SBEL Simulation Based Engineering Lab
University Of Wisconsin - Madison

An Analysis of Primal-Dual Interior Point Method for Computing Frictional Contact Forces in a Differential Inclusion-based Approach for Multibody Dynamics

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8/14/2014

Motivation

Background - Differential Variational Inequality approach (DVI)

kinematic constraints $\boldsymbol{g}(\boldsymbol{q}, t) = \mathbf{0}$

E.O.M

$$\underbrace{\boldsymbol{M}(q)\,\dot{\boldsymbol{v}}}_{\text{mass} * \text{acceleration}} = \underbrace{\boldsymbol{f}(t, \boldsymbol{q}, \boldsymbol{v})}_{\text{applied force}} - \underbrace{\boldsymbol{g}_q^T(\boldsymbol{q}, t)\,\lambda}_{\text{reaction force}} + \sum_{i=1}^{N_c} \underbrace{\left(\hat{\gamma}_{i,n}\boldsymbol{D}_{i,n}^T + \hat{\gamma}_{i,u}\boldsymbol{D}_{i,u}^T + \hat{\gamma}_{i,w}\boldsymbol{D}_{i,w}^T\right)}_{\text{contact force}}$$

solved through **Coulomb friction model**

$$0 \leq \Phi_i(\boldsymbol{q}, t) \perp \hat{\gamma}_{i,n} \geq 0 \qquad i = 1, 2, \ldots, N_c$$

$$(\hat{\gamma}_{i,u}, \hat{\gamma}_{i,w}) = \underset{\sqrt{\gamma_{i,u}^2 + \gamma_{i,w}^2} \leq \mu_i \hat{\gamma}_{i,n}}{\arg\min} \left(\gamma_{i,u}\boldsymbol{v}^T\boldsymbol{D}_{i,u} + \gamma_{i,w}\boldsymbol{v}^T\boldsymbol{D}_{i,w}\right)$$

gap: $\Phi_i(\boldsymbol{q}, t)$

normal contact force: $\hat{\gamma}_{i,n}$

# of contacts: N_c

tangential contact forces: $(\hat{\gamma}_{i,u}, \hat{\gamma}_{i,w})$

Friction cone: $\sqrt{\gamma_{i,u}^2 + \gamma_{i,w}^2} \leq \mu_i \hat{\gamma}_{i,n}$

friction coefficient: μ_i

Background - Differential Variational Inequality approach (DVI)

Discretized over time with a stabilization term:

stabilization term

normal contact force impulse

$$0 \le \left(\frac{1}{h}\Phi_i\left(\boldsymbol{q}^{(l)}, t\right) + \boldsymbol{D}_{i,n}^T \boldsymbol{v}^{(l+1)} - \mu_i \sqrt{\left(\boldsymbol{D}_{i,u}^T\boldsymbol{v}^{(l+1)}\right)^2 + \left(\boldsymbol{D}_{i,w}^T\boldsymbol{v}^{(l+1)}\right)^2}\right) \perp \gamma_{i,n} \ge 0$$

time step

$$\left(\gamma_{i,u},\gamma_{i,w}\right) = \underset{\sqrt{\gamma_{i,u}^2+\gamma_{i,w}^2}\le\mu_i\gamma_{i,n}}{\arg\min} \left(\gamma_{i,u}\boldsymbol{v}^{(l+1)^T}\boldsymbol{D}_{i,u} + \gamma_{i,w}\boldsymbol{v}^{(l+1)^T}\boldsymbol{D}_{i,w}\right)$$

tangential contact force impulses

Background - Differential Variational Inequality approach (DVI)

Form into a **cone** complementarity problem:

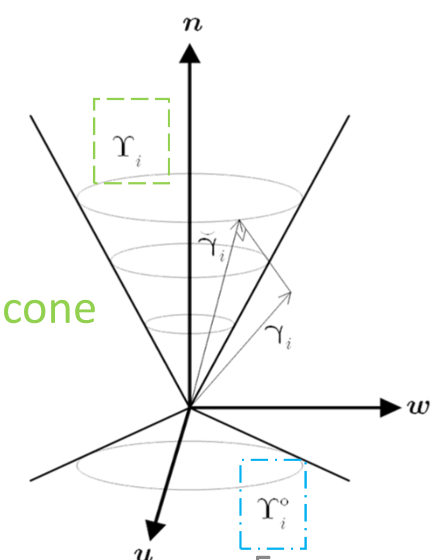
$$\text{Find } \gamma_i^{(l+1)}, \text{ for } i = 1, \ldots, N_c$$

$$\text{such that } \Upsilon_i \ni \gamma_i^{(l+1)} \perp -\left(N\gamma^{(l+1)} + \mathbf{r}\right)_i \in \Upsilon_i^\circ$$

$$\Upsilon_i = \{[\gamma_{i,n}, \gamma_{i,u}, \gamma_{i,w}]^T \in \mathbb{R}^3 | \sqrt{\gamma_{i,u}^2 + \gamma_{i,w}^2} \leq \mu_i \gamma_{i,n}\}$$

$$\Upsilon_i^\circ = \{[\gamma_{i,n}, \gamma_{i,u}, \gamma_{i,w}]^T \in \mathbb{R}^3 | \gamma_{i,n} \leq -\mu_i \sqrt{\gamma_{i,u}^2 + \gamma_{i,w}^2}\}$$

Friction cone



Background - Differential Variational Inequality approach (DVI)

Cone constrained convex optimization problem

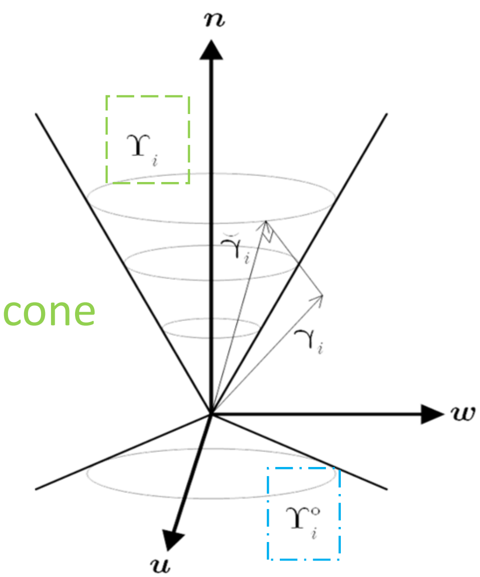
$$\min f(\gamma) = \frac{1}{2}\gamma^T \mathbf{N}\gamma + \mathbf{r}^T\gamma$$
$$\text{subject to } \gamma_i \in \Upsilon_i$$
$$\text{for } i = 1, 2, \ldots, N_c$$

$$\begin{aligned} \mathbf{N} &= \mathbf{D}^T\mathbf{M}^{-1}\mathbf{D} \\ \mathbf{r} &= \mathbf{b} + \mathbf{D}^T\mathbf{M}^{-1}\boldsymbol{k} \\ \boldsymbol{\gamma} &= \left[\gamma_1^{\mathbf{T}}, \gamma_2^{\mathbf{T}}, \cdots, \gamma_{\mathbf{N_c}}^{\mathbf{T}}\right]^T \in \mathbb{R}^{3N_c} \end{aligned}$$

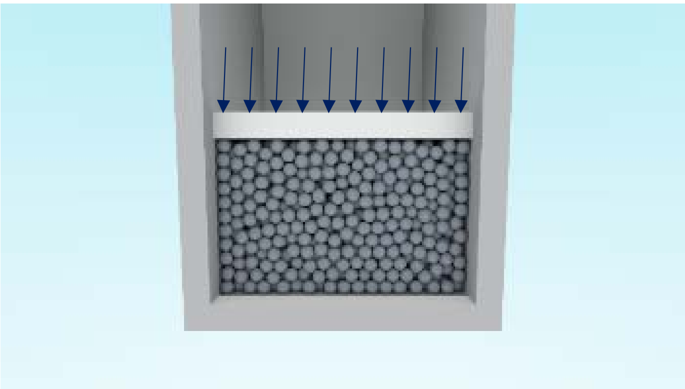
Existing 1-st order numerical methods for solving optimization problem:

- Projected Jacobi
- Projected Gauss – Siedel
- Accelerated Projected Gradient Descent (APGD)

Friction cone



Background – Performance of existing numerical methods



- 4000 Rigid spheres
- Heavy block rests on packed spheres

solver performance at one time step
solver tolerance: 7e-6

Solver	Residual	Iterations	Time [s]
Jacobi	7.54×10^{-6} (UtC)	500 000	24 300
Gauss Seidel	6.99×10^{-6}	11 485	494.8
APGD	6.97×10^{-6}	202	10.6

Best 1-st order method

What about 2nd order method?

Background – Primal-dual interior point method algorithm

$$\min \boldsymbol{f}(\boldsymbol{x})$$

$$\text{subject to } c_i(\boldsymbol{x}) \leq 0,\ i = 1, \ldots, m$$

KKT condition w/ dual variable $\boldsymbol{\lambda}$

$$\begin{aligned} c_i(\boldsymbol{x}) &\leq 0, \quad i = 1, \ldots, m \\ \boldsymbol{\lambda}_i &\geq 0, \quad i = 1, \ldots, m \end{aligned}$$

Line search applied to find suitable step size α

$$\begin{aligned} \nabla f(\boldsymbol{x}) + \sum_{i=1}^{m} \boldsymbol{\lambda}_i \nabla c_i(\boldsymbol{x}) &= 0 \\ -\boldsymbol{\lambda}_i c_i(\boldsymbol{x}) - \frac{1}{t} &= 0, \quad i = 1, \ldots, m \end{aligned}$$

$$\boldsymbol{A}(\boldsymbol{x}, \boldsymbol{\lambda}) \begin{bmatrix} \Delta \boldsymbol{x} \\ \Delta \boldsymbol{\lambda} \end{bmatrix} = \boldsymbol{r}(\boldsymbol{x}, \boldsymbol{\lambda}, t)$$

Update the primal $\boldsymbol{x}$ and the dual $\boldsymbol{\lambda}$ of constraints $(x, \lambda) \rightarrow (x + \alpha \Delta x, \lambda + \alpha \Delta \lambda)$ by computing a sequence of linear system of equations

Application to friction and contact problem

$$\min \boldsymbol{f}(\boldsymbol{x})$$
$$\text{subject to } c_i(\boldsymbol{x}) \le 0,\ i = 1, \ldots, m$$

Coulomb friction constraints

$$c_i(\boldsymbol{\gamma}) = \begin{cases} \frac{1}{2}\left(\gamma_{i,u}^2 + \gamma_{i,w}^2 - \mu_i^2 \gamma_{i,n}^2\right) & i = 1, \ldots, N_c \\ -\gamma_{(i-N_c),n} & i = N_c + 1, \ldots, 2N_c \end{cases}$$

Compute linear system

$$\begin{bmatrix} \boldsymbol{N} + \boldsymbol{M} & \boldsymbol{B}^T \\ \boldsymbol{B} & \boldsymbol{C} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\gamma} \\ \Delta \boldsymbol{\lambda} \end{bmatrix} = - \begin{bmatrix} \boldsymbol{r}_d \\ \boldsymbol{r}_g \end{bmatrix}$$

Size of linear system: $5Nc \times 5Nc$

apply Schur complement

$$(\boldsymbol{N} + \boldsymbol{M} - \boldsymbol{B}^T \boldsymbol{C}^{-1} \boldsymbol{B}) \Delta \boldsymbol{\gamma} = \boldsymbol{B} \boldsymbol{C}^{-1} \boldsymbol{r}_g - \boldsymbol{r}_d$$

Size of linear system: $3Nc \times 3Nc$

Direct solver or Iterative solver (using preconditioned CG)

preconditioner:

$$\boldsymbol{M} - \boldsymbol{B}^T \boldsymbol{C}^{-1} \boldsymbol{B}$$

$$diag(\boldsymbol{N}) + \boldsymbol{M} - \boldsymbol{B}^T \boldsymbol{C}^{-1} \boldsymbol{B}$$

Convergence test

- Generate contact info matrices of settled-spheres simulation
- For one time step, compare the results between APGD solver and PDIP solver
  - PDIP with Schur Complement, using iterative solver
  - PDIP with Schur Complement, using direct solver
  - PDIP without Schur Complement (KKT), using direct solver
- Stopping criteria
  - No more changes in objective function
  - No more changes in contact force magnitude

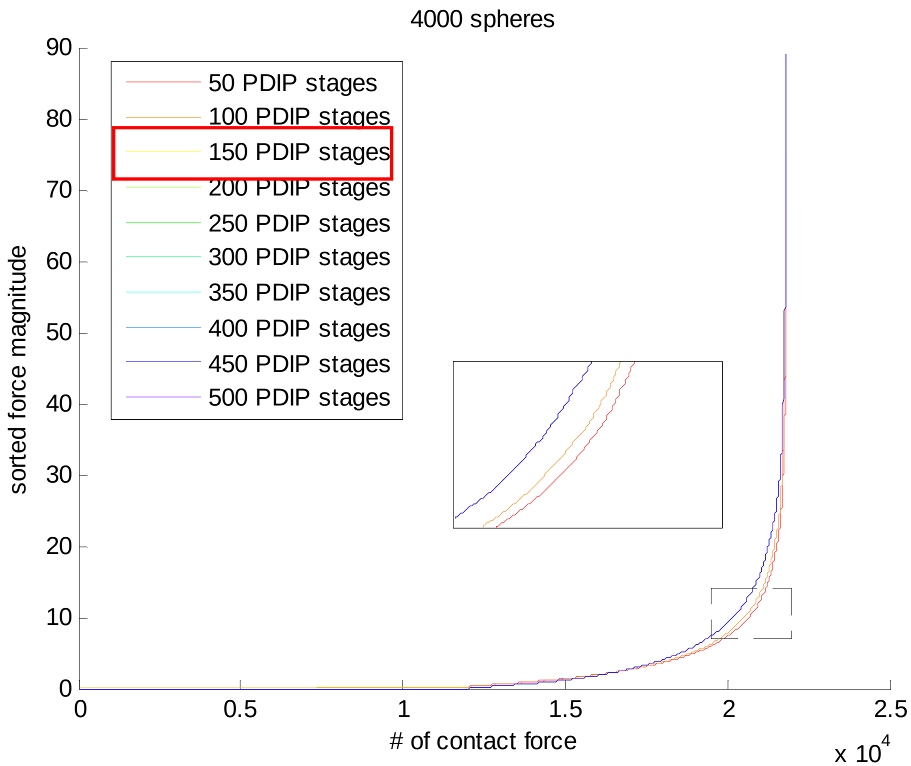
Convergence result
– no more changes in objective function

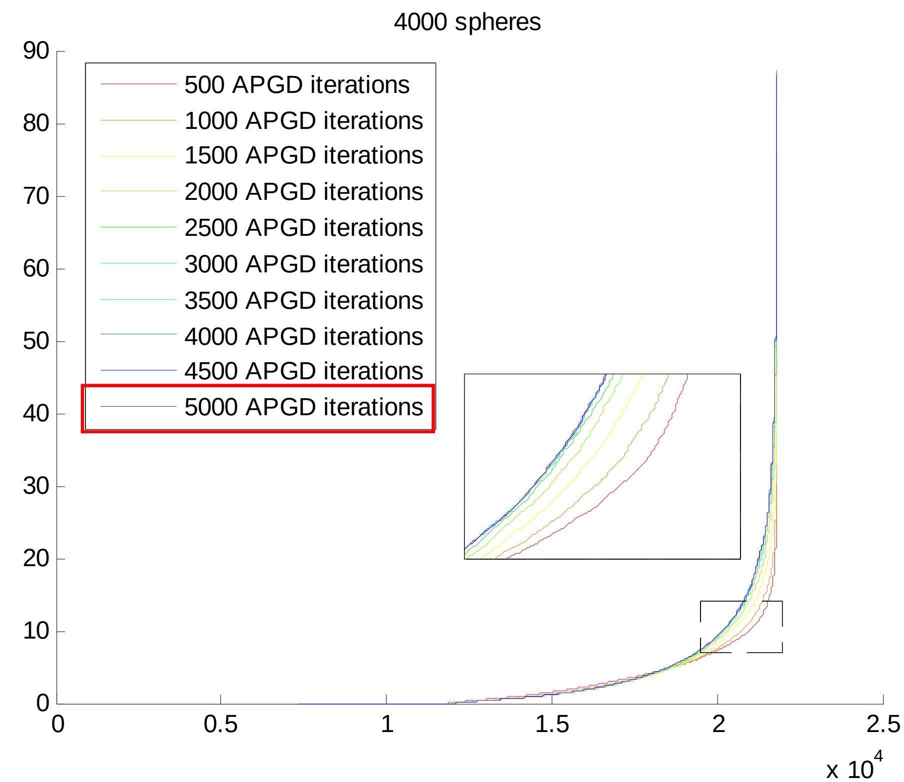
Method (512 bodies)		Size of linear system	# of stages (iterations)	objective function
PDIP	Schur precondtioned	8K * 8K	75	-14.918723
	Schur direct	8K * 8K	74	-14.918728
	KKT	14K * 14K	56	-14.918725
APGD			28265	-14.9181

@ each PDIP stage: compute a linear system

@ each APGD iteration: matrix-vector multiplication

Convergence result
– no more changes in contact force magnitude





objective function -121.9280 -121.9231

Timing result

- for reaching the same objective function
- run in Matlab

spheres	size of linear system	method	stages/iterations	overall Time (sec)
512	8K * 8K	PDIP	65	39.14
	NA	APGD	30000	466.35
1024	18K * 18K	PDIP	64	192
	NA	APGD	30000	1808
2048	36K * 36K	PDIP	67	751
	NA	APGD	30000	3256

Conclusion

- A second-order method, Primal-Dual Interior Point Method (PDIP) is implemented to calculate the frictional contact forces,
- A series of linear systems of equations are computed to find the solution, and an efficient linear solver is needed for large problem,
- PDIP exhibits better performances compared with the best first-order method, APGD, in terms of
  - Better accuracy (smaller objective function)
  - Overall less computation time

Acknowledgements

- SBEL
  - Advisor Dan Negrut
  - Scientist Radu Serban
  - Grad students: Aaron Bartholomew, Makarand Datar, Felipe Guttierrez, Toby Heyn, Daniel Kaczmarek, Naresh Khude, Ang Li, Justin Madsen, Hammad Mazhar, Dan Melanz, Francisco Mercado, Spencer O’Rourke, Arman Pazouki, Andrew Seidl, Rebecca Shotwell

ASME SETTING THE STANDARD

SBEL Simulation Based Engineering Lab University Of Wisconsin - Madison

Talks that you should **NOT** miss…

Topic	Presented by	Time	Rm
A GPU-Based Preconditioned Newton-Krylov Solver for Flexible Multibody Dynamics	Radu Serban	Mo, 10:30am	104
Chrono::Render - A Graphical Visualization Pipeline for Multibody Dynamics Simulation	Daniel Kaczmarek	Mo, 10:30am	104
Gauging Military Vehicle Mobility Through Many-body Dynamics Simulation	Dan Melanz	Tu, 15:30pm	109
Evaluation of Tire/Soil Contact Models to Predict Vehicle Mobility and Soil Compaction on a Reduced-order Terrain Model	Justin Madsen	Tu, 15:30pm	109
A Fluid-solid Interaction Approach for the Simulation of Rigid and Deformable Bodies in Newtonian Fluid	Arman Pazouki	We, 10:30am	108

Questions?