ME751 - Assignment 3

Problem 1. Problem 9.2.5 out of Haug's textbook.

Problem 2. [Related to Implicit Function Theorem] An implicit dependency is established between x and y through the relation $u(x,y) = \sin x + \cos^2 y - 1$. Find all the points (a,b) where the Implicit Function Theorem does not help you express x as a function of y. Comment on what happens at all these points when you try to express the dependency mentioned.

Problem 3. Consider the dependency between the three variables x, y, and z that is implicitly induced by the relation $\mathbf{u}(x,y,z) = \mathbf{0}_2$, where

$$\mathbf{u}(x,y,z) = \begin{bmatrix} x + 2y - 3\sin(z^2) \\ x - y + 3\exp(z) \end{bmatrix} \tag{1}$$

a) For this problem, indicate the value of the scalar m and n that we introduced in relation with the Implicit Function Theorem

b) I will consider here z to be the independent variable. For this simple relation defined in Eq. (1) it is actually possible to find the expressions of $x(z)$ and $y(z)$ (this is not generally possible for complicated $\mathbf{u}$ relations). Find $x(z)$ and $y(z)$, and explain why for this problem they capture *globally* the dependence of x on z and of y on z. That is, for any value of z, there is a unique and globally valid expression of $x(z)$ and of $y(z)$ induced by the specified relation. Is there any contradiction here, given that the Implicit Function Theorem only guarantees locally an expression for say $x(z)$ yet you are able to find a global one?

NOTE: In the future, you'll see that the independent variable z actually will change as a function of time: $z = z(t)$. Consequently, since x and y depend on z, which now depends on time, the relation in Eq. (1) will implicitly dictate how x and y change as functions of time.

Problem 4. Take care of the work labeled as HOMEWORK in the following slides:

- Slide 9 of 09/16 lecture (the font is messed up on that slide, some strange character shows up instead of ϕ)
- Slide 6 of 09/21 lecture
- Slide 27 of 09/21 lecture (do this only if you can, challenge homework)