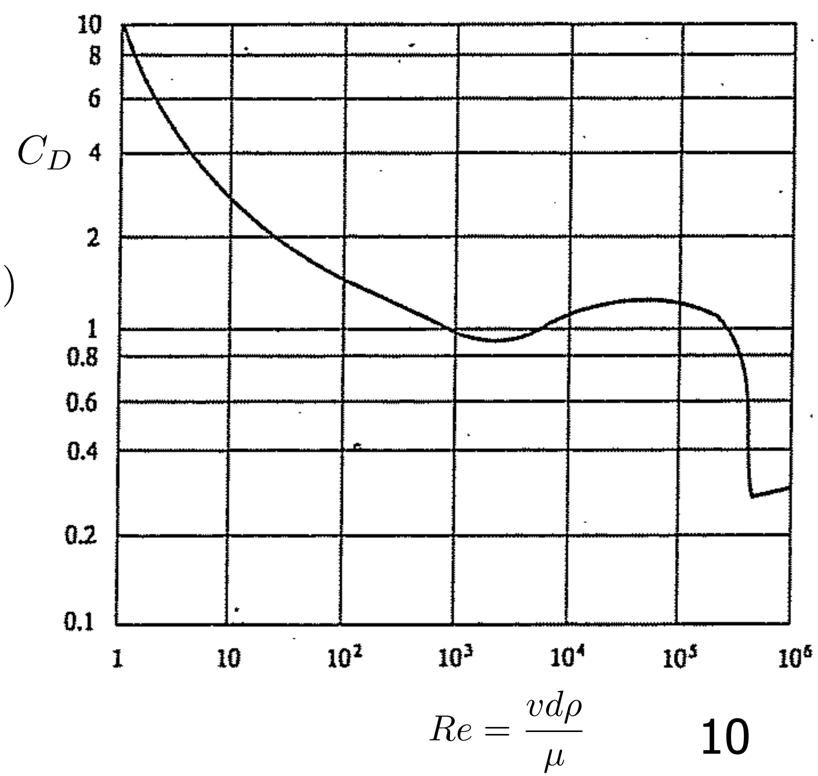
Vortex Shedding: Lift Force Magnitude

- Lift force turns out to be a harmonic function:

$$F(t) = \frac{C_D \rho v^2 A}{2} \sin(2\pi f\, t) \qquad \Leftrightarrow \qquad F(t) = \frac{C_D \rho v^2 A}{2} \sin(\omega\, t)$$

- Nomenclature:

- v – fluid velocity
- A – projected area of the cylinder (perpendicular to v)
- ρ - mass density of the fluid
- C_D – drag coefficient (dimensionless).



- Note:

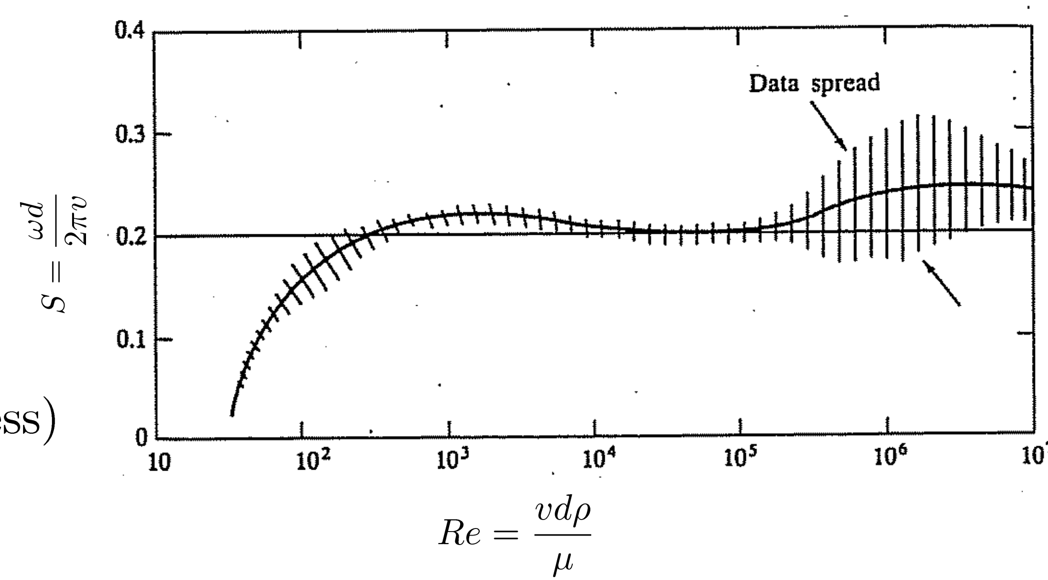
$$C_D \approx 1 \quad \text{for} \quad 10^3 \leq \text{Re} \leq 2 \cdot 10^5.$$

Vortex Shedding: Excitation Frequency

- Frequency of vertex shedding

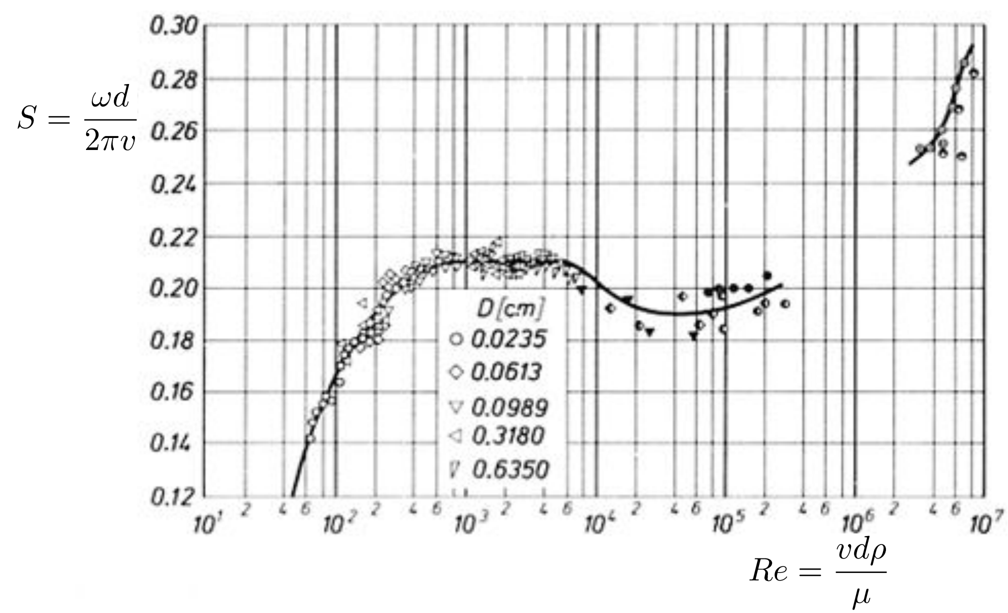
$$f = \frac{S \cdot v}{d}$$

- S – Strouhal number (dimensionless)
- d – diameter of cylinder
- v – velocity of fluid



- Dependency of S on Re illustrated in the right plots
- For a large range of values, Re between 400 and 300,000 one can assume that

$$S \cong 0.20$$



Vortex Shedding Dictated

Design Considerations

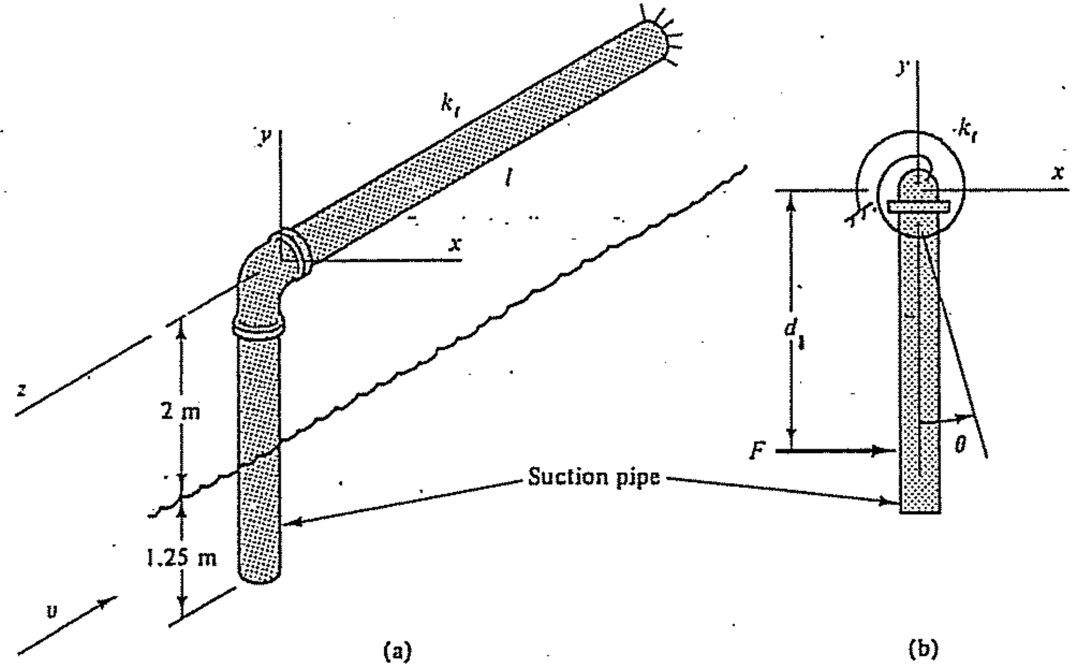
- The magnitude of the force exerted on the cylinder (F_0) should be less than the static failure load.
- Even if the magnitude of the force F_0 is small the frequency of oscillation (f) should not cause fatigue failure during the expected lifetime of the structure (or cylinder).
- The natural frequency of the structure or cylinder should not coincide with the frequency of vortex shedding (f) to avoid resonance

Example: Steel Chimney [AO]

- Steel chimney has height of 20 m
- Inner/outer diameters of 0.75/0.80 m

- Find velocity of wind flowing around chimney that induces maximum transverse vibration of the chimney (resonance condition)

Example: Submerged Pipe [AO



A vertical suction pipe 3.25 m long with an outside diameter of 0.15 m is submerged to a depth of 1.25 m in a river that is flowing at 5 mph as shown in the accompanying figure. It is attached to a horizontal pipe that has a torsional spring constant of $k = 15000\ \mathrm{N \cdot m/rad}$. A simplified model of the two pipes is shown in part b of the figure. Vortex shedding subjects the system to a moment M, about the z axis shown due to the force

$$F = \frac{C\rho v^2 A}{2} \sin(2\pi f t)$$

where C = 1.0 and $\rho = 1000\ \mathrm{kg/m^3}$.
Determine the steady-state amplitude of vibration θ_0 of the suction pipe if the fundamental natural frequency of the system is 3.1 Hz and the damping factor is $\zeta = 0.25$.
Ans: $\theta_0 = 0.17$ rad.

End Chapter 3: Response to Harmonic Loading

Begin Chapter 4: Vibration Under General Forcing Conditions