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Assignment 11: Due April 15, 2010.

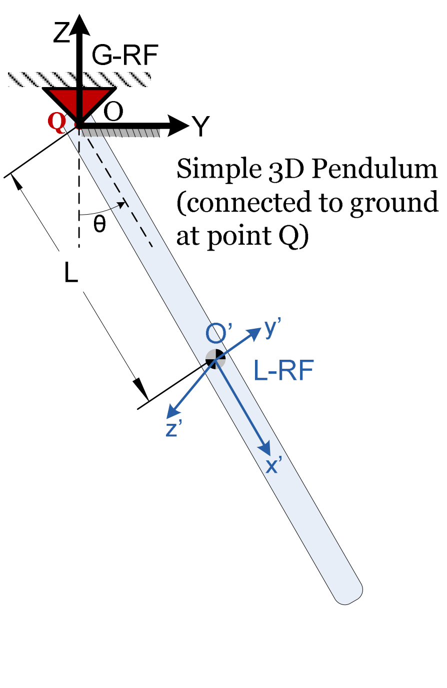
Problem 1. Use the approach discussed in class to derive the BDF method of order 3. To this end, follow the same steps as in the example covered in class (available also online) with the caveat that this time you will have to consider one more point, (t_{n-3}, y_{n-3}), when constructing the interpolation polynomial.

Problem 2. Compute $C_n^x(l)$ and $C_n^v(l)$ for the BDF of order $l = 1$ and then $l = 3$.

Problem 3 [BONUS PROBLEM]. Prove that the iteration matrix $\mathbf{\Psi}$ introduced on slide 27 of April 8 lecture for the Quasi-Newton approach is nonsingular.

Problem 4. Consider the mass-spring-damper model discussed in the April 6 lecture. For that problem, take $m = 2$, $c = 200$, $k = 400$. The initial conditions at time $t = 0$ are $x_0 = 1$, $v_0 = -1$. In two separate plots, plot the location and velocity of the mass as a function of time for $t \in [0,10]$. All units are SI. To find an approximate solution of the IVP use the BDF formula of order 3 that you derived in Problem 1. Remarks: (a) use the BDF formula of order 1; i.e., Backward Euler, to prime the order 3 integrator; (b) use your judgment to choose a time step small enough so that the approximate solution of the IVP looks accurate enough. Solve this problem first with Newton-Raphson, then with Modified-Newton, and finally with a Quasi-Newton approach as discussed in class (MATLAB code for each of these three approaches is available online). Provide a plot with the number of iterations for Newton convergence in each of these three cases (like the one provided in lecture of April 08).

Problem 5. This problem builds on simple pendulum problem of previous assignments. The schematic of the mechanism is shown in the figure.



You will have to carry out a Dynamics Analysis for the mechanism for 10 seconds of its evolution using a BDF method of order 1 in conjunction with a Quasi-Newton method for solving the resulting discretization nonlinear system of equations. For initial conditions consider the pendulum to be horizontal; i.e., according to the figure, $\theta = \frac{\pi}{2}$. The pendulum has zero velocity at time $t = 0$.

In the solution folder include a movie of the time evolution of this mechanism, along with six plots. The first three will display the location of point O' in the G-RF as a function of time, the second one its velocity in the G-RF as a function of time, and the third one will display its acceleration in the G-RF. The last set of three plots will display to same information for the tip of the pendulum located at Q.