ME 440
Intermediate Vibrations

Th, Feb. 26, 2009

Single DOF Harmonic Excitation

Before we get started…

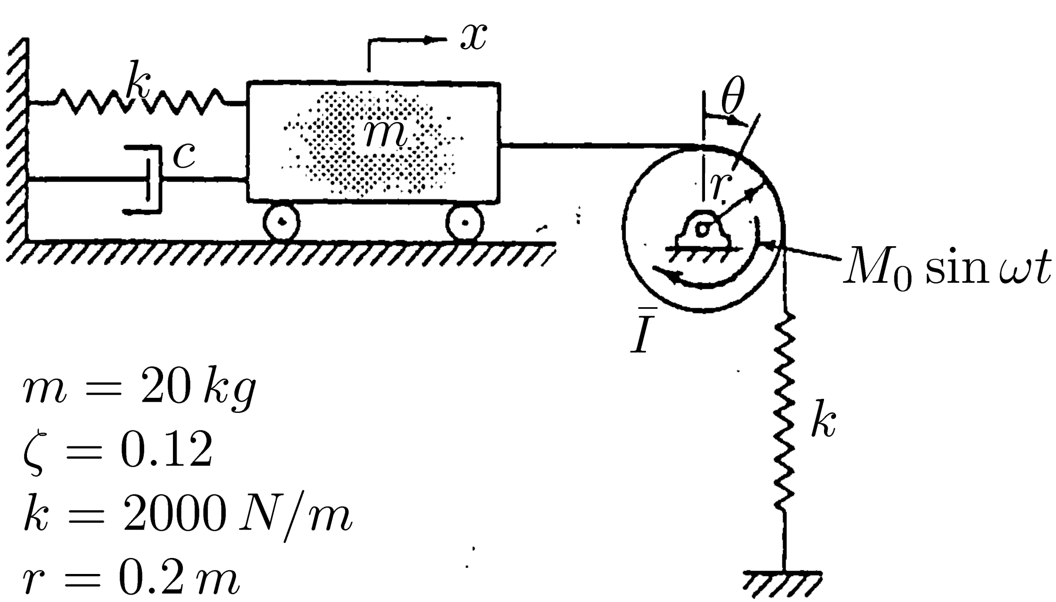
- Last Time (before exam):
  - Harmonically Excited Vibration

- Today:
  - HW Assigned: 3.30, 3.56 (due on Mar. 5)
  - Material Covered:
    - Unbalanced mass in rotation
    - Beating phenomena
    - Support excitation

- Comments on exam:
    - Multiple choice questions of chapter 3: you will get full credit for them
    - Return date: most likely one week from today

Example [AO1]

- No slip between light cable and cylinder. Interested in:
  - EOM for the system
  - |X| amplitude of the forced response if ω=10 rad/s



$$
\begin{cases}
m = 20\,kg \\
\zeta = 0.12 \\
k = 2000\,N/m \\
r = 0.2\,m \\
\bar{I} = 0.2\,kg \cdot m^2
\end{cases}
$$

Comments:
Frequencies associated with a m-c-k system

- Natural frequency
  - Always exists (as soon as you have an "m" and a "k" to speak of):

$$\omega_n = \sqrt{\frac{k}{m}}$$

- Damped natural frequency
  - Exists only for underdamped systems, $\zeta<1$
  - Too much damping prevents system from oscillating, no ω_d to speak of…)

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

- Resonance frequency
  - Exists only when $\zeta < 0.707$

$$\omega_r = \omega_n \sqrt{1-2\zeta^2}$$

- When all these three frequencies exist, note that

$$\omega_r < \omega_d < \omega_n$$

Comments on the Design HW Problem

- The easy way out:
  - Pick up lucky c and k, use appropriate equation to get X:

$$X = \frac{F/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

  - If c and k really lucky, then $X < X_{limit}$ – all DONE.

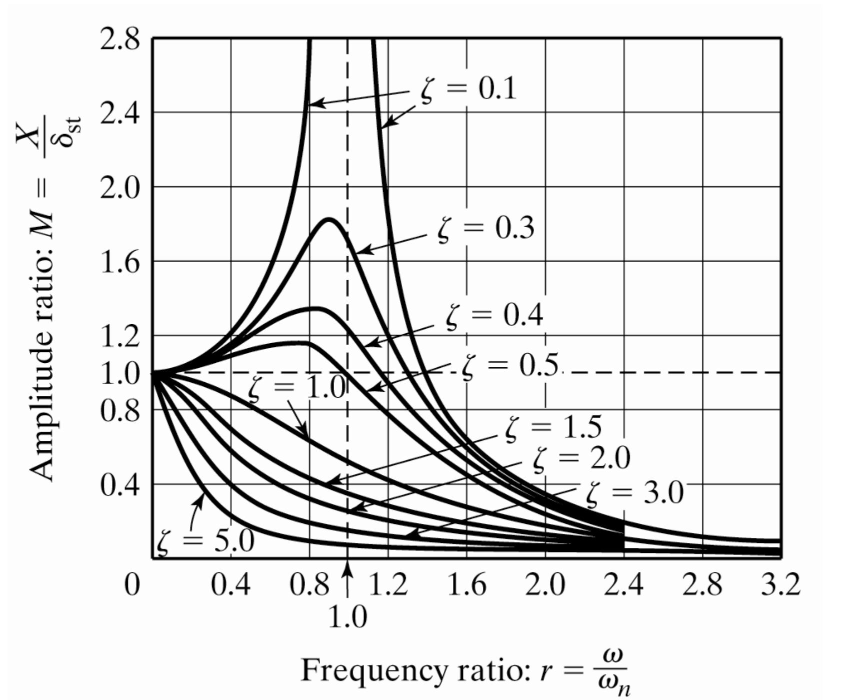
- The problem with taking the easy way out:
  - If you change the excitation frequency, X will change
    - There is no way to know if your design constraint is satisfied
  - Ideally, you'd like a robust design, where even when the excitation frequency changes, you still satisfy the design constraint

Comments on the Design HW Problem [Cntd.]

- Conservative design alternative:
  - Make sure you are covered in the worst case scenario
    - The frequency of oscillation turns out to be (unfortunately) precisely the resonance frequency
      - Note that this requires that $\zeta<0.707$ (!)

- Robust design: if you satisfy the design constraint in the worst case scenario, you'll satisfy it everywhere
- At resonance, amplification is (pp. 231)

$$X = \frac{F/k}{2\zeta\sqrt{(1-\zeta^2)}} < X_{limit}$$



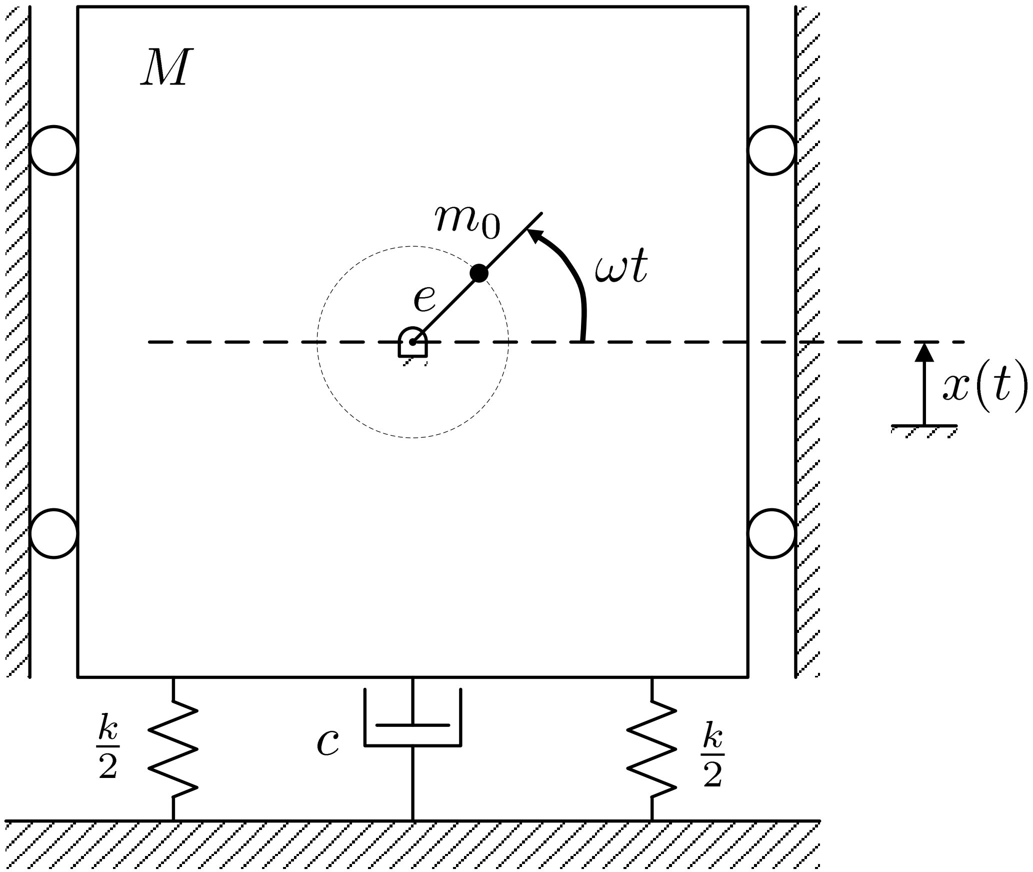
- What's left at this point?
  - Choose a ζ (say 0.5)
  - Compute k based on relation above
  - Compute c

$$c = 2\zeta\sqrt{km}$$

New Topic:

Excitation Due to Unbalanced Rotating Mass

- Unbalance in rotating machines: common source of vibration excitation



Excitation Due to Unbalanced Rotating Mass (Cntd)

- One degree of freedom system, determine EOM (use N2L, motion in the vertical direction):

$$Mg + m_0 g + (kx - W) + c\dot{x} = -M\ddot{x} - m_0\ddot{x} + m_0 e\omega^2 \sin\omega t$$

$$\Downarrow$$

$$m\ddot{x} + c\dot{x} + kx = m_0 e\omega^2 \sin\omega t$$

$$\Downarrow$$

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t$$

- Notation used above:

$$\begin{cases} F_0 &= m_0 e\omega^2 \\ m &= M + m_0 \end{cases}$$

- Note: The EOM is of the same form as the one for the "single degree of freedom system experiencing harmonic excitation"

Example: Francis Water Turbine [AO2]

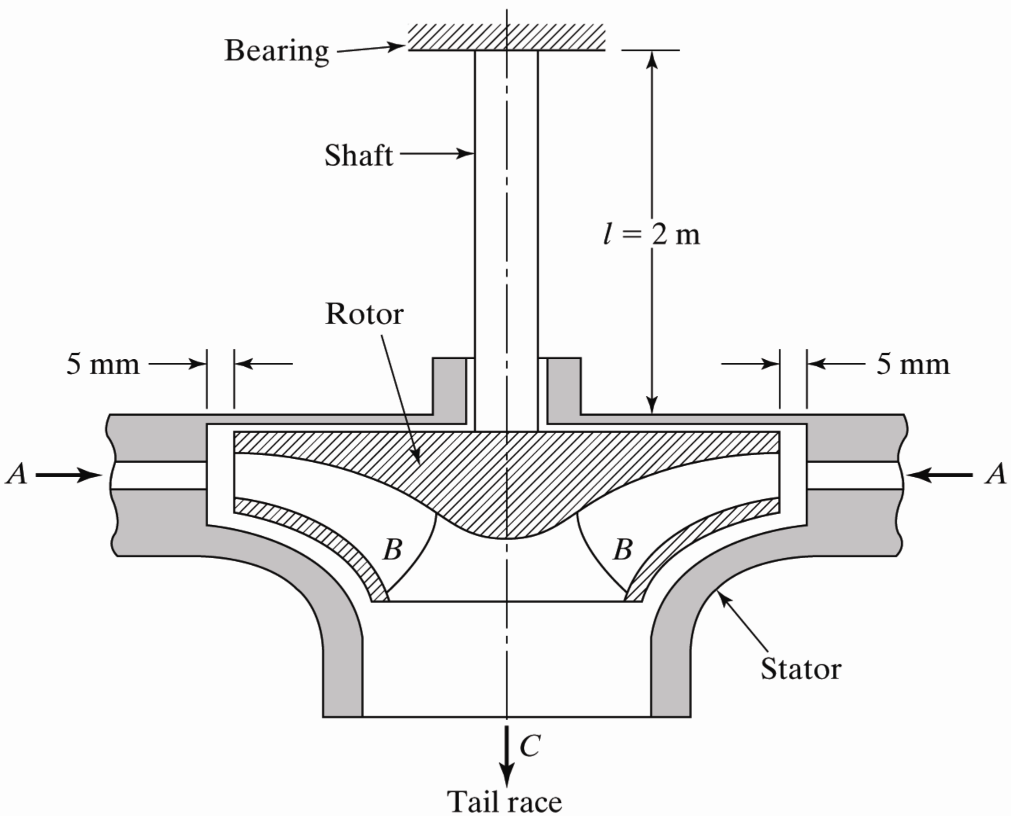


Figure 3.20
Francis water turbine.

- Rotor has mass 250 kg
- Unbalance in rotor 1 kg-mm
  - This is m_0e
- Radial clearance (rotor-stator): 5 mm
- Shaft
  - E = 200 GPa (steel)
  - Diameter: 10 cm
  - Considered clamped at the bearings
- Rotor operated at resonance
  - Initial radial deflection of rotor: 1 mm
- How long does it take before rotor hits stator?
  - Assume starts from rest
- Neglect damping and assume mass of turbine much grater than mass of shaft