Assignment 4: Due February 18.

Problem 1. Do Exercise on slide 4 of the 02/09/2010 lecture.

Problem 2. Provide two proofs to the fact that the orientation matrix $\mathbf{A}$ obtained when using Euler Angles is orthonormal.

Problem 3. Prove that the expression of the angular velocity ω when using Euler Angles to express the orientation of a body in 3D space is as indicated on slide 22 (or 23, for that matter) of the 02/09/2010 lecture.

Problem 4. Prove that the determinant of the $\mathbf{B}$ matrix on slide 23 of the 02/09/2010 lecture is indeed $\sin\theta$. More specifically, prove that $|\det(\mathbf{B})| = |\sin\theta|$. Explain why this is relevant when one tries to link the time derivative of the Euler Angles to the value of the angular velocity of the rigid body.

Problem 5. Assume that $e_0^2 + \mathbf{e}^T\mathbf{e} = 1$, in other words, $\mathbf{p}^T = [e_0, e_1, e_2, e_3]$ represents a valid set of Euler Parameters. Prove that the matrix defined as $\mathbf{A} = (2e_0^2 - 1)\mathbf{I} + 2(\mathbf{e}\mathbf{e}^T + e_0\tilde{\mathbf{e}})$ is an orthogonal matrix. This goes to show that a healthy $\mathbf{p}$ leads to a healthy orientation matrix $\mathbf{A}$.

Problem 6. Prove the identity on slide 22 of Feb. 11 presentation. Given $\mathbf{E}$ and $\mathbf{G}$ therein, prove that $\mathbf{A} = \mathbf{E}\mathbf{G}^T$.

Problem 7. Prove identities 3 and 4 on slide 22 of Feb. 11 presentation.

Problem 8. Prove identity 4 on slide 23 of Feb. 11 presentation.

Problem 9. [Bonus Problem]: Prove that the Euler Parameter $\mathbf{p}$ obtained following the procedure outlined on slide 13 (when $e_0 \neq 0$) and then slide 18 (when $e_0 = 0$) is consistent. That is, it satisfies the normalization constraint $\mathbf{p}^T\mathbf{p} = 1$.