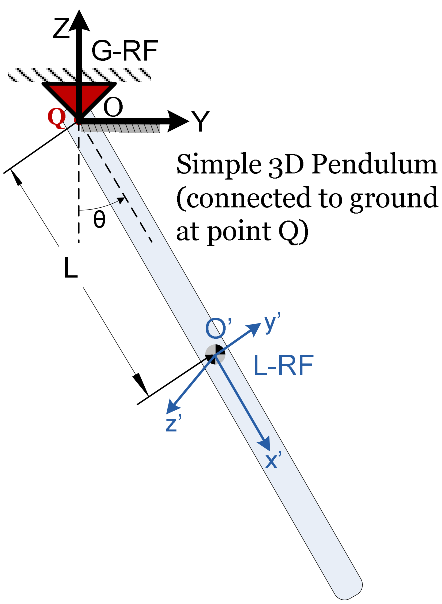
Assignment 10: Due April 8, 2010.

Problem 1. This problem builds on Problem 3 of Assignment 8. The schematic of the mechanism is shown in the figure. The rigid body is subjected to a motion specified as $\theta(t) = \frac{\pi}{4}\sin(2t)$.



You will have to carry out a Kinematics Analysis for the mechanism for 10 seconds of its evolution *yet this time around you'll have to use the* $\mathbf{r}-\mathbf{p}$ *formulation*. For your Kinematics Analysis use a time grid with time steps of $\Delta t = 10^{-3}$ seconds. In the folder solution, include a movie of the time evolution of this mechanism, along with six plots. The first three will display the location of point O' in the G-RF as a function of time, the second one its velocity in the G-RF as a function of time, and the third one will display its acceleration in the G-RF. The last set of three plots will display to same information for the tip of the pendulum located at Q. Once you implement this analysis, post on the forum under the topic "**Timing Results, Kinematics Analysis**" the amount of time your code required to complete the analysis using the $\mathbf{r}-\bar{\omega}$ implementation (out of HW8) and then this HW's $\mathbf{r}-\mathbf{p}$ implementation.

Problem 2. For the mechanism in Problem 1, perform an Inverse Dynamics Analysis to compute the amount of torque that you would have to apply to the pendulum to make it move as indicated by the specified motion. Assume that $L = 2$, and the section of the bar is a square of width 0.05. The density of the material is $7,800$. All units are SI. For this problem please provide a plot that displays the value of the torque as a function of time for $t \in [0,10]$.

Problem 3. Consider the test problem that is used to define the region of stability.

$$\begin{cases} \dot{y} = \lambda y \\ y(0) = 1 \end{cases}$$

Use Forward Euler to solve this IVP three times for $\lambda = -10, -100, -1000$. For each value of λ identify the smallest step size h at which your numerical solution loses stability. Does this come in line with what we discussed in class on March 23?

Problem 4. Consider the following IVP (discussed in class, see also handout example on the computation of the Jacobian):

$$\begin{cases} \dot{x} = \alpha - x - \dfrac{4xy}{1+x^2} \\ \dot{y} = \beta x(1 - \dfrac{y}{1+x^2}) \end{cases} \qquad \& \qquad \begin{cases} x(0) = 0 \\ y(0) = 2 \\ t \in [0, 20] \\ NOTE: \alpha \,\&\, \beta \text{ are given parameters} \end{cases}$$

Apply Backward Euler to find an approximation of the exact solution of this IVP. Use $\alpha = 0$ and $\beta = 1$ to generate two plots of x and y, respectively, that you return as part of your HW. Experiment with other α and β values as well to gauge the sensitivity of the solution of these two parameters.

Problem 5. Consider the following IVP:

$$\dot{y} = -y^2 - \frac{1}{t^4} \qquad \& \qquad \begin{cases} y(1) = 1 \\ t \in [1, 10] \end{cases}$$

a) Prove that the exact solution of this IVP is $y(t) = \dfrac{1}{t} + \dfrac{1}{t^2}\tan\left(\dfrac{1}{t} + \pi - 1\right)$ by showing that it satisfies both the scalar ODE above and the IC specified.
b) Generate the Backward Euler convergence plot for the above IVP
c) Generate the BDF convergence plot for the above IVP. Note: (*i*) display the convergence plot in the same figure you used for the Backward Euler analysis; (*ii*) use the fourth order BDF formula in this exercise; (*iii*) use the exact solution above to generate the required starting points for the BDF formula
d) Measure the slope of the two plots and comment whether the two values come in line with your expectations