ME751: Assignment 10

Problem 1. In class, we partially derived the mass matrix and quadratic inertia terms for the FFR (10/28 lecture). Derive all the inertia terms of the finite element (FE)/FFR (that is, considering the kinematic expressions for finite elements that involve the intermediate coordinate system) in detail, in 3D, assuming that the shape functions and all relative initial orientations are known.
Step 1. Review slides from lecture10/31 (from the course website)
Step 2. Start with the kinematics of the FE/FFR. Remember:

$$\mathbf{r}^i = \mathbf{R}^i + \mathbf{A}\mathbf{N}^{ij}\left(\mathbf{q}_0^i + \mathbf{B}_2^i\mathbf{q}_f^i\right) \quad (1)$$

Step 3. Assume the rotational parameters are Euler parameters (or unit quaternions), just like in class
Step 4. Using pen and paper, derive the expressions that define the mass matrix and the quadratic velocity terms. To that end, follow the steps taken in class:

- Derive the velocity expression
- Write the kinetic energy of the deformable body
- Write the kinetic part of the Lagrange equations
- Make explicit all the expressions for the mass matrix
- Make explicit all the expressions that are quadratic in the velocities (this is the actual problem). To help you in this derivation, look at the paper attached and review the rules of differentiation of vectors and matrices.

Observations: Note that not all the derivations were made explicit in the slides. Please do make all the derivations explicit in this exercise and write every step and define all the variables. You may need to take a look at the rules for differentiating vectors and matrices.
References:
Ahmed Shabana, Dynamics of Multibody Systems, 3rd edition;
Sherif and Nachbagauer, "A Detailed Derivation of the Velocity-Dependent Inertia Forces in the Floating Frame of Reference Formulation", Journal of Computational and Nonlinear Dynamics, October 2014, Vol. 9 / 044501-1-Sections 3 and 4.

Problem 2. Draw and define with equations the following joints between one ANCF cable element node and one shear deformable 3-node ANCF node:

- Spherical
- Revolute
- Prismatic

Step 1. Review the definitions of constraints from the first part of the course

Step 2. Revisit the ANCF cable element and the 3-node shear deformable element: what their coordinates are and what direction their gradients are oriented along (lecture 11/07).

Step 3. Draw a connection between one ANCF cable node and one node corresponding to the 3-node shear deformable element. Define the joint axes for each of the 3 mechanical joints: spherical, revolute, and prismatic. Note that you have freedom to define the orientation of the joint axes; but you can choose to have a similar set up as seen in class.

Step 4. Write the constraint equations for each of the 3 joint cases. If you need to make assumptions or find any limitation, explain it.

Step 5. Write the Jacobian of the revolute joint in terms of the coordinates of the two ANCF finite elements.

Problem 3 (bonus). As an extension to Problem 1, derive the FE/FFR deformable body inertia tensor in terms of FFR coordinates, inertia shape integrals, and other integrals that may be needed. Start from the following expression:

$$\bar{\mathbf{I}}^i_{\theta\theta} = \int_{V^i} \rho^i \tilde{\bar{\mathbf{u}}}^{iT} \tilde{\bar{\mathbf{u}}}^i \mathrm{d}V^i \qquad (2)$$

Step 1. Review slides 10/28, 10/31

Step 2. Transform the expression given above into a function of the system's –FFR- coordinates. Hint: Start by substituting the local displacement vector $\bar{\mathbf{u}}$ by its definition in terms of coordinates (see Equation (1)).

Step 3. Briefly discuss the functional dependency of the inertia tensor of a deformable body. Is there any term that can be computed only once at the beginning of the simulation? Why?