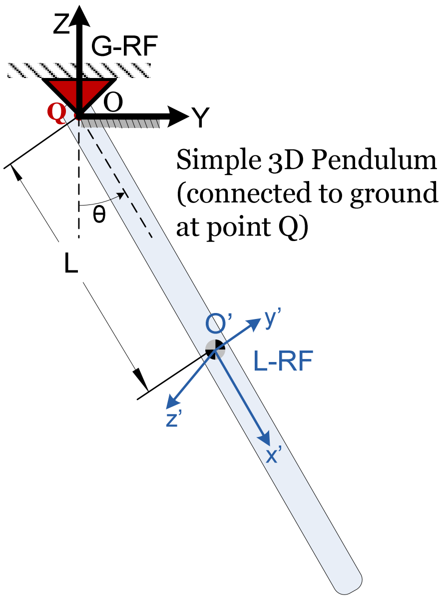
ME751: Assignment 7

Problem 1 [simEngine3D track]. This problem builds on the problem in the previous assignment. The schematic of the mechanism is shown below. The rigid body is subjected to a motion specified as $\theta(t) = \frac{\pi}{4}\cos(2t)$. Recycle as much as possible of the code you already generated.



Perform an Inverse Dynamics Analysis to compute the amount of torque that you would have to apply to the pendulum to make it move as indicated by the specified motion. Assume that $L=2$, and the section of the bar is a square of width 0.05. The density of the material is $7,800$. All units are SI. For this problem please provide a plot that displays the value of the torque as a function of time for $t \in [0,10]$.

Upload your results in a zip file at Learn@UW.

Post any questions on the ME751 Forum.

Problem 2 [Chrono track]. Go to http://lim.ii.udc.es/mbsbenchmark/docs/specifications.html. Pick up the four bar example (from under N-four-bar) and carry out the analysis whose results are reported under the "View results" link. Take N=1; i.e., a classical four bar mechanism, see, for instance, page 168 of Ed Haug's book. For this problem:

- Generate a movie of the motion of the left most bar
- Generate a plot that shows the magnitude of the reaction force in the joint connecting the leftmost bar with the horizontal bar
- Generate a plot of the time evolution of the tip of the rightmost bar (x, y, and z as a function of time)
- Generate a plot of the velocity of the tip of the rightmost bar (three plots, for x, y, and z)

Upload your results in a zip file at Learn@UW.

Post any questions on the Chrono Forum.

Problem 3. Consider the test problem that is used to define the region of stability.

$$\begin{cases} \dot{y} = \lambda y \\ y(0) = 1 \end{cases}$$

Use Forward Euler to solve this IVP three times for $\lambda = -10, -100, -1000$. For each value of λ identify the smallest step size h at which your numerical solution loses stability. Does this come in line with what we discussed in class?

Problem 4. Consider the following IVP (discussed in class, see also handout example on the computation of the Jacobian):

$$\begin{cases} \dot{x} = \alpha - x - \dfrac{4xy}{1+x^2} \\ \dot{y} = \beta x(1 - \dfrac{y}{1+x^2}) \end{cases} \quad \& \quad \begin{cases} x(0) = 0 \\ y(0) = 2 \\ t \in [0, 20] \\ NOTE: \alpha \,\&\, \beta \; are\ given\ parameters \end{cases}$$

Apply Backward Euler to find an approximation of the exact solution of this IVP. Use $\alpha = 0$ and $\beta = 1$ to generate two plots of x and y, respectively, that you return as part of your HW. Experiment with other α and β values as well to gauge the sensitivity of the solution of these two parameters.

Problem 5. Consider the following IVP:

$$\dot{y} = -y^2 - \frac{1}{t^4} \quad \& \quad \begin{cases} y(1) = 1 \\ t \in [1, 10] \end{cases}$$

a) Prove that the exact solution of this IVP is $y(t) = \dfrac{1}{t} + \dfrac{1}{t^2}\tan\left(\dfrac{1}{t} + \pi - 1\right)$ by showing that it satisfies both the scalar ODE above and the IC specified.
b) Generate the Backward Euler convergence plot for the above IVP
c) Generate the BDF convergence plot for the above IVP. Note: (*i*) display the convergence plot in the same figure you used for the Backward Euler analysis; (*ii*) use the fourth order BDF formula in this exercise; (*iii*) use the exact solution above to generate the required starting points for the BDF formula
d) Measure the slope of the two plots and comment whether the two values come in line with your expectations