ME 440
Intermediate Vibrations

Tu, Feb. 17, 2009

Section 2.5

Before we get started…

- Last Time:
  - Motion of pendulum (inversted pendulum, its stability)
  - Torsional vibration
  - Examples

- Today:
  - HW Assigned: 2.73, 2.82 (due on Feb. 24)
    - For 2.73, only derive EOM using Newton's second law and conservation of energy

  - Energy methods (applied herein for conservative systems)
    - For determining EOM
    - For determining the natural frequency of a system (Rayleigh's method)

- Next Tu (02/24): exam, covers chapters 1 and 2
  - Please point out missing material from the website
  - Review session: Monday evening, 7:15PM, in this room

New Topic:

Energy Methods

- The so called "Energy Methods" draw on the interplay between kinetic and potential energies associated with a conservative system

- Kinetic Energy
  - Discrete masses:
    - **Point mass**: Has translation only, therefore kinetic energy is

$$\text{Kinetic Energy} = T = \tfrac{1}{2} m v^2$$

    - **Rigid body**: Has both translation and rotation, therefore kinetic energy is

$$\text{Kinetic Energy} = T = \tfrac{1}{2} m v^2 + \tfrac{1}{2} J \dot{\theta}^2$$

Note: J - mass moment of inertia.

For rotation about a fixed axis (point O), $\quad T = \tfrac{1}{2} J_O \omega^2.$

Potential Energy Component

- U – change in potential energy of the system from its value in the static-equilibrium configuration

$$
\begin{aligned}
U &= mgh \quad \text{(weight)} \qquad \text{(watch sign here...)} \\
U &= \tfrac{1}{2}kx^2 \quad \text{(spring)}
\end{aligned}
$$

- Conservation of Energy
  - Can use with conservative systems only
  - At any two different moments of time t_1 and t_2 we have that

$$T_1 + U_1 = T_2 + U_2 = \text{constant}.$$

Derivation of EOM

- For the free vibration of an undamped system the energy is partly kinetic and partly potential.

- Kinetic energy T is stored in the mass by virtue of its velocity

- Potential energy U is stored in the form of strain energy in elastic deformation or work done in a force field such as gravity.

$$T + U = \text{constant} \qquad \Rightarrow \qquad \boxed{\frac{d}{dt}(T+U) = 0}$$

- The expression obtained above after taking the time derivative is (after some massaging) precisely the EOM

Derivation of Natural Frequency ~ Rayleigh's Energy Method ~

- Natural frequency can be obtained starting with

$$T_1 + U_1 = T_2 + U_2 = \text{constant}.$$

- Recall that system assumed to be oscillating about static equilibrium configuration, no damping present (harmonic oscillation)

- Let time t_1 be the time when the mass is passing through its static equilibrium configuration.
  - Choose $U_1=0$ (that is, this represents the reference configuration)

- Let time t_2 be the time that corresponds to the maximum displacement of the mass relative to the reference configuration defined above.
  - In this configuration, the velocity of the mass is zero ($T_2 = 0$)
    - Recall that the velocity is first time derivative of position to understand why it's zero where the displacement is the largest

Derivation of Natural Frequency (Cntd)

- Therefore, we are left with

$$T_1 + 0 = 0 + U_2$$

- If the system is undergoing harmonic motion though, then T_1 and U_2 are maximum, hence

$$T_{\max} = U_{\max}$$

- The condition above leads to an equation that can be solved for ω_n

- Important remark
  - Since motion assumed harmonic, the maximum velocity is obtained by multiplying the maximum amplitude by ω_n (recall that velocity is the time derivative of position…)

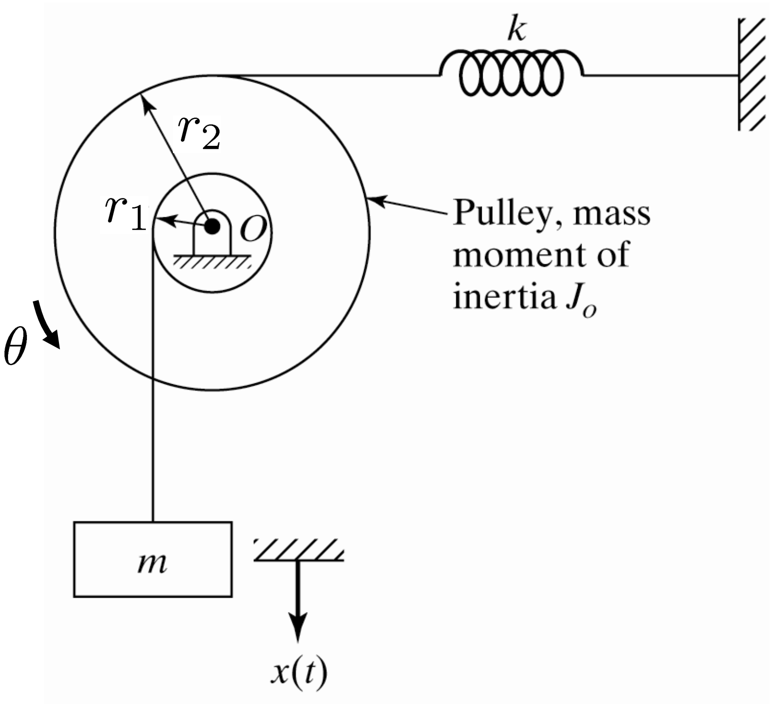
Example 1 [AO]

- Determine the EOM for a mass-spring system

- Determine the natural frequency of the mass motion

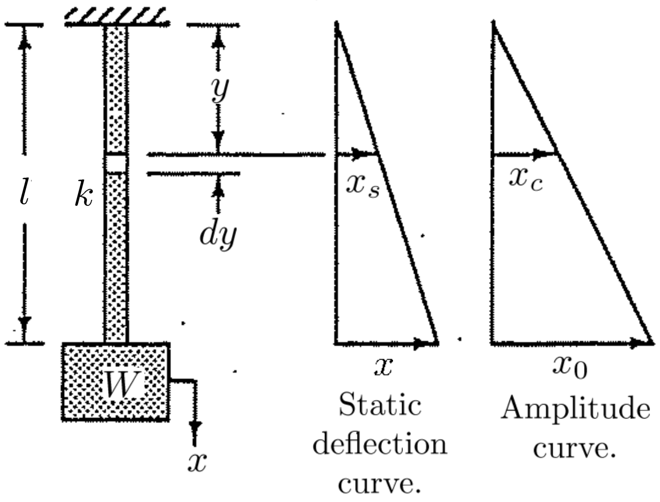
Example 2 [AO]

- Determine the natural frequency of the system shown



Example 3 [AO]

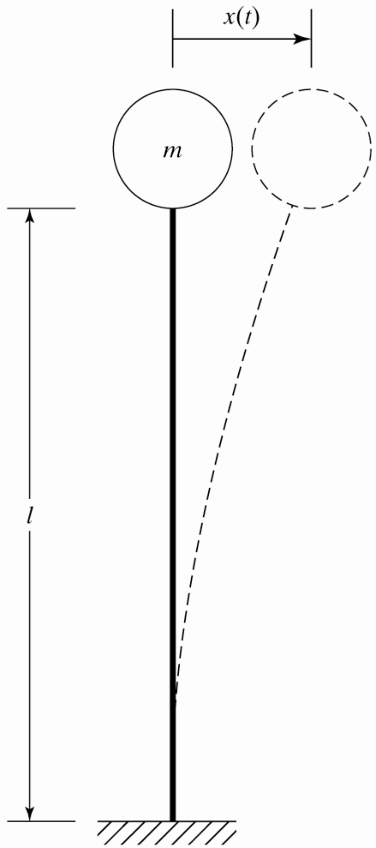
- Determine the natural frequency of the system shown



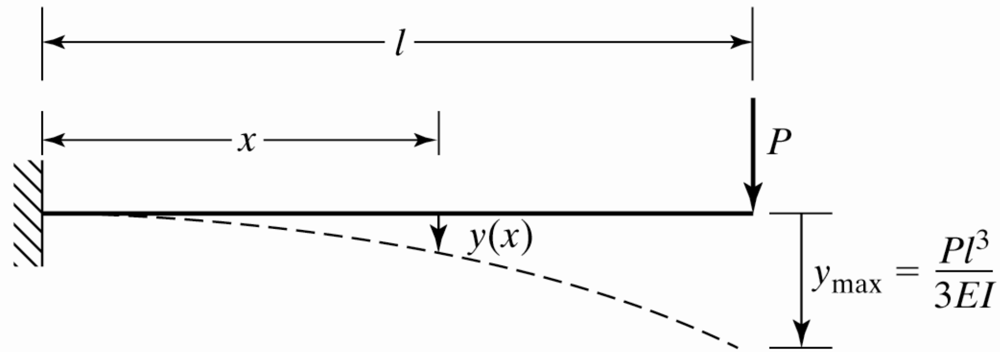
- See attached handout for natural frequencies of other beam configurations

Example 4 [text, pp.138]

- Find natural frequency for transverse rotation of water tank after including the mass of the column



Example 4 [text, pp.138]



- Approach: find first equivalent mass, and then use the spring stiffness associated with transversal motion of beam

- Key relations:

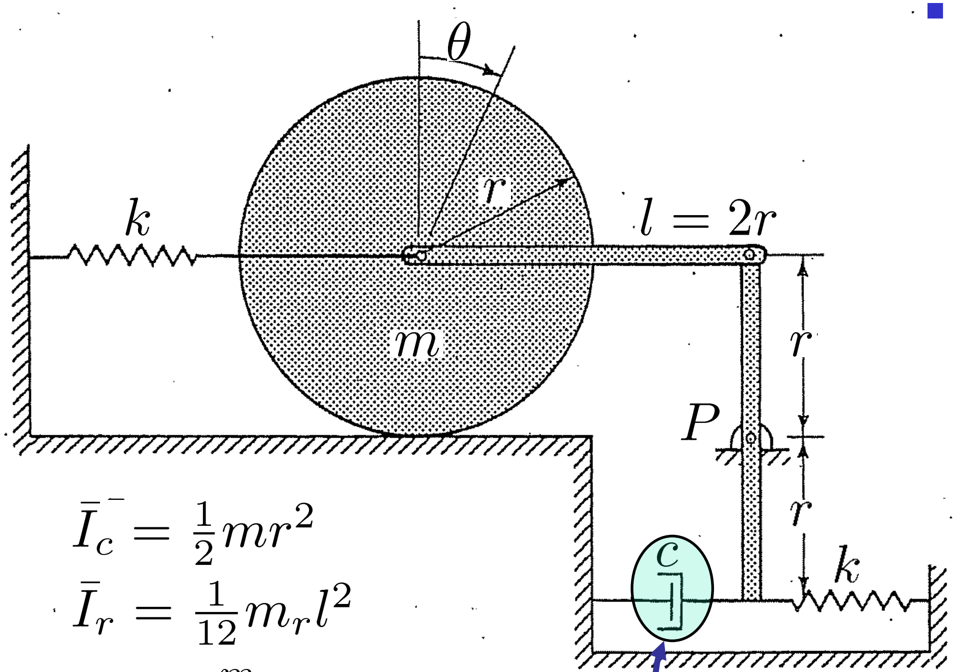
$$y(x) = \frac{Px^2}{6EI}(3l - x) = \frac{y_{\max}x^2}{2l^3}(3l - x) = \frac{y_{\max}}{2l^3}(3lx^2 - x^3)$$

$$T_{\max} = \frac{1}{2}\int_0^l [\dot{y}(x)]^2 \frac{m}{l} dx$$

$$\dot{y}(x) = \frac{\dot{y}_{\max}}{2l^3}(3lx^2 - x^3)$$

Example 5 [AO]

- Cylinder of radius r rolls without slip. Mass of each rod is m_r=m/4
- Assume small oscillation and ignore the very small rotational effect of the horizontal bar



$\bar{I}_c = \frac{1}{2}mr^2$

$\bar{I}_r = \frac{1}{12}m_r l^2$

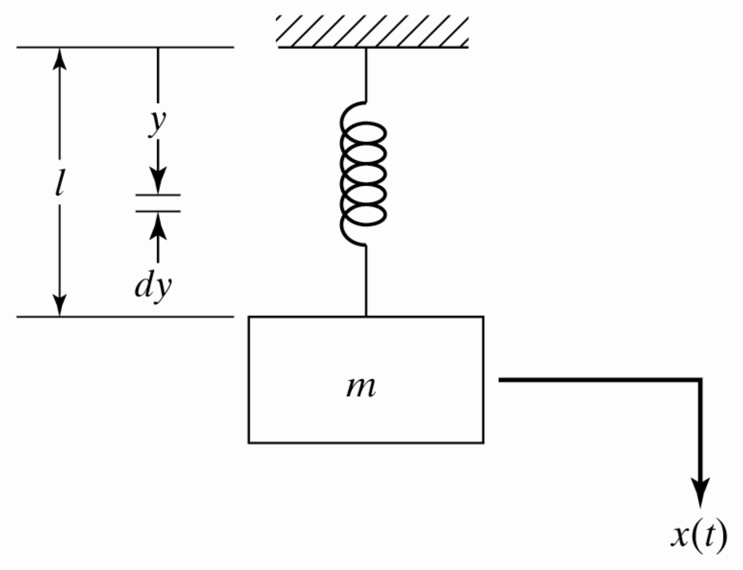
$m_r = \frac{m}{4}$

- Use T_{max}=U_{max} to determine undamped natural frequency ω_n for the system

Assume no damper present in system…

Example 6 [text, pp. 136]

- Determine the effect of the mass of the spring on the natural frequency of the system shown (mass-spring system)



- Key relation:

$$dT_s = \frac{1}{2}\left(\frac{m_s}{l}dy\right)\left(\frac{y}{l}\dot{x}\right)^2$$

End Chapter 2: Free Response

Begin Chapter 3: Response to Harmonic Loading