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Lugre Tire Model for HMMWV

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Abstract

The objective of this report is to provide a short introduction regarding simple tire models that could be used when modeling the military vehicle HMMWV. Drawbacks and advantages associated with the tire models are briefly discussed. The report also introduces a simple three-dimensional tire model that has been implemented to Chrono software [3].

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1. Background

A tire plays an important role in the handling and dynamic performance of a vehicle. In the case of vehicles intended for both road and off-road usage such as a military vehicle HMMWV, tires can be in contact with the unmade surfaces, emphasizing the importance of tire behavior. Such vehicles are typically equipped with all-terrain tires that have stiffer sidewalls as compared to tires used in passenger cars. Stiff sidewalls offer a protection against puncture when traveling off-road. In all-terrain tires, the tread patterns are wide to allow mud to travel from the tire surface. An example of an all-terrain tire that can be used on HMMWV is depicted in figure 1.



Figure 1. A Goodyear tire 37/12.50R17LT (<http://www.goodyear.com/>).

Tires can be modelled in a number of ways in computer simulation [1]. As a tire transmits forces between the road and the rim, contact description between a tire and ground is critical when a tire is modelled. A realistic model can be obtained by considering the contact patch interaction between the tires and soil by employing a finite element method [2]. Detailed finite element models needed in the contact patch description are, however, associated with high computing

costs, making it difficult for finite element based models together with a full vehicle model. It is for this reason that a number of simplified tire models are introduced in the literature.

The objective of this report is to provide a short introduction regarding simple tire models that could be used when modeling the military vehicle HMMWV. Drawbacks and advantages associated with the tire models are briefly discussed. The report also introduces a simple three-dimensional tire model that has been implemented to Chrono software [3].

2. Static friction models

A simple one wheel system is depicted in figure 2. Commonly used simplification in the tire modeling is to assume that tire-ground friction (F_s in figure 2) follows a slip-force relationship described with algebraic functions. This approach provides a friction force applied to the tire as a function of slip rate used as input of the model. Longitudinal slip rate, s , can be defined as follows:

$$s = \begin{cases} s_b = \frac{r\omega}{v} - 1 & \text{if } v > r\omega, \quad v \neq 0 \quad \text{for braking} \\ s_d = 1 - \frac{v}{r\omega} & \text{if } v < r\omega, \quad \omega \neq 0 \quad \text{for driving} \end{cases} \quad (1)$$

where r is the tire radius, ω is angular velocity of tire and v is ground velocity as depicted in Figure 2.

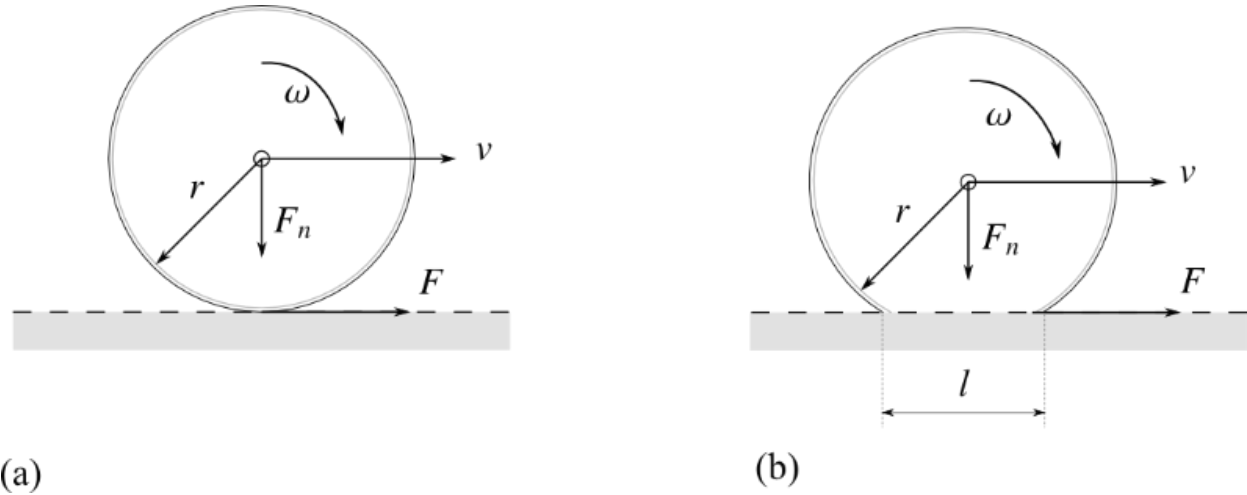


Figure 2. A tire system with a) lumped and b) distributed friction description.

As can be seen from equation 1, the slip rate can have numerical values between 0 and 1. In case of slipping value 0, it is assumed that the tire undergoes pure rolling and, consequently, there is no slipping. Slipping value 1, on the other hand, indicates that there is a full sliding case.

A widely used approach in this category of static models is called the “Magic Formula” [4]. The general form of the “Magic Formula” can be written as follows:

$$F_s = c_1 \sin(c_2 \arctan(c_3 s - c_4 (c_3 s - \arctan(c_3 s)))) \quad (2)$$

where c_1 , c_2 and c_3 are tire parameters of the tire that can be obtained through practical experiments. Accordingly, usage of the “Magic Formula” requires experimental data that can be obtained using an apparatus capable of analyzing tires under particular conditions of constant linear and angular velocity. The “Magic Formula” is easy to program and efficient to solve. The “Magic Formula” provides accurate prediction of friction under a steady-state condition. Unfortunately, a steady-state condition is not common in off-road. The “Magic Formula” is difficult to use at low speeds, another drawback associated with usage of the approach in off-road applications.

The alternative for “Magic Formula” is explained in [5]. In this model, friction between the tire and road is expressed as:

$$F_s = c_1(1 - e^{c_2 v}) - c_3) e^{c_4 v} \quad (3)$$

where $c_1 \dots c_4$ are constants describing the tire. This tire model differs from “Magic Formula” approach in the sense that friction is a function of velocity. A number of additional static tire models can be found in the literature [6].

3. Dynamic friction models

A number of dynamic friction models have been introduced in the literature aiming to capture the transient behavior of the tire-road contact forces under time-varying velocity conditions. Dynamic models can be categorized based on the friction contact description. Dynamic models are called lumped if the friction model assumes a point tire-road friction contact as shown in Figure 2(a). In case of lumped assumption, the mathematical model describing such a friction can be written in a form of an ordinary differential equation that can be solved by a time integration scheme. An alternative to the lumped approach is distributed friction models that accounts for the existence of a contact patch between the tire and the ground. Figure 2(b) depicts the distributed friction model. Distributed friction models can be computationally expensive due to the fact that friction models must be solved in terms of both time and space.

Lumped Lugre model

The Lugre tire model is a commonly used approach based on the description of an elastic bristle at the microscopic level. At the contact point, the tangential force will affect the bristles by deflecting like springs and create the friction force [7, 8]. When a large tangential force is imposed, the bristles deflect excessively and start to slip. A concept of bristle deformation is depicted in figure 3.

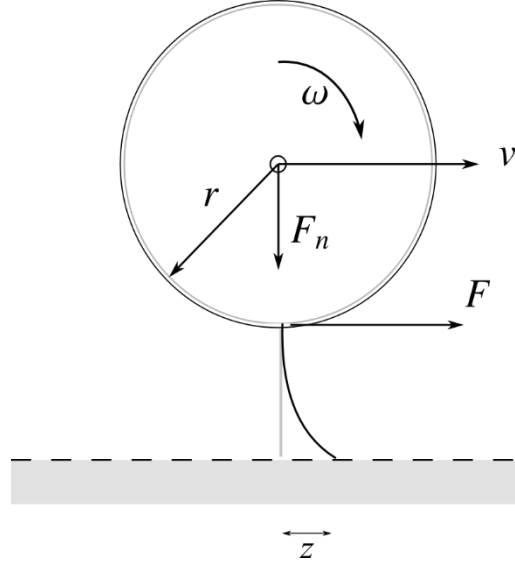


Figure 3. A bristle deformation during an acceleration.

The Luge friction model can be seen as an enhancement of the Dahl model as it accounts for the Stribeck effect. With this approach, many important aspects of friction including stiction, the Stribeck effect, stick slip, zero slip displacement and hysteresis can be accounted. When the tire rotates, the bristle will deflect with respect to time and the deflection of the bristles denoted by z can be evaluated as follows:

$$\frac{dz}{dt} = v_r - \frac{\sigma_0 |v_r|}{g} z \quad (4)$$

where g is friction, σ_0 is the rubber longitudinal lumped stiffness and v_r is the relative velocity between two sliding surfaces defined as follows (see also figure 2)

$$v_r = r\omega - v \quad (5)$$

Friction, g , needed in equation 4 can be written as

$$g = \mu_c + (\mu_s - \mu_c) e^{-\left| \frac{v_r}{v_s} \right|^{\frac{1}{2}}} \quad (6)$$

where μ_c is normalized Coulomb friction, μ_s normalized static friction and v_s is the Stribeck relative velocity. The friction, g , must always be positive and depends on material properties, lubricants, temperature and other factors. Friction force based on the bristle deformation can be expressed as follows:

$$F_s = (\sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v_r) F_n \quad (7)$$

where σ_1 is the longitudinal lumped damping coefficient, σ_2 is viscous relative damping and F_n is the normal force (see figure 2).

The steady-state characteristics of the Lugre can be obtained by setting $dz/dt=0$ and by assuming that translational and angular velocities are constant. Under these assumptions, equations 4 can be written as:

$$z = \frac{g}{\sigma_0} \text{sgn}(v_r) \quad (8)$$

Where sgn is a sign function. Equation 7 can be rewritten as

$$F_s = (g \text{sgn}(v_r) + \sigma_2 v_r) F_n = \left(\left(\mu_c + (\mu_s - \mu_c) e^{-\left| \frac{v_r}{v_s} \right|^{-\frac{1}{2}}} \right) \text{sgn}(v_r) + \sigma_2 v_r \right) F_n \quad (9)$$

Using relations $r\omega s_d = v s_b$ and $s_d = s_b / (s_b + 1)$, equation 9 can be plotted as a function of s (see equation 1) and either v or ω .

Distributed Lugre model

Distributed Lugre model accounts for the existence of an area of contact between the tire and the road as depicted in figure 2(b). The area of contact is a representation of the projection of the tire

part that is touching the ground. By denoting length of contact area as L , the following partial differential equation for the internal friction state $z(\zeta, t)$ can be obtained:

$$\frac{dz}{dt}(\zeta, t)r\omega + \frac{dz}{dt}(\zeta, t)(g|v_r| + \sigma_2 v_r) = v_r - \frac{\sigma_0 |v_r|}{g} z \quad (10)$$

Which can be solved with help of equation 6. In the distributed model, the friction force can be expressed as:

$$dF(\zeta, t) = \left(\sigma_0 z(\zeta, t) + \sigma_1 \frac{dz}{dt}(\zeta, t) + \sigma_2 v_r \right) dF_n(\zeta, t) \quad (11)$$

where $dF(\zeta, t)$ is the differential friction force and $dF_n(\zeta, t)$ is the differential normal force. Equation 10 assumes that the contact velocity within the entire contact patch is equal to v_r . As in the case of the lumped Lugre model, the distributed Lugre model can be expressed in steady state form [6]. It is worth noting that the steady state form of the distributed Lugre friction model can be compared against the ‘‘Magic Formula’’. To shed light on this property, figure 4 shows steady-state dependence on the vehicle velocity in case of braking when the steady-state form of distributed Lugre model and data given in Table 1 is used.

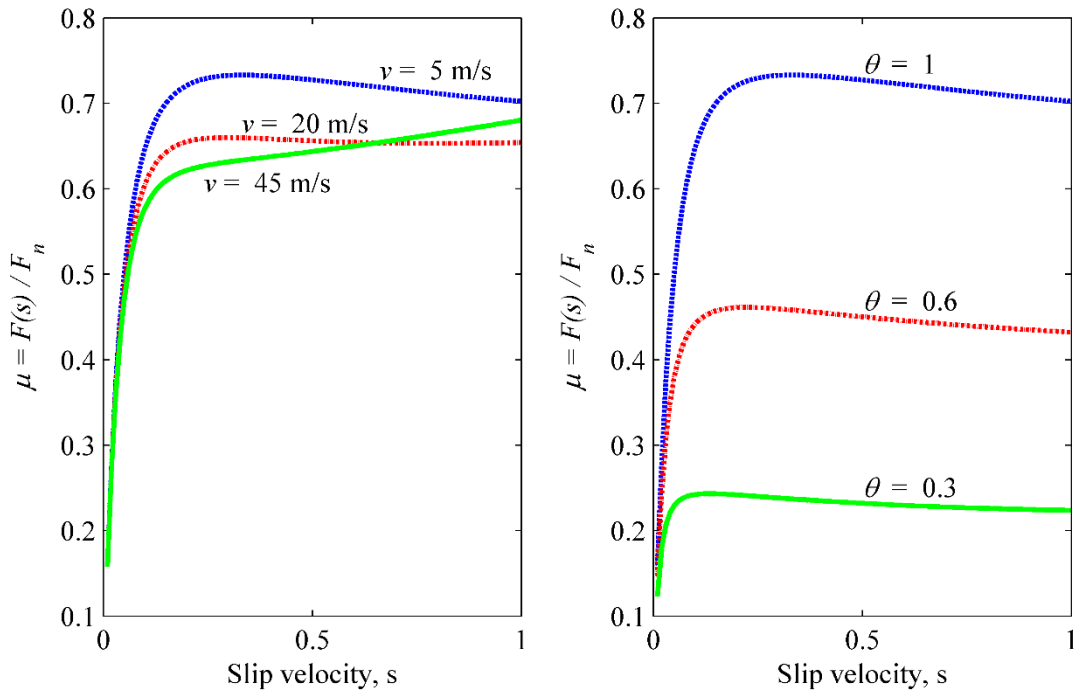


Figure 4. Steady state representation of the lumped LuGre model (during braking).

Table 1. Steady state parameters

Parameter	Value	Units
σ_0	181	[1/m]
σ_2	0.002	[s/m]
μ_c	0.6	[-]
μ_s	1.0	[-]
v_s	3.5	[m]
L	0.2	[m]

4. Chrono implementation of Lugre tire model

To obtain a computationally efficient and accurate tire model, a tire of the HMMWV can be presumed as a series of discs as shown in Figure 5(a). In this approach, disks can be assumed as simple rigid bodies whereas the profile of the ground can be constructed depending on the required environment (off-road profile). In the Lugre tire implementation, all forces are applied to the hub of the tire, denoted as A in figure 5(a).

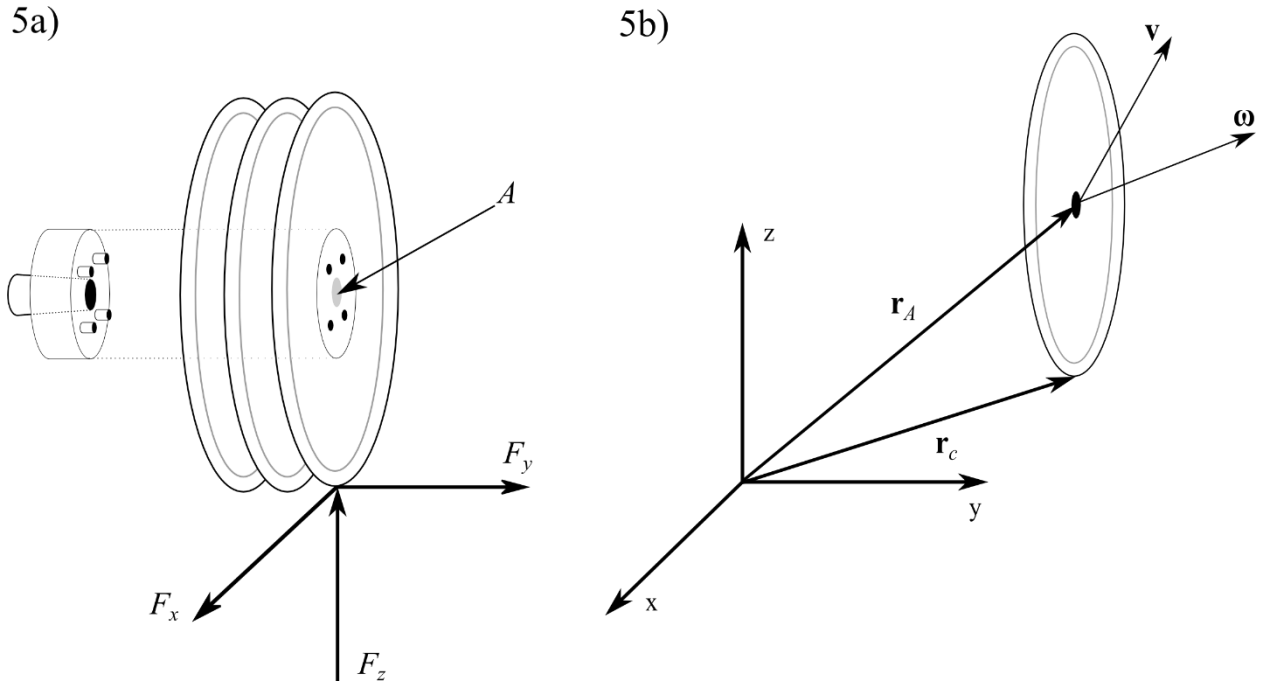


Figure 5. A tire model implemented to Chrono.

Description of one disk is illustrated in figure 5a. In the Chrono implementation, relative velocity between the contact point of the tire and ground is defined as follows:

$$\mathbf{v}_r = \mathbf{v} - \boldsymbol{\omega} \times \mathbf{r} \quad (12)$$

where \mathbf{v} is velocity vector of the tire hub, $\boldsymbol{\omega}$ is the angular velocity vector of the tire and \mathbf{r} is the vector that defines the radius of the tire. Vector \mathbf{r} can be defined by employing positions of disk

center and contact point (vectors \mathbf{r}_A and \mathbf{r}_c in figure 5(b)). Relative velocity as defined in equation 5 could be used in the Lugre friction model. This approach, however, assumes that tire properties in lateral and longitudinal direction are identical. In practice this is not the case and for this reason relative velocity should be expressed in the contact coordinate system, in which x -axis defines disk direction of travelling, y -axis defines disk lateral and z -axis defines ground normal. By denoting the rotation matrix of the contact coordinate system as matrix \mathbf{A} , relative velocities within the contact coordinate system can be written as:

$$\bar{\mathbf{v}}_r = \mathbf{A}^T \mathbf{v}_r \quad (13)$$

Components of vector $\bar{\mathbf{v}}_r$ can be employed to define relative velocities of longitudinal, \bar{v}_{rx} , lateral, \bar{v}_{ry} , and vertical \bar{v}_{rz} directions. Longitudinal relative velocity can be now used in equation 5 to solve bristle deformation according to equation 4. As equation 4 is a linear differential equation, its solution needs to be carried out using a time integration scheme. In terms of practical implementation, it is advisable to solve equation 4 separately from the solution of the vector of generalized coordinates. In the Chrono implementation the time integration was conducted using a trapezoidal method. Friction force in the longitudinal direction can be finally obtained with help of equation 7. Similarly, transverse relative velocity can be used to solve corresponding friction in lateral direction. In the Chrono implementation, friction calculations in the longitudinal and lateral direction are decoupled, making it possible to use different tire parameters in two directions.

In the Chrono implementation, the contact between a disk and ground is described by employing the penalty method. In this approach, the kinematics of collision points are described using a global reference frame. In the penalty method, the contact force at the contact point can be formulated as

$$\mathbf{F} = -K\mathbf{x} - C(\mathbf{n} \cdot \mathbf{v}) \quad (14)$$

where \mathbf{n} is the contact normal, \mathbf{x} is the penetration depth, K is the coefficient of elasticity and C is the damping factor. The dot product of velocity \mathbf{v} and contact normal \mathbf{n} is the component of the relative velocity in the direction of the collision normal. The choice of value for the two coefficients K and C is vital and may result in a range of desired collision response types.

5. Conclusion

This report explains a simple three-dimensional tire model that can be applied to a military vehicle, HMMWV. The tire model employs lumped Lugre friction description and is implemented to Chrono software by assuming a tire to be constructed with a number of interconnected disks. The tire model based on lumped Lugre friction can be expressed in steady state form, allowing it to be compared against methods based on the static friction such as the “Magic Formula.” If experimental tire data is available, the steady state form helps set up the parameters needed for the Lugre tire model.

The implemented tire model could be enhanced by using a distributed Lugre description instead of the lumped approach. This will improve the accuracy with the price of increased computational burden.

6. References

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