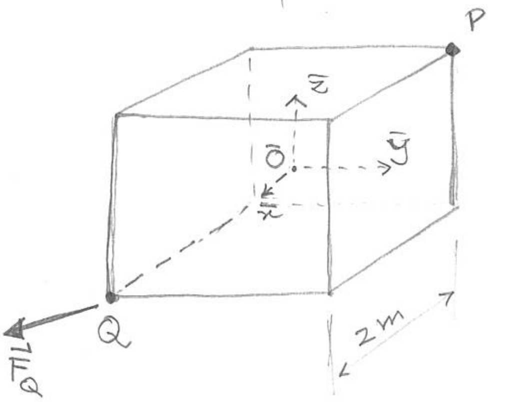
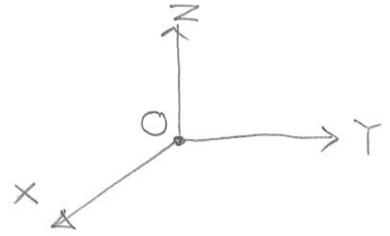
Equations of Motion $\mapsto$ dangling cube.

Cube of mass $m = 6\,kg$ Edge length 2 m.





$\downarrow g = 9.81 \ \frac{m}{s^2}$

Corner P connected to origin O through a spherical joint.

A force $\vec{F}_Q$ acts on cube at corner Q.

$$F_Q = \begin{bmatrix} \cos t \\ \sin t \\ 0 \end{bmatrix}$$

Need to get all ingredients to compute

$$\begin{bmatrix} M & 0 & \Phi_r^T \\ 0 & \bar{J} & \bar{\Pi}^T(\Phi) \\ \Phi_r & \bar{\Pi}(\Phi) & 0_{nc \times nc} \end{bmatrix} \begin{bmatrix} \ddot{r} \\ \dot{\bar{\omega}} \\ \lambda \end{bmatrix} = \begin{bmatrix} F \\ n - \tilde{\bar{\omega}} \bar{J} \bar{\omega} \\ \gamma \end{bmatrix} \qquad (*)$$

$nb = 1 \qquad nc = 3 \qquad M \in \mathbb{R}^{3nb \times 3nb} \qquad M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

Centroidal ⊕ Principal LRF $\Rightarrow \bar{J} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

(see pp. 435)

Spherical joint constraint: $\Phi(r, A) = r^P = 0$

$r^P = r + A\bar{s}^P \quad w/ \quad \bar{s}^P = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

$$\Phi(r, A) = r + A\bar{s}^P = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathring{\Phi} = \mathring{r} + \mathring{A}\bar{s}^P = \mathring{r} + A\tilde{\bar{\omega}}\bar{s}^P = \mathring{r} - A\tilde{\bar{s}}^P\bar{\omega} = [I \quad -A\tilde{\bar{s}}^P]\begin{bmatrix}\mathring{r}\\ \bar{\omega}\end{bmatrix}$$

$$\Rightarrow \bar{R} = [I \quad -A\tilde{\bar{s}}^P] \in \mathbb{R}^{3\times 6} \Rightarrow \boxed{\Phi_r = I_{3\times 3}} \quad \& \quad \boxed{\bar{\Pi}(\Phi) = -A\tilde{\bar{s}}^P \in \mathbb{R}^{3\times 3}}$$

$$\mathring{\mathring{\Phi}} = \mathring{\mathring{r}} - A\tilde{\bar{\omega}}\tilde{\bar{s}}^P\bar{\omega} - A\tilde{\bar{s}}\mathring{\bar{\omega}} = 0 \quad \Rightarrow \boxed{\gamma = -A\tilde{\bar{\omega}}\tilde{\bar{\omega}}\bar{s}^P}$$

Virtual work of $\vec{F}_Q$: $\delta W_Q = (\delta r^Q)^T \cdot F_Q$

$$\delta r^Q = \delta r + A\widetilde{\delta\bar{u}}\,\bar{s}^Q = \delta r - A\tilde{\bar{s}}^Q\delta\bar{u}$$

$$\Rightarrow (\delta r^Q)^T \cdot F_Q = (\delta r^T + \delta\bar{u}^T \cdot \tilde{\bar{s}}^Q A^T)F_Q = \delta r^T \cdot F + \delta\bar{u}^T \cdot \tilde{\bar{s}}^Q A^T F_Q$$

Since the gravitational acceleration is distributed all over the mass and the LRF is centroidal there is no resulting torque and the resultant force is

$$\begin{bmatrix}0\\0\\-6\cdot g\end{bmatrix}$$

Then $F = \begin{bmatrix}\cos t\\ \sin t\\ 0\end{bmatrix} + \begin{bmatrix}0\\0\\-6g\end{bmatrix} = \begin{bmatrix}\cos t\\ \sin t\\ -6g\end{bmatrix}$, and

$$n = -\tilde{\bar{\omega}}\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}\bar{\omega} + \tilde{\bar{s}}^Q A^T \cdot \begin{bmatrix}\cos t\\ \sin t\\ 0\end{bmatrix} \qquad \text{where } \bar{s}^Q = \begin{bmatrix}1\\-1\\-1\end{bmatrix}$$

At this point, we have all the ingredients to assemble the Linear system in Eq (*):

- the coefficient matrix is known
- the RHS is known
- we assume the r, A, $\mathring{r}$ and $\bar{\omega}$ are given (initial conditions).