ME751: Assignment 11

Problem 1. In 11/09's lecture, we talked about the ANCF shell element and introduced the sort of computations needed to obtain the Green-Lagrange vector of strains, both with respect to a curvilinear (unmodified) reference and with respect to an orthonormal basis.
Given the initial coordinates of one bilinear ANCF shell element (see Fig. 1) as follows (superscript refers to node number and $\mathbf{r}_z = \partial\mathbf{r}/\partial z$):

$$\begin{aligned}
&\mathbf{r}^1 = (0,0,0),\ \mathbf{r}_z^1 = (0,0,1) \\
&\mathbf{r}^2 = (1,-0.1,0),\ \mathbf{r}_z^2 = (-0.1407125, 0.1407125, 0.98) \\
&\mathbf{r}^3 = (1.1,1.1,0.1),\ \mathbf{r}_z^3 = (-0.1407125, -0.1407125, 0.98) \\
&\mathbf{r}^4 = (0,1,0),\ \mathbf{r}_z^4 = (0,0,1),
\end{aligned} \tag{1}$$

And considering the material point in the local parameterized space $[\xi,\eta,\zeta] = \left[0, \sqrt{3/5}, \sqrt{3/5}\right]$, follow the steps:

Step 1. Review slides from lecture11/09 (from the course website)
Step 2. For the element coordinates and material point given in the problem statement, obtain the Green-Lagrange strain vector in the curvilinear system assuming that the initial coordinates are:
$\mathbf{r}^1 = (0,0,0)$, $\mathbf{r}_z^1 = (0,0,1)$, $\mathbf{r}^2 = (1,0,0)$, $\mathbf{r}_z^2 = (0,0,1)$, $\mathbf{r}^3 = (1,1,0)$, $\mathbf{r}_z^3 = (0,0,1)$, $\mathbf{r}^4 = (0,1,0)$, $\mathbf{r}_z^4 = (0,0,1)$

Step 3. For the element coordinates and material point given in the problem statement, obtain the Green-Lagrange strain vector with respect to an orthonormal frame (expression given on slide 27, lecture1109 on website). Use, as initial configuration, the initial configuration given in the step 2.
Step 4. Repeat the step 3 using the reference frame resulting from the problem statement's coordinates as initial configuration.
Step 5. Discuss the difference in the results obtained for the steps 2 to 4. In addition, discuss: Why would it be needed to refer the strains to a material, local orthonormal frame?
Step 6 (bonus). Repeat steps 3 and 4 using as material point that of node 1. Discuss the results.

Observations/hints: You can solve the computational part of this problem using Chrono, C++, Matlab, calculator or pen and paper. Looking at Chrono's ANCF shell implementation file can help you figure out which shape functions and/or strain expressions need to be used to solve this problem, regardless of how you choose to compute the strains. If you need to make assumptions to solve these problems, e.g. in terms of element dimensions, please mention and explain it.

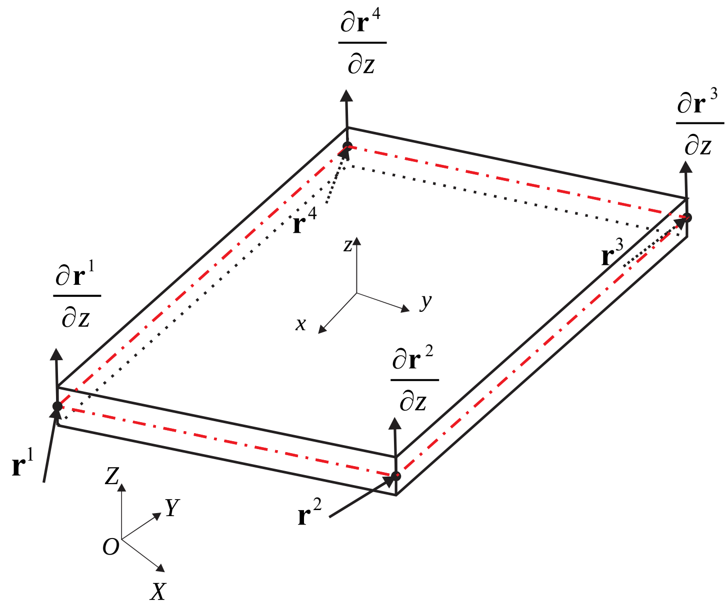

Figure 1. Coordinates of the ANCF shell element