ME451
Kinematics and Dynamics of Machine Systems

Post Dynamics

Inverse Analysis

Equilibrium Analysis

December 11, 2014

- Quote of the day: "My mother said to me, 'If you are a soldier, you will become a general. If you are a monk, you will become the Pope.' Instead, I was a painter, and became Picasso."

-- Pablo Picasso

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Before we get started…

- Last time
  - Solving the constrained equations of motion using the Newmark integration formulas

- Today
  - Inverse Dynamics Analysis
  - Equilibrium Analysis

- Project 2 due on 12/16 at 11:59 PM

- End of semester here, all scores in Learn@UW
  - Please let me know now if you notice any mistakes

- I will travel next week, Dr. Pazouki will proctor the Final Exam
  - Tuesday 12/16, 2:45PM - 4:45PM
  - Room : 2109ME (computer lab)

Final Exam Structure/Content

- Comprehensive exam
- Same format as the Kinematics and Dynamics midterm exams
  - First problem made up of several short questions
  - Second problem – you have to do some busy work
  - In last problem you will be asked to generate an acf/adm file
    - This will require you to have access to a tablet/laptop/workstation
- Bonus: Running simEngine2D with your acf/adm combo and producing the right plots for a reaction force in a joint

Inverse Dynamics: The idea

- First of all, what does (forward) dynamic analysis mean?
  - Apply some forces and/or torques on a mechanical system and look at how the configuration of the mechanism changes in time
  - The mechanism evolution also depends on the specified initial conditions
  - This is a forward process: forces $\Longrightarrow$ motion

- In inverse dynamics, the situation is quite the opposite:
  - Specify driving constraints on the mechanical system (that is specify the desired motion) and find the set of forces and/or torques that should have been applied to the mechanical system to lead to this motion
  - Note that the ICs are implicitly defined by the specified desired motion
  - Note that we need to include as many driver constraints as kinematic degrees of freedom there are (exactly like in Kinematics, we require NDOF = 0)
  - This is a reverse (inverse) process: forces $\Longleftarrow$ motion

- Inverse dynamics is useful in controls
  - Example: controlling the motion of a robot – we know how we want this robot to move and we need to figure out what joint torques we should apply to make it move that way

Inverse Dynamics: The Math

- When can one talk about Inverse Dynamics?
  - Given a mechanical system, a prerequisite for Inverse Dynamics is that the number of degrees of freedom associated with the system is **zero**
    - We must have as many generalized coordinates as constraints (THIS IS KEY)
  - This effectively makes the problem a Kinematics problem
- The two stages of the Inverse Dynamics analysis
  - First solve for accelerations (recall the acceleration equation):

$$\mathbf{\Phi_q}\ddot{\mathbf{q}} = \gamma$$

  - Next solve for the Lagrange multipliers and then the reaction forces:

$$\mathbf{\Phi_q}^T\lambda = \mathbf{Q}^A - \mathbf{M}\ddot{\mathbf{q}}$$

Inverse Dynamics: Conclusion

- Are we done once we computed the reaction forces?
  - Yes, because among the reaction forces we computed, we get all the forces/torques that are necessary to enforce the specified driving constraints Φ^D which were used to specify the desired motion

$$\mathbf{F}_i^D = -\left(\mathbf{\Phi}_{\mathbf{r}_i}^D\right)^T \lambda^D$$

$$T_i^D = \left(\mathbf{\Phi}_{\mathbf{r}_i}^D \mathbf{B}_i \mathbf{s}'^P_i - \mathbf{\Phi}_{\phi_i}^D\right)^T \lambda^D$$

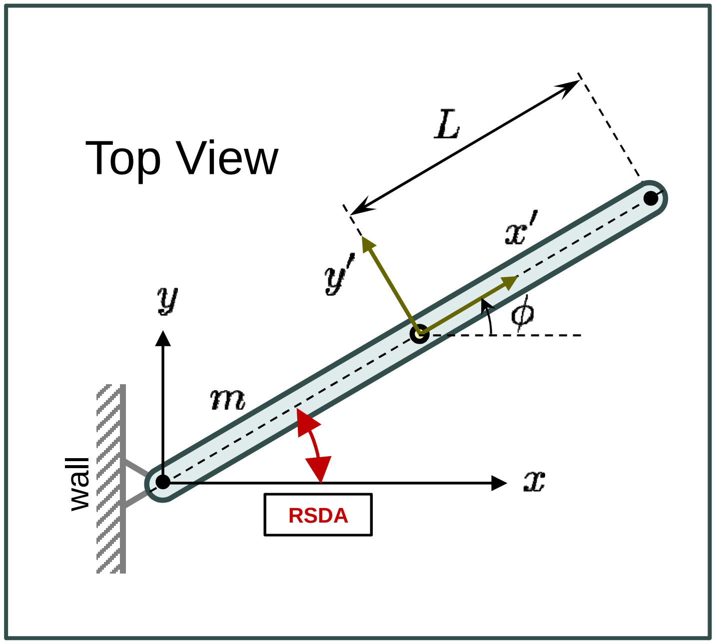
  - Here, the driving constraint $\mathbf{\Phi}^D$ acts between body i and some other body. Reaction forces (induced by the corresponding Lagrange multiplier $\boldsymbol{\lambda}^D$) are computed as "experienced" by body i
  - This gives us the forces/torques that need to be applied to get the prescribed motion

Example: Inverse Dynamics

- Compute the torque that the electrical motor must produce to open the door for 2 seconds following the prescribed motion:

$$\phi = \frac{\pi}{2} \sin\left(\frac{\pi}{4} t\right)$$

- Half-length: $L = 0.5$
- Mass: $m = 30$
- Polar moment of inertia: $J' = 2.5$
- RSDA spring coefficient: $k = 8$
- RSDA free angle: $\theta_0 = 0$
- RSDA damping coefficient: $c = 1$
- All units are SI



Equilibrium Analysis

Equilibrium Analysis: The Idea

- A mechanical system is in equilibrium if the following conditions hold:

$$\dot{\mathbf{q}} = \mathbf{0} \quad \& \quad \ddot{\mathbf{q}} = \mathbf{0}$$

- Equivalently, the system is at rest with zero acceleration

- So what does it take to be in this state of equilibrium?
  - The system must be in a certain configuration $\mathbf{q}$
  - The reaction forces (in other words, the Lagrange Multipliers $\boldsymbol{\lambda}$) should assume certain values
  - What does "certain" mean?

Equilibrium Analysis: The Math

- Equations of Motion:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{T}\lambda = \mathbf{Q}^{A} \quad \Rightarrow \quad \boxed{\mathbf{\Phi}_{\mathbf{q}}^{T}\lambda = \mathbf{Q}^{A}}$$

- Position Constraint Equations:

$$\boxed{\mathbf{\Phi}(\mathbf{q}, t) = \mathbf{0}}$$

- Velocity Constraint Equations:

$$\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}} = -\mathbf{\Phi}_{t} \triangleq \nu \quad \Rightarrow \quad \boxed{\mathbf{\Phi}_{t} = \mathbf{0}}$$

- Acceleration Constraint Equations:

$$\mathbf{\Phi}_{\mathbf{q}}\ddot{\mathbf{q}} = -\left(\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}}\right)_{\mathbf{q}}\dot{\mathbf{q}} - 2\mathbf{\Phi}_{\mathbf{q}t}\dot{\mathbf{q}} - \mathbf{\Phi}_{tt} \triangleq \gamma \quad \Rightarrow \quad \boxed{\mathbf{\Phi}_{tt} = \mathbf{0}}$$

Equilibrium Analysis: The Math

- Approach 1
  - Simply solve the nonlinear system to find $\mathbf{q}$ and $\boldsymbol{\lambda}$

$$\mathbf{\Phi}_{\mathbf{q}}^{T}\lambda = \mathbf{Q}^{A}$$

$$\mathbf{\Phi}(\mathbf{q}) = \mathbf{0}$$

  - One pretty big issue: we need a good initial guess

- Approach 2
  - Add damping in the system and perform dynamic analysis until it stops
  - Brute force, but effective

- Approach 3
  - Cast it as an optimization problem (minimize potential energy)
  - One big issue: works for conservative systems only

Example: Equilibrium Analysis

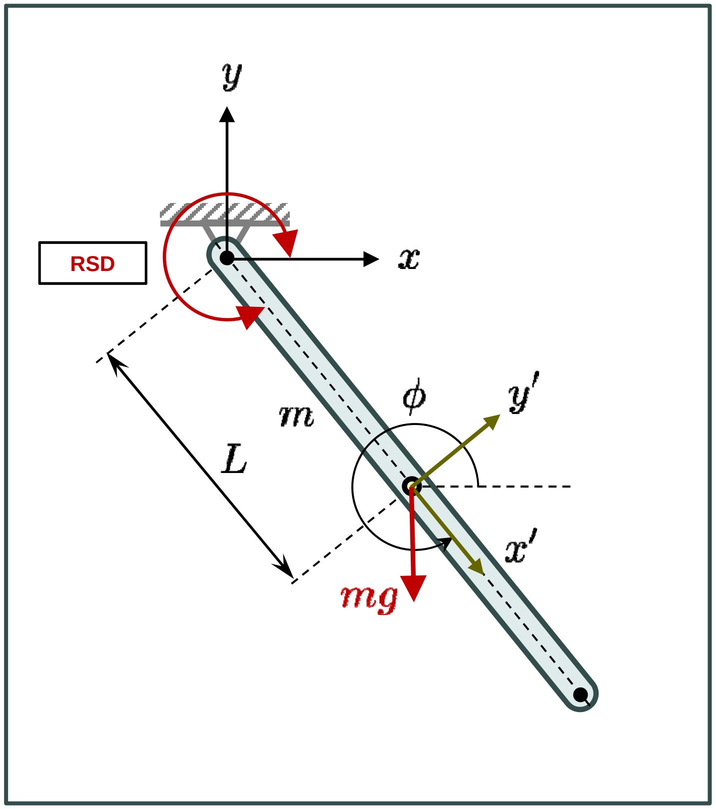
- Consider a pendulum connected to ground through a revolute joint and a rotational spring-damper.

- Length: $L = 1$
- Mass: $m = 10$

- RSDA spring coefficient: $k = 25$
- RSDA free angle: $\theta_0 = 0$
- RSDA damping coefficient: c

- Gravitational acceleration: $g = 9.81$

- All units are SI



ME451: Putting Things in Perspective

Summary of ME451

- Pick a set of Cartesian generalized coordinates
- Formulate the following sets of equations
  - Equations of Motion: $$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{T}\lambda = \mathbf{Q}^{A}$$
  - Position Constraint Equations: $$\mathbf{\Phi}(\mathbf{q}, t) = \mathbf{0}$$
  - Velocity Constraint Equations: $$\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}} = -\mathbf{\Phi}_{t} \triangleq \nu$$
  - Acceleration Constraint Equations: $$\mathbf{\Phi}_{\mathbf{q}}\ddot{\mathbf{q}} = -\left(\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}}\right)_{\mathbf{q}}\dot{\mathbf{q}} - 2\mathbf{\Phi}_{\mathbf{q}t}\dot{\mathbf{q}} - \mathbf{\Phi}_{tt} \triangleq \gamma$$
- Solve the appropriate set of equations for Kinematics or Dynamics

Purpose of ME451

- Learn how to characterize the motion of any 2D mechanism:
  - Learn how to pose a Kinematics/Dynamics problem (modeling)
  - Learn how to solve a Kinematics/Dynamics problem (simulation)
- Improve your MATLAB programming skills

The End.