Validation of a Single Contact Point Tire Model Based on the Transient Pacejka Model in the Open-Source Dynamics Software ProjectChrono

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Technical Report TR-2014-16

November 19, 2014

Abstract

A single contact point model was implemented in the open-source dynamics engine Chrono to provide a more accurate calculation of input slip quantities to the Magic Formula equations for non steady-state, or transient, slip cases. The model is formulated using a consistent ISO z-up x-forward coordinate frame, where the slip angle was shown to be invariant with the choice of coordinate system, but the lateral force was curve not. Similar to a previous tech report by the author, the results of the model are validated against a commercially available Pac2002 model in Adams/Car. Pure lateral slip and combined lateral/longitudinal slip cases were tested, with slip ranges of $\kappa = \pm 1.0$, and $\alpha = \pm 15$ degrees, with no camber or spin slip. Simulation were performed in 8 seconds, using a fixed integration step size of 0.01 seconds.

It was shown that the single contact point model with the Magic Formula equations implemented in ChronoT gives accurate output for a single tire under pure and combined slip operating conditions when compared to a model in the commercial package ADAMS/Car. Reaction longitudinal and lateral forces, as well as aligning moment curves as a function of input slip rate matched nearly exactly in all cases. Even an extremely transient maneuver, a powered lane change that transitions from 100% skid to 100% longitudinal slip, showed excellent agreement between the plots of the highly non-linear aligning moment as a function of lateral slip angle.

Keywords: Tire Modeling, Tire Simulation, Vehicle Dynamics, Magic Formula, Single Contact Point Tire, Tire Model Validation, ADAMS/Car, Chrono Engine, Open-source software

Contents

1   Introduction

A previous report by the author compared two similar tire models under three slip cases to simulate distinct handling maneuvers. The model implemented the steady-state Magic Formula, as desribed by Pacejka [1], in the open-source dynamics engine Chrono [2], available on github [3]. Comparisons were drawn with the Pac2002 implementation in the commercial software ADAMS/Car, where the tire testrig tool was used to generate tire output. For pure longitudinal and lateral slip cases where the input slip rates were changed gradually, the two models matched nearly exactly. When both longitudinal and lateral slips were varied simultaneously to achieve combined slip, the lateral force and aligning torque outputs began to show non-negligible differences.

Transient effects were seen in the combined slip case, which occurs during common driving scenarios such as a powered lane change, or a sudden turn of the steering wheel. This prompted the implementation of a single contact point tire model, to more accurately calculate slip input values to the steady-state Magic Formula equations. This model becomes especially important during transient events. A similar model is present in ADAMS/Car, which allows for a more precise comparison.

First, the coordinate system and variables used in the single contact point tire model are introduced. The model to calculate slip displacements as Ordinary Differential Equations (ODE) of slip velocities is based on the approach by Pacejka [1].

Comparison of the pure lateral slip case as well as the combined slip case is performed. The pure steady-state Magic formula model from the previous report, as well as the ADAMS/Car tire test rig outputs are used in the comparison.

2   Tire coordinate system and important variables

The tire coordinate system is shown in figure 1, where the local tire axis is defined with x- in the longitudinal direction, y- in the lateral direction, and z- in the vertical direction. It should be noted that this coordinate system differs from that used in the reference model, presented in [1]. The most noticeable difference is that the z-axis now points up, rather than toward the ground, resulting in the y-axis also switching directions. Although the slip angle is invariant with respect to the selection of coordinate system, the definition of the lateral slip rate does switch directions. All velocity and force vectors are either parallel or normal to the flat

ground, and the tire vertical velocity $V_z^C = 0$.

Some of the variables are defined with respect to the global coordinate system, for example the roll angular velocity $\dot{\psi}$. All the variables in figure 1 are described in the following list.

- $\vec{V}^C$ velocity of the wheel center parallel to the ground, global coordinates

- V_x^C x-velocity of the wheel center parallel to the ground, tire coordinates

- V_y^C y-velocity of the wheel center parallel to the ground, tire coordinates

- $\omega_x, \omega_y, \omega_z$ wheel angular velocities, tire coordinates

- r_0 unloaded tire radius

- $\dot{\psi}$ yaw rate

- α lateral slip angle, the angle between V_x^C and $\vec{V}^C$

- γ tire camber angle, the angle between the vertical and the tire spin axis

- ϕ tire spin slip

- κ (not shown) longitudinal slip ratio

2.1 Slip definitions

All quantities in this section are listed in Figure 1. The main slip quantities are the longitudinal slip ratio, κ, lateral slip angle α and camber angle γ. Equations 1, 2 and 3 are referred to as the "input", "steady-state" or "kinematic" slips, as they can be calculated from wheel rigid body state information alone, and can be used directly as input to the Magic Formula equations during steady-state operating conditions. However, these slip values were shown to be inaccurate in situations where the slips do not remain in the steady-state range, which was discussed in the previous technical report by the author.

The longitudinal slip rate κ is a scalar quantity, the ratio of the tire forward slip velocity, $V_{s,x}^C$, over the tire linear velocity magnitude, V_x^C,

$$\kappa = \frac{-V_{s,x}^C}{|V_x^C|} = \frac{r_{\text{eff}}\omega_y - V_x^C}{|V_x^C|} \tag{1}$$
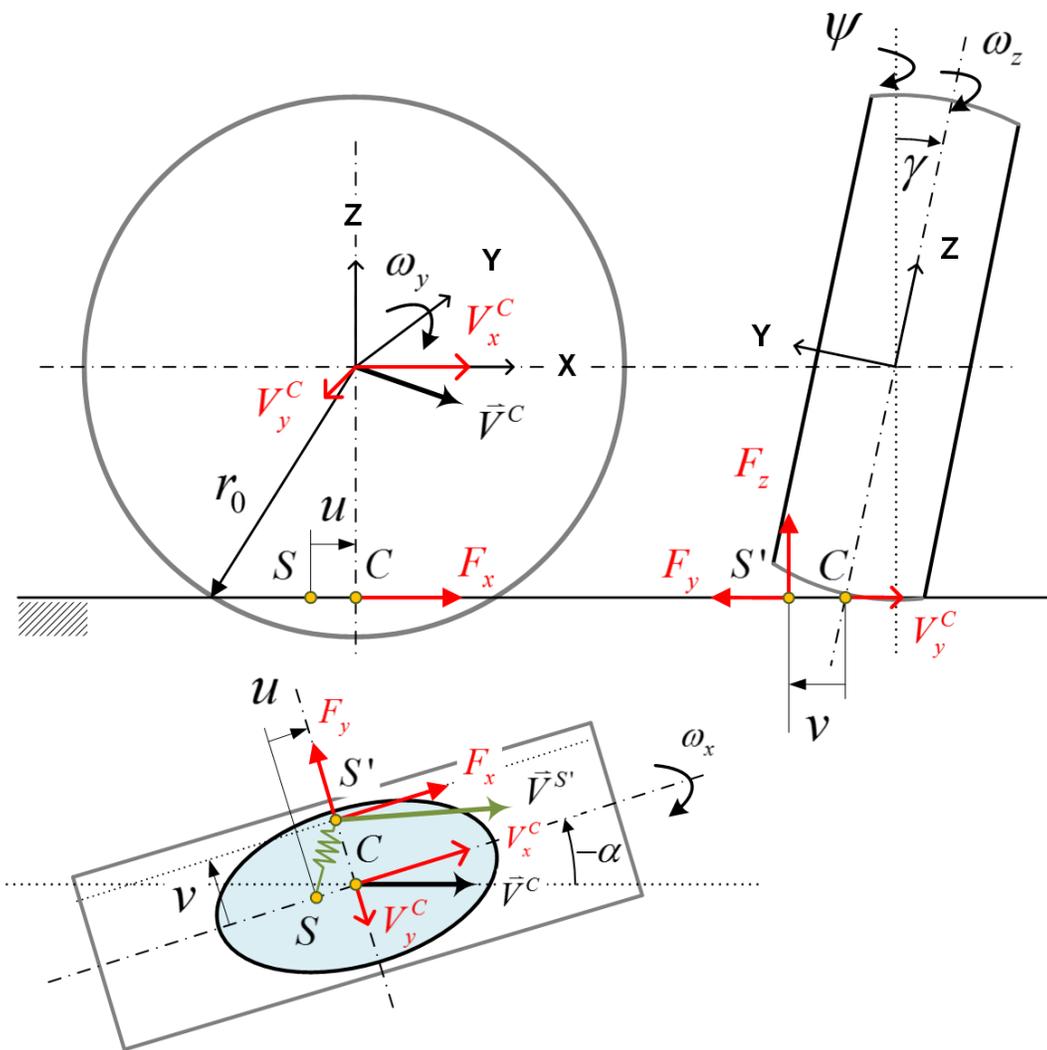
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Figure 1: Tire coordinate system and generalized coordinates. Top Left - right-side view. Top right - rear view. Bottom - top view, normal to road surface

The tangent of the lateral slip angle, α, is used to define the ratio of the tire center linear velocities in the forward and lateral directions, V_x^C and V_y^C, respectively.

$$\tan \alpha = \frac{V_y^C}{V_x^C} \tag{2}$$

Spin slip is the portion of the tire yaw angular velocity that affects the slip angle,

$$\dot{\phi} = \frac{\omega \sin \gamma - \dot{\psi}}{V_x^C} \tag{3}$$

where $\dot{\psi}$ is the tire yaw rate about a vector normal to the ground, and corresponds to the global z-axis in the case of flat ground.

2.2 Magic Formula Equations

Pacejkas Magic Formula is a trigonometric function and has the general form [1]

$$F = F(\alpha, \kappa, \gamma, \phi, F_z) \tag{4}$$

$$M_z = M_z(\alpha, \kappa, \gamma, \phi, F_x, F_y) \tag{5}$$

where F represents either the longitudinal or lateral force, F_x, F_y, and M_z is the aligning moment. The characteristics of each curve are heavily dependent on empirically derived constants, which are found experimentally. Overturning moment, M_x and rolling resistance M_y, can also be computed using the same type of equation, but are omitted as this study focuses on the main contributions to a tire's performance during handling maneuvers, namely F_x, F_y and M_z.

When tire forward velocity V_x^C goes to zero, the kinematic definitions of slip are undefined. Even with an acceptable tire velocity, any maneuver performed too quickly leads to inaccurate results. This problem is exasperated when multiple input slip rates are changed simultaneously. A solution is to compute the input slip values using a more sophisticated model, which considers the compliance of the tire carcass very simply in two dimensions, and and avoids the issues from using equations 1 - 3.

3 Single contact point tire model description

The single contact point tire model is an extension of the Pacejka Magic Formula for non-steady or transient driving maneuvers. Either linear or non-linear models are available depending on the frequency range of operation, and in this section both are described and implemented in the Chrono class ChPacejkaTire. This model provides improved calculation of slip values and is used to calculate the input slip quantities to the Magic Formula equations to more provide more realistic reaction forces and moments on the tire.

Consider a step input to the steering wheel; the steering wheel is connected to the wheel spindle through a series of rigid bodies and idealized kinematic constraints, resulting in an instantaneous change in α with steer input. In reality the slip values in the contact patch do not change instantaneously, but transition to a new value due to the compliance of the tire carcass. Tire carcass stiffness and relaxation lengths determine the compliance characteristics of the tire. Single contact point models are accurate to frequencies of up to 8 and 15 Hz for the linear and non-linear models, respectively.

3.1 Linear single contact point model

The models described in this section follow those found in Pacejka [1], with the notable exception of the coordinate system change to a z-up orientation, resulting in the y-axis switching directions. Longitudinal and lateral velocities of the tire center, V_x^C and V_y^C, are projected down to the road surface to the point C, shown in figure 1. Point C is located on a rigid disk centered along the centerline of the tire, with an inclination angle γ, projected down along the tire Z-axis to the intersection with the road surface.

Spring displacements are found in terms of the longitudinal u and lateral v components. The point S' is constrained to only translate parallel the tire y-axis, while staying in contact with the ground. Point S is constrained to only translate along the tire x-axis which is parallel to the ground. Forces generated by the displacement of the tire carcass (represented by displacements u and v) are in equilibrium with the forces generated by the input slip definitions as linear functions of stiffness with respect to each input slip quantity. The input slip definitions can be expressed in terms of slip velocities, and is related to the time rate of change of the displacements u and v of the contact spring as

$$\frac{\mathrm{du}}{\mathrm{dt}} = V_{s,x}^S - V_{s,x}^C \tag{6}$$

$$\frac{\mathrm{dv}}{\mathrm{dt}} = V_y^{S'} - V_y^C \tag{7}$$

where $V_{s,x}^C = V_x^C - \omega_y r_{\mathrm{eff}}$ is the longitudinal slip velocity for the tire center point.

Introduce the longitudinal tire stiffness for the spring between points S and S' as C_{Fx}, and the longitudinal slip stiffness $C_{F\kappa}$, to define the longitudinal relaxation length,

$$\sigma_\kappa = \frac{C_{F\kappa}}{C_{Fx}} \tag{8}$$

Equilibrium of forces can be interpreted from the slip rate or the slip displacement view, as:

$$F_x = \kappa C_{F\kappa} = C_{Fx} u \tag{9}$$

Assume $\kappa \approx \frac{-V_{s,x}^C}{|V_x^C|}$, substitute into equation 9 to solve for the longitudinal slip velocity of the contact point S,

$$V_{s,x}^C = \frac{-u|V_x^C|}{\sigma_k} \tag{10}$$

This is inserted into the rate of change of longitudinal slip in the top of equation 7 to give the linear first order ODE for the longitudinal slip displacement as

$$\frac{\mathrm{du}}{\mathrm{dt}} = -V_{s,x}^C - \frac{|V_x^C| u}{\sigma_\kappa} \tag{11}$$

Introduce the lateral tire stiffness at road level for the spring between points S and S' as C_{Fy}, and also the tire carcass stiffness to cornering as $C_{F\alpha}$, to define side slip relaxation length as a ratio of these two quantities,

$$\sigma_\alpha = \frac{C_{F\alpha}}{C_{Fy}} \tag{12}$$

Equilibrium of forces from the lateral slip rate or the lateral slip displacement leads to,

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$$F_y = \tan(\alpha)C_{F\alpha} = C_{Fy}v \tag{13}$$

For small lateral slip angles, $\tan(\alpha) \approx \frac{V_y^C}{|V_x^C|}$, which is substituted into the equation for lateral displacement, the bottom of equation 7, leading to a linear first order ODE,

$$\frac{\mathrm{d}v_\alpha}{\mathrm{dt}} = V_y^C - \frac{|V_x^C|v}{\sigma_\alpha} \tag{14}$$

Wheel camber, γ, is assumed to induce a lateral slip angle α', the force equilibrium in the lateral direction is

$$F_y = C_{Fy}v_\gamma = C_{F\gamma}\gamma + C_{F\alpha}\alpha' \tag{15}$$

where $C_{F\gamma}$ is the tire carcass stiffness to camber angle. The linear first order ODE for the change in lateral slip deflection with γ is

$$\frac{\mathrm{d}v_\gamma}{\mathrm{dt}} = \frac{C_{F\gamma}}{C_{F\alpha}}|V_x^C|\gamma - \frac{|V_x^C|v_\gamma}{\sigma_\alpha} \tag{16}$$

Also consider the effect of camber on the tire spin, ϕ,

$$\phi = \frac{-1}{V_x^C}[\dot{\psi} - (1 - \epsilon_\gamma)\omega_y \sin(\gamma)] \tag{17}$$

where ϵ_γ is the camber reduction ratio. Spin also contributes to the lateral deflection,

$$\frac{\mathrm{d}v_\phi}{\mathrm{dt}} = \frac{C_{F\phi}}{C_{F\alpha}}|V_x^C|\phi - \frac{|V_x^C|v_\phi}{\sigma_\alpha} \tag{18}$$

In the validation exercises carried out, γ was set to zero to match the output of the Adams/Car tire test rig, and therefore the contributions from camber and spin on lateral slip, equations 16 and 18 can be neglected.

3.2   Non-Linear single contact point model

The solutions to the ODEs for $u, v_\alpha, v_\gamma, v_\phi$ are used to find modified values for the slip and spin used as input to the Magic Formula equations,

$$\kappa' = \frac{u}{\sigma_\kappa} \tag{19}$$

$$\alpha' \approx \tan(\alpha') = \frac{v_\alpha}{\sigma_\alpha} \tag{20}$$

$$\gamma' = \frac{C_{F\alpha} v_\gamma}{C_{F\gamma} \sigma_\alpha} \tag{21}$$

$$\phi' = \frac{C_{F\alpha} v_\phi}{C_{F\phi} \sigma_\alpha} \tag{22}$$

where α' is valid for small values of slip. These values are substituted for the kinematic slips found from the wheel rigid body state alone, and allow a much closer comparison to the Pac2002 tire model used for validation. The reaction forces and moments are calculated with the same Magic Formula equations that were presented in the previous report. This linear model is valid up to frequencies of 8 Hz.

At low velocities, especially starting from a stop, can lead to undamped vibrations in the response. Further, there is still a forward velocity term in the denominator of the ODE for spin slip, equation 18, which leads to non-realistic values of ϕ at low forward velocities.

The result of calculating the forces based on slip velocities leads to the tire behaving as a damping element. When the tire is at stand-still or switching directions (at low velocity), the tire should act more like a spring force element. An artificial damper can be used below a certain cutoff forward velocity, V_{low}; typically the method by Besselink is used. If the tire velocity $|V_x^C|$ drops below this cutoff a damping component will be subtracted to all the input slip values,

$$D_{low} = 385(1 + \cos(\pi |V_x^C|/V_{low})) \tag{23}$$

The slip values in equation 22 can be modified, for example the slip angle is changed to

$$\alpha' = \left(\frac{v_\alpha}{\sigma_\alpha} - \frac{D_{low}}{C_{F\alpha}} V_{x,y}^C \right) \tag{24}$$

The same is done for the modified longitudinal slip κ'.

To increase the accuracy of higher frequency events up to 15 Hz, the relaxation lengths needs to be solved iteratively, as they are dependent on the modified slip angle α' as,

$$\frac{\mathrm{d}\tan(\alpha')}{\mathrm{d}t}\sigma_\alpha + |V_x^C|\tan(\alpha') = -V_y^C \tag{25}$$

where $\sigma_\alpha = \frac{\partial F_y}{C_{Fy}\partial \tan\alpha'}$.

This interprets the growth of the sliding region of the contact patch at higher lateral slip angles.

Two more modifications can be made to help certain cases become more realistic. First, there is a non-lagging portion of the lateral force due to camber having an instant effect from deforming the shape of the tire belt in the radial cross section. Second, tire belt mass inertia effects can have an effect on the aligning moment at higher wheel spin rates.

4   Model validation

ADAMS/Car is a widely used and validated commercial multibody dynamics simulation software tool, and has an implementation of the Pac2002 model that includes the Magic Formula steady-state equations as well as transient single contact point tire models. It will be used to compare and validate the performance of the single contact point tire model implemented in ChronoT. It additionally can consider the contact patch dynamically by associating a mass with the contact point S' in figure 1, which may cause slight differences when compared to this model.

It should be noted that the implementation of the Pac2002 model in Adams/Tire does not exactly follow the approach described by Pacejka [1], for example the lateral and longitudinal relaxation lengths are calculated quite differently. Further, the Adams/Car model typically discards terms that are a function of second order inputs. To aid in making the validation more comparable, the Macic Formula equations and single contact point transient slip models in ChronoT were modified according to the Pac2002 model described in the Adams/Car documentation. Beware the typo in that document describing the transient behavior in Pac2002, the extremely important equation for longitudinal relaxation length should have a negative in the exponential (Eq. (109) in the 2013 Adams/Car documentation).

The single contact point tire model was implemented in the ChronoT class ChPacejkaTire and used for the validation studies. Two slip scenarios are used for comparison, pure lateral and combined slip. Pure longitudinal slip was omitted due to good agreement shown using only the steady-state Magic Formula in the previous report. It was tested and showed exact agreement between the two models. The simulation parameters of the ADAMS/Car tire test rig are described,
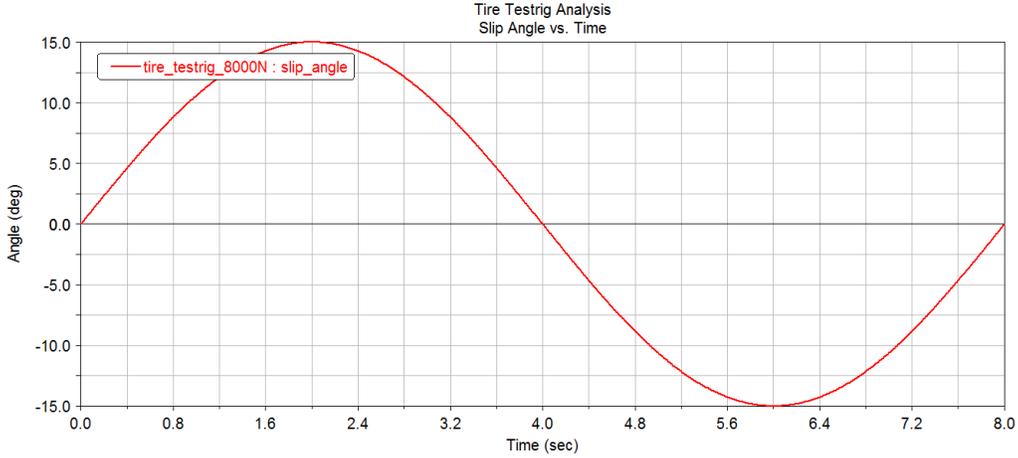
Figure 2: Lateral slip angle vs. time as a sine wave in ADAMS/Car

and a similar test is run using the model in ChronoT. Outputs are recorded and compared for forces and moments as a function of slip.

Common for both simulations are:

- A tire parameter file corresponding to a 235 60R 16 type tire is used.

- Tire vertical load is 8000 N.

- Maneuvers are performed in 8 seconds, with 800 steps and a constant step size, 0.01.

4.1 Pure lateral slip case

Pure lateral slip occurs when the wheel is freely spinning, $\kappa \approx 0$. The tire center velocity magnitude, $|\vec{V}^C|$, remains constant at 16.6 m/s while the wheel is turned. Initially, the Adams/Car tire test rig tire is facing straight forward with lateral slip angle $\alpha = 0$. The wheel is swept to the right until $\alpha = \frac{\pi}{12}$ radians, then to the left to $\alpha = \frac{-\pi}{12}$, and then back to zero. The lateral slip angle is varied in time as a sine curve, shown in figure 2. Camber angle is usually assumed for small values of lateral slip to be $\gamma = \frac{\alpha}{10}$, but in ADAMS/Car it is held constant at zero. For consistency, camber is matched in the ChronoT model.

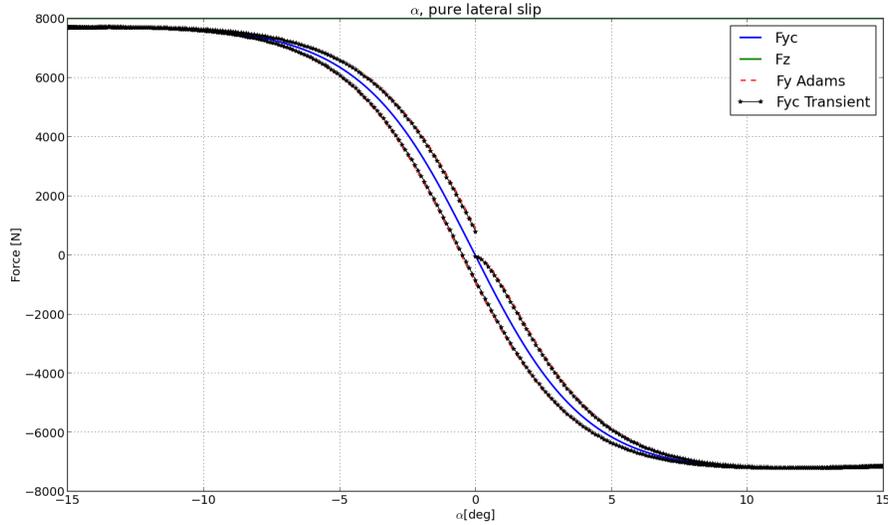Lateral force F_y as a function of lateral slip angle is shown in figure 3,

Figure 3: Lateral force vs. lateral slip angle, pure lateral slip

It can be seen in figure 3 that the lateral tire force results in a near exact match between the Adams and ChronoT output. Adams/Car calculates both relaxation lengths as a function of the vertical load as

$$\sigma_\kappa* = F_z(P_{Tx1} + P_{Tx2}df_z)e^{-P_{Tx3}df_z}(R_0/F'_{z0})\lambda_{\sigma_\kappa} \tag{26}$$

$$\sigma_\alpha* = P_{Ty1}\sin\left(2\arctan\left(\frac{F_z}{P_{Ty2}F_{z0}}\right)\right)(1 - P_{Ky3}|\gamma_y|)R_0\lambda_{\sigma_\alpha} \tag{27}$$

Compare this to the relaxation lengths according to Pacejka [1], which requires the knowledge of the tire carcass stiffness with respect to displacements of the tire contact point, C_{Fx} and C_{Fy},

$$\sigma_\kappa = \frac{C_{F\kappa}}{C_{Fx}} = F_z(P_{Kx1} + P_{Kx2}df_z)e^{P_{Kx3}df_z}\lambda_{Kx\kappa}/C_{Fx} \tag{28}$$

$$\sigma_\alpha = \frac{C_{F\alpha}}{C_{Fy}} = |P_{Ky1}F_{z0}\sin\left(2\arctan\left(\frac{F_z}{P_{Ky2}F_{z0}}\right)\right)\zeta_3\lambda_{Ky\alpha}|/C_{Fy} \tag{29}$$

Using a vertical test load $F_z = 8000$ N in equation 27 results in Adams/Car calculating relaxation lengths for the longitudinal and lateral directions of $\sigma_\kappa =$
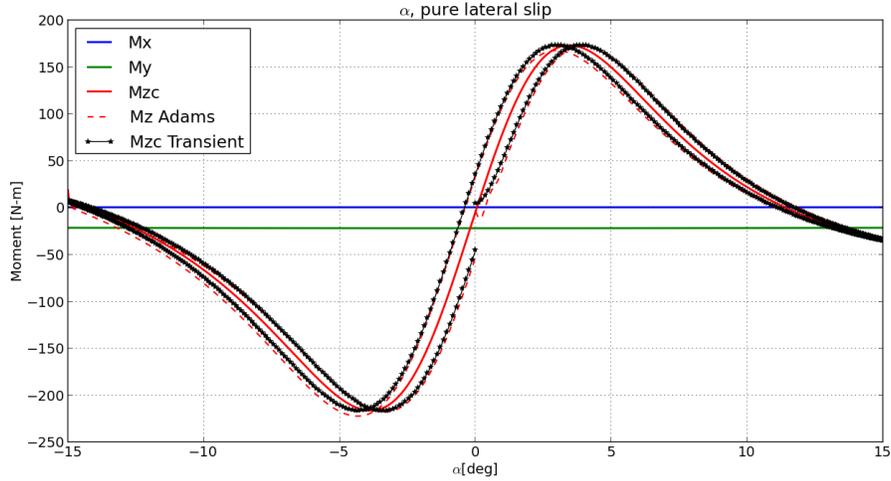
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Figure 4: Aligning torque vs. lateral slip angle, pure lateral slip.

1.29 and $\sigma_\alpha = 0.725$. To match the Adams/Car values of relaxation lengths, the tire carcass stiffness values in equation 29 need to be set to $C_{Fx} = 161,000[N/m]$ and $C_{Fy} = 144,000[N/m]$.

Aligning torque M_z as a function of lateral slip angle is shown in figure 4, where the results now match nearly exactly.

In the ChronoT model it is possible to force the longitudinal slip rate to zero, resulting in much smaller longitudinal force that results only from the horizontal and vertical offsets of the longitudinal force Magic Formula Equation. For the pure lateral slip validation case, Adams/Car reported an average value of $\kappa = -0.005$, resulting in a near constant $F_x = -75$ N, needed to balance the rolling resistance of the tire. This would slightly lessen the lateral combined reaction force calculation, but may have a non-negligible effect on the aligning moment because the albeit small longitudinal force is applied to a large moment arm when the lateral forces are also large. As shown in figure 1, a left turn due to $-\alpha$ causes the contact point S' to move in that direction laterally, and vice versa. Due to a constant negative longitudinal force, when the contact point is to the right of the wheel (positive α) the aligning moment will reduced, and the moment is increased on a left turn. This intuitive difference between the aligning moment curves can be seen in figure 4, where the ChronoT aligning moment begins slightly above that from Adams/Car, but as the right turn is executed, the Adams/Car curve begins to dip below the ChronoT results. Due to contact-point inertia, this trend continues as

the right turn transitions to the left turn. However, as the left turn is executed, the Adams/Car aligning moment begin to increase with respect to the ChronoT curve, which is expected. There is an expected symmetry in this difference in aligning moment values since the longitudinal force acts for an equal amount of time in each moment direction due to the equal time for each right and left turn. The overall effect is that the difference between the ChronoT and Adams/Car model should go to zero at the end of the maneuver, which can be seen in figure 4 as the curves match exactly again at $\alpha = 0$ as the left turn is completed.

4.2   Combined slip case

The lateral slip angle event described in the previous section is combined with a longitudinal slip ratio varied over the range $\kappa = \pm 1$. In this case, it is a simple linear progression from $\kappa = -1$ to $\kappa = 1$, so the value of the input longitudinal slip is proportional to the simulation time. Longitudinal force, F_x, is shown in figure 5.

Results match very closely, which is not surprising considering the longitudinal force showed the least deviation from the steady-state curve for the combined slip case. Differences are seen as κ is close to zero, where the more advanced Pac2002 tire model may have switched to a dynamic contact point model since the sign of both the lateral and longitudinal forces switch half-way through the simulation, when $\kappa = 0$, creating a very transient region for the slip calculations.

Lateral force F_y as a function of lateral slip angle for the combined slip case is shown in figure 6. The lateral force is surprisingly comparable to the ADAMS/Car results in not only curve shape and peak forces, but the phase-lag effect is almost an exact match also. A huge improvement can be seen when calculating lateral force using transient slips rather than the kinematic slips as input to the steady-state Magic Formula, 'Fyc' in figure 6.

Aligning torque M_z as a function of longitudinal slip for the combined slip case is shown in figure 7. It is predicted that the aligning moment, being largely a function of the lateral and longitudinal forces, will be a near match in all places except for the areas near $\kappa = 0$, due to the slight deviations of F_x and F_y. The individual contributions of each force to the aligning moment are also plotted, where it can be seen that the lateral force contribution, 'Mz,y', seems to match the shape of the results from Adams/Car; however, the overall aligning moment in ChronoT is slightly larger just to the left of $\kappa = 0$, due to the differences in the longitudinal force occurring in that area. The shapes of the curves match closely outside of the problem area, and with some small changes to the longitudinal force
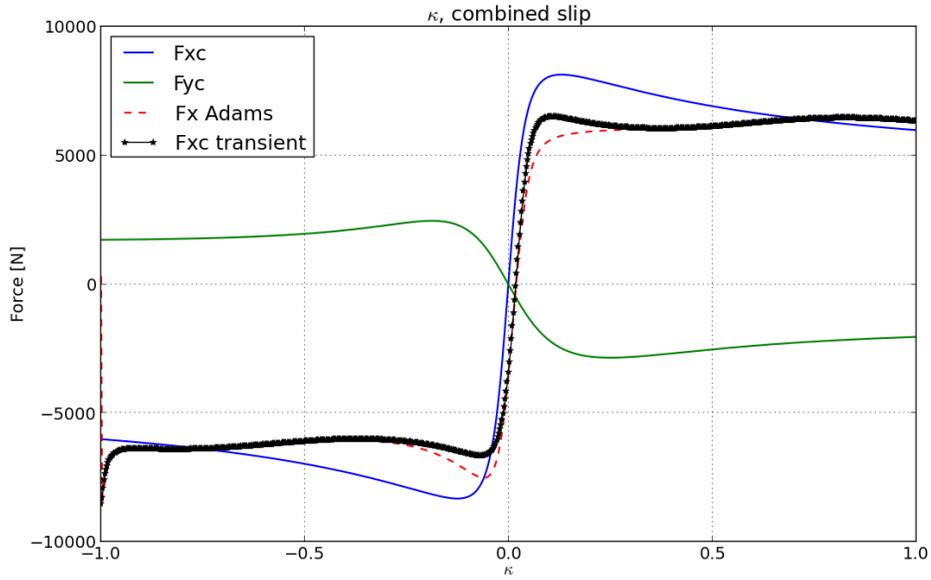
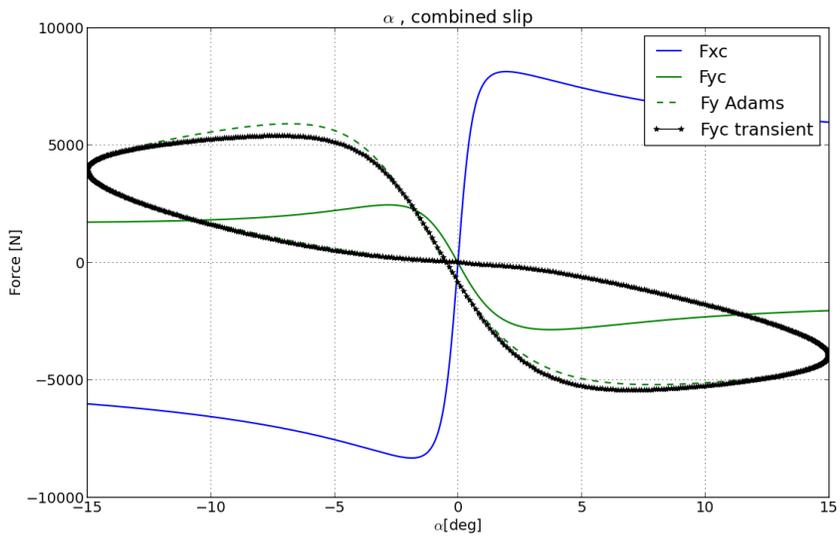Figure 5: Longitudinal force vs. longitudinal slip, combined slip

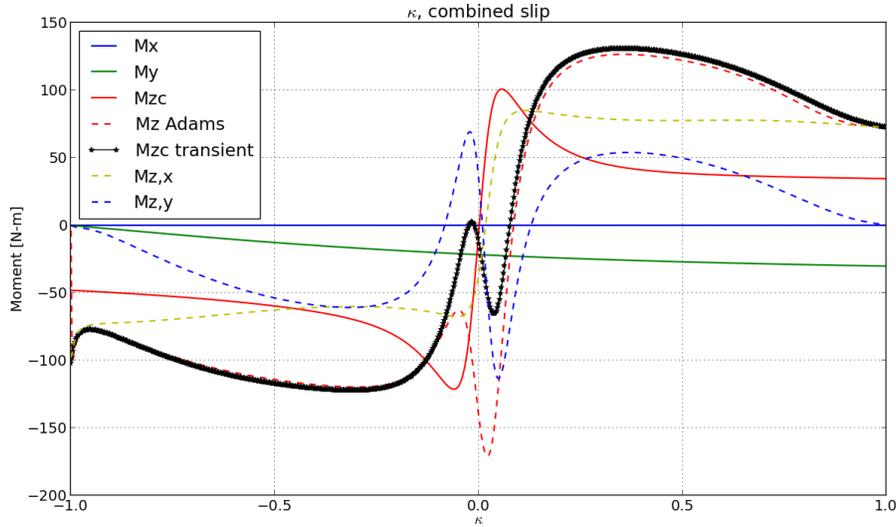Figure 6: Lateral force vs. lateral slip angle, combined slip

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Figure 7: Aligning torque vs. lateral slip angle, combined slip

calculated for combined slip, a near exact match would be expected.

A closer inspection of the lateral slip angle used for the Adams/Car test rig reveals the curve is not a true sinusoid, figure 9. Further studies should be run where the time step taken by Adams/Car is exactly matched in order to easily use the input slips reported by Adams/Car rather than making a closer approximation as was done in this report.

Finally, the combined lateral and longitudinal forces can be plotted to ensure they do not exceed physically realistic values. Results for the steady-state, single contact point and Adams/Car tire simulations are shown in figure 10. A clear vertical (F_y) offset can be seen as the forces transition from the third to second quadrants.

A similar relationship is seen when plotting the aligning moment with the longitudinal force, figure 11. The vertical offset between the ChronoT and Adams/-Car lateral force in figure 10 during the transition between quadrants is manifested as a lower aligning torque as seen in figure 11.

Tire Testrig Analysis
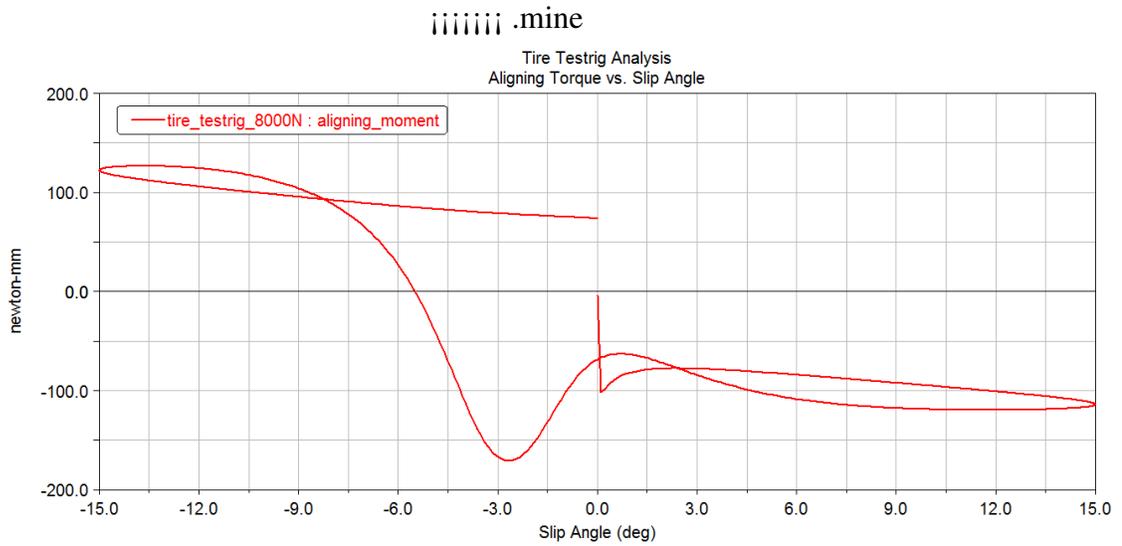Aligning Torque vs. Slip Angle

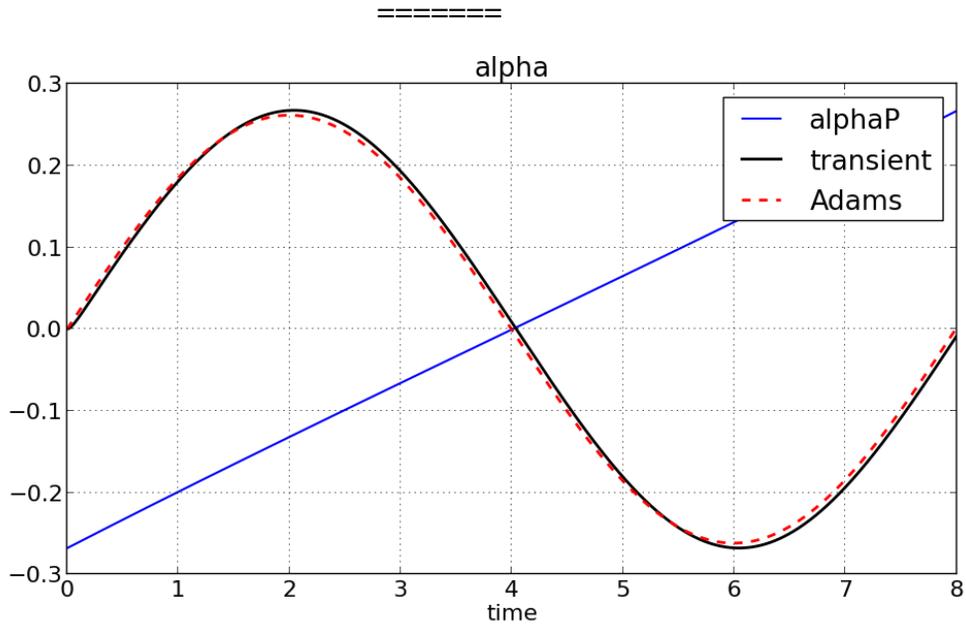Figure 8: ADAMS/Car tire aligning torque vs. lateral slip angle, combined slip case

Figure 9: lateral slip angle is not exactly matched by using a sine wave, combined slip
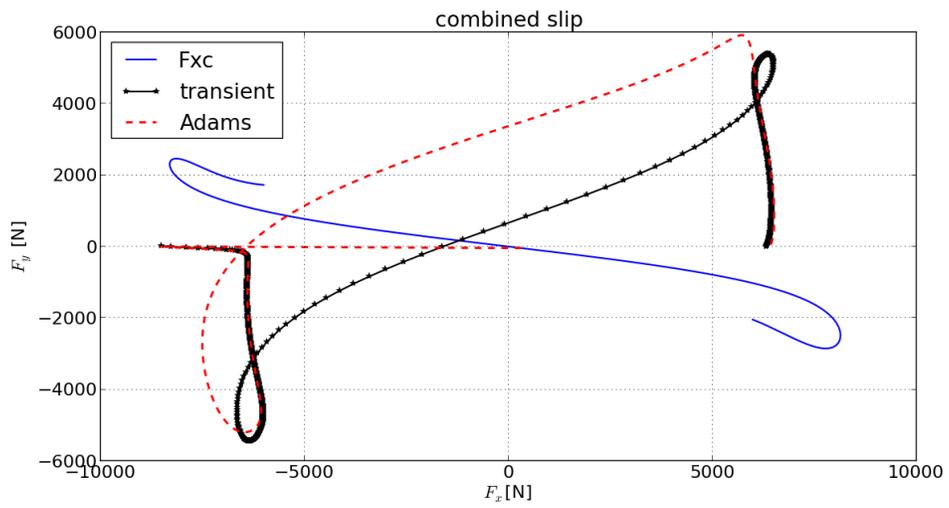
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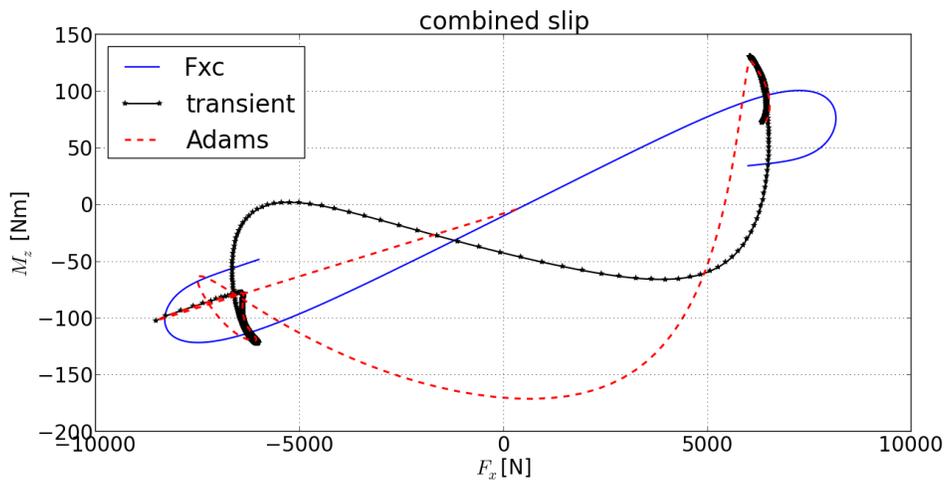Figure 10: longitudinal vs. lateral force, combined slip

Figure 11: longitudinal force vs. aligning moment, combined slip

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5 Conclusion

Simulating vehicle dynamics models in a virtual environment requires considering any possible combination of driver inputs, including: throttle, braking and steer angle. Changes in the driver inputs directly affect the slip rate, slip angle, camber and spin rate of the wheel rim and tire. It has been shown by Pacejka [1], among others, that the tire reaction forces and moments can be correlated to these slip values, given a flat and rigid road surface where the input slip values are not changed too quickly. A previous technical report documented the output from the Magic Formula equations implemented in the ChPacejkaTire class in ChronoT, using kinematic (i.e., steady-state) slip inputs, for three different slip maneuver cases. The results were validated with output from the Adams/Car Pac2002 tire model, which indicated that quickly performed maneuvers or combined changes to input slip values do not fall in the steady-state range.

As suggested by Pacejka [1], tire carcass effects can be considered using a single contact point tire model that relates the tire carcass stiffness with a combined displacement to the stiffness with slip input. The solution of a pair of ODEs results in the longitudinal and lateral contact point displacements, which are used to calculate the input slips α', κ', γ' and ϕ'. These transient slips are used as inputs to the steady-state Magic Formula equations, described in the previous report and also in Chapter 4 of Pacejka [1]. Operation at low forward velocities and transient changes to input slip (up to frequencies of 8Hz) are now possible, given the lateral slip angle does not become too large.

In the previous report, pure lateral and combined slip cases showed non-negligible differences between the ADAMS/Car and steady-state Magic Formula implemented in ChronoT. Validation tests are re-run for these two cases using the single contact point model implemented in the ChPacejkaTire class in ChronoT, and a comparison of output forces and moments show marked improvement in all cases. A majority of the curves match nearly exactly, and the model can handle input frequencies in the range of 8 Hz, and can be extended to 15 Hz by considering the variation of relaxation lengths with the slip rate. Here, slight differences between the results can be attributed to slight differences between the input slip values, or perhaps due to Adams/Car switching to a more advanced model during rapid changes in calculated forces.

The single contact point model with the Magic Formula equations implemented in ChronoT gives accurate output for a single tire under pure and combined slip operating conditions when compared to a model in the commercial package ADAMS/Car. Slips tested were in the range $\kappa = \pm 1.0$, and $\alpha = \pm 15$

degrees, with no camber or spin slip. Each simulation was performed in 8 seconds, using a fixed integration step size of 0.01 seconds. Further improvements to the tire model accuracy can be achieved by considering the contact point to have mass, leading to a set of second order ODEs that relate the contact point accelerations to the slip displacements used to find the transient slip values.

References

[1] H. Pacejka. *Tire and vehicle dynamics*. Elsevier, 2005.

[2] H. Mazhar, T. Heyn, A. Pazouki, D. Melanz, A. Seidl, A. Bartholomew, A. Tasora, and D. Negrut. Chrono: a parallel multi-physics library for rigid-body, flexible-body, and fluid dynamics. *Mech. Sci.*, 4(1):49–64, 2013.

[3] A. Tasora, D. Negrut, and et al. Project chrono. github.com, 2014.