ME451
Kinematics and Dynamics of Machine Systems

Elements of 2D Kinematics

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Dan Negrut
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University of Wisconsin-Madison

- Quote of the day: “Wanting to be someone else is a waste of the person you are.” – Kurt Cobain

Before we get started…

- Last time
  - Constraints, KDOF, NDOF, conditions required for Kinematics analysis
  - Position, Velocity, and Acceleration Analysis

- Today
  - Discuss kinematic absolute constraints and how they are expressed in equations
    - Learn how to compute the essential ingredients required by a Kinematics analysis
  - Discuss kinematic relative constraints and how they are expressed in equations

- HW:
    - Haug's book: 3.1.3, 3.3.2
    - MATLAB 2: available online at class website
  - Due on Th, 10/02, at 9:30 am
  - Post questions on the forum
  - Drop MATLAB assignment in learn@uw dropbox

3.2, 3.3, 3.4, 3.5

SYSTEMATIC DERIVATION OF CONSTRAINTS

[What comes next:]

Focus on Geometric Constraints

- Learn how to write kinematic constraints that specify that the location and/or orientation of a body w.r.t. the global (or absolute) RF is constrained in a certain way
  - A.k.a. absolute constraints

- Learn how to write kinematic constraints that restrict the relative motion of two bodies
  - A.k.a. relative constraints

The Drill…

- Step 1: Identify a kinematic constraint (revolute, translational, relative distance, etc., i.e., the *physical* thing) acting between two bodies of a mechanism

- Step 2: Formulate the algebraic equations that capture that constraint, $\mathbf{\Phi}(\mathbf{q}) = \mathbf{0}$
  - This is the actual modeling stage in Kinematics

- Step 3: Compute the Jacobian $\mathbf{\Phi}_{\mathbf{q}}$ (needed in P, V, A analyses)

- Step 4: Compute $\boldsymbol{\nu}(\mathbf{q}, t)$, right-hand side of the velocity equation (V analysis)

- Step 5: Compute $\boldsymbol{\gamma}(\dot{\mathbf{q}}, \mathbf{q}, t)$, right-hand side of acceleration equation (A analysis)

This is what we do time and again in Chapter 3

3.2

ABSOLUTE CONSTRAINTS

Absolute Constraints (1)

- Called “Absolute” since they express constraint between a body in a system and the absolute (global, ground) reference frame.

- Types of Absolute Constraints:
  - Absolute position constraints
  - Absolute orientation constraints
  - Absolute distance constraints

Absolute Constraints (2)

- Absolute position constraints
  - x-coordinate of P_i

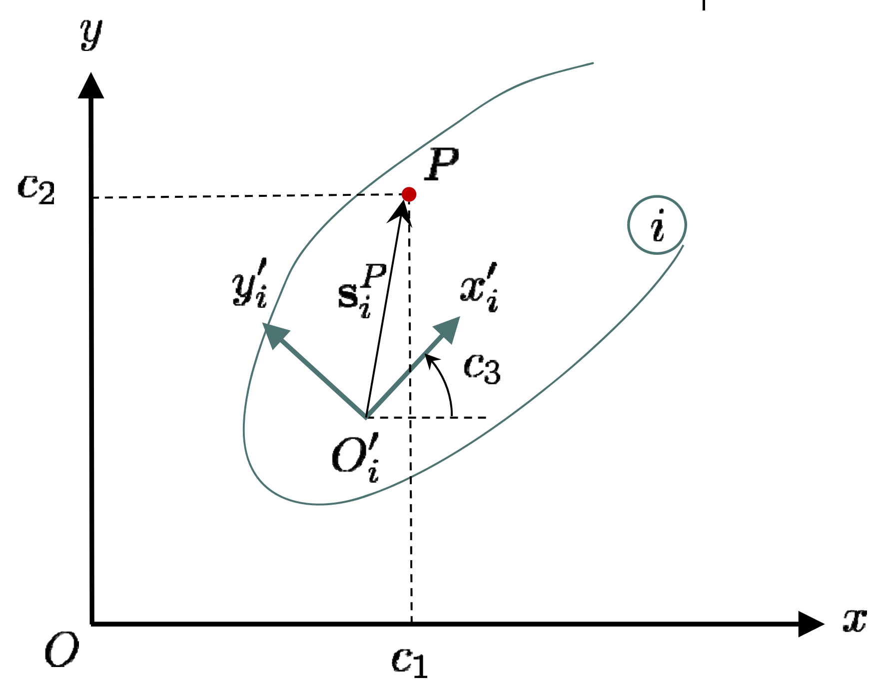
$$x_i^P - c_1 = 0$$

  - y-coordinate of P_i

$$y_i^P - c_2 = 0$$

- Absolute orientation constraint
  - Orientation ϕ of body

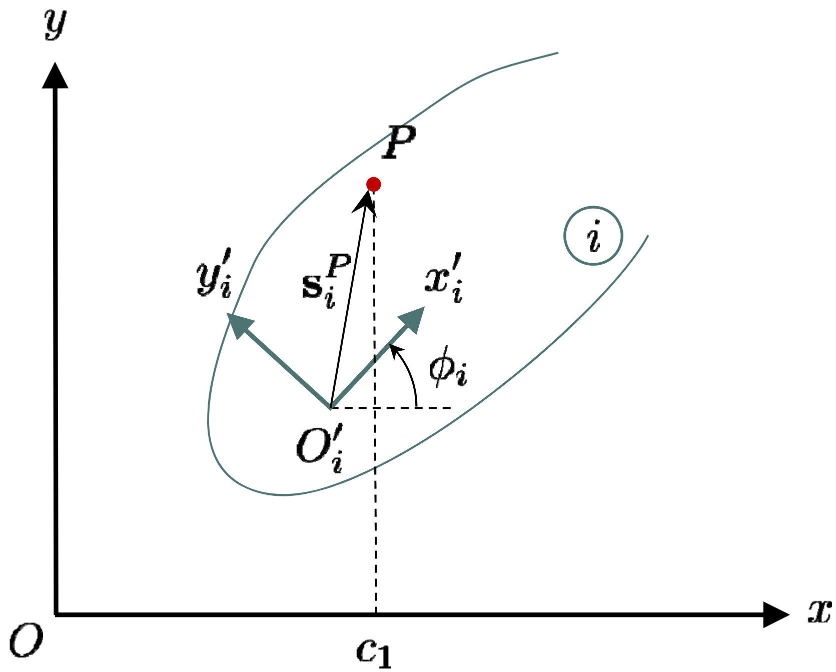
$$\phi_i - c_3 = 0$$



$$\mathbf{r}_i^P = \mathbf{r}_i + \mathbf{A}_i \mathbf{s'}_i^P \quad \Leftrightarrow \quad \begin{bmatrix} x_i^P \\ y_i^P \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} \cos\phi_i & -\sin\phi_i \\ \sin\phi_i & \cos\phi_i \end{bmatrix} \begin{bmatrix} x'^P_i \\ y'^P_i \end{bmatrix}$$

Absolute x-constraint

- Step 1: the absolute x component of the location of a point P on body i stays constant and equal to some known value c_1

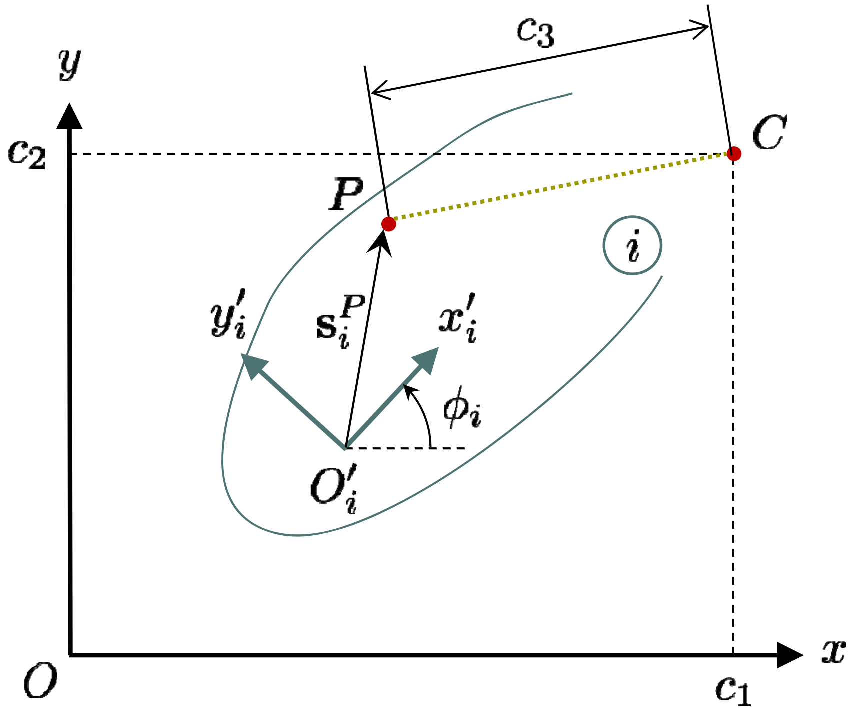


- Step 2: Identify $\Phi^{ax(i)} = 0$
- Step 3: $\Phi_{\mathbf{q}}^{ax(i)} = ?$
- Step 4: $\nu^{ax(i)} = ?$
- Step 5: $\gamma^{ax(i)} = ?$

NOTE: The same approach is used to get the y- and angle-constraints

Absolute distance-constraint

- Step 1: the distance from a point P on body i to a point C defined in the GRF stays constant and equal to some known value c_3



- Step 2: Identify $\Phi^{ad(i)} = 0$
- Step 3: $\Phi_{\mathbf{q}}^{ad(i)} = ?$
- Step 4: $\nu^{ad(i)} = ?$
- Step 5: $\gamma^{ad(i)} = ?$