Assignment 3: Due February 11.

Problem 1. Do Problem 9.2.5 out of Haug's textbook.

Problem 2. Explain in greater details all the steps that I quickly went through in class to show that the rows of the Jacobian matrix $\mathbf{u}_{\mathbf{q}}$ are indeed linearly independent, that is, that the rank of this matrix is 6.

Problem 3. An implicit dependency is established between x and y through the relation $u(x,y) = \sin x + \cos^2 y - 1$. Find all the points (a,b) where the Implicit Function Theorem does **not** help you to express y as a function of x. Comment on what happens at these points when you try to express the dependency mentioned.

Problem 4. An implicit dependency is established between x and y through the relation $u(x,y) = \sin x + \cos^2 y - 1$. Find all the points (a,b) where the Implicit Function Theorem does not help you express x as a function of y. Comment on what happens at all these points when you try to express the dependency mentioned.

Problem 5. Consider the rigid body in Figure 1, which is connected to ground through a spherical joint located at its center of mass $\mathbf{O}$. Let $\mathbf{P}$ be a point of the rigid body, and use the notation $\vec{\mathbf{s}}^{\mathbf{P}} = \overrightarrow{\mathbf{OP}}$.

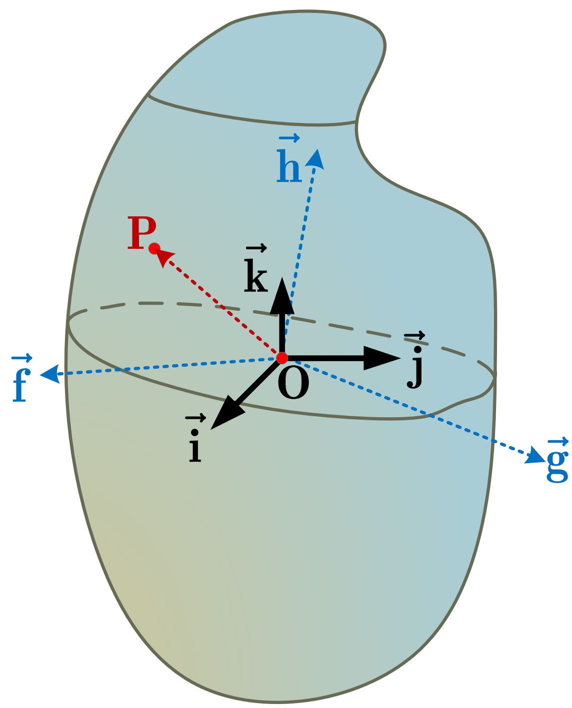

Figure 1. Rigid body

From ME240 you know that the velocity of point $\mathbf{P}$ is the cross product between the angular velocity of the body and the location of the point $\mathbf{P}: \vec{\mathbf{v}}^{\mathbf{P}} = \dot{\vec{\mathbf{s}}}^{\mathbf{P}} = \vec{\omega} \times \vec{\mathbf{s}}^{\mathbf{P}}$. Although back then you probably didn't insist too much on the origin of $\vec{\omega}$, now you can use the definition we introduced in ME751 to prove that the above expression is true. In other words, using the definition of the angular velocity as the vector whose cross product matrix satisfies the condition $\tilde{\omega} = \dot{\mathbf{A}}\mathbf{A}^T$, prove that the velocity of point $\mathbf{P}$, that is, the time derivative of $\mathbf{s}^{\mathbf{P}}$, is computed as

$$\mathbf{v}^{\mathbf{P}} = \dot{\mathbf{s}}^{\mathbf{P}} = \tilde{\omega}\mathbf{s}^{\mathbf{P}} \qquad (1)$$

Note that the geometric vector representation of the identity in Eq. (1) is precisely the formula with which you are familiar from ME240.

Problem 6. Consider the dependency between the three variables x, y, and z that is implicitly induced by the relation $\mathbf{u}(x,y,z)=\mathbf{0}_2$, where

$$\mathbf{u}(x,y,z)=\begin{bmatrix} x+2y-3\sin(z^2) \\ x-y+3\exp(z) \end{bmatrix} \tag{2}$$

a) For this problem, indicate the value of the scalar m and n that we introduced in relation with the Implicit Function Theorem

b) I will consider here z to be the independent variable. For this simple relation defined in Eq. (2) it is actually possible to find the expressions of $x(z)$ and $y(z)$ (this is not generally possible for complicated $\mathbf{u}$ relations). Find $x(z)$ and $y(z)$, and explain why for this problem they capture *globally* the dependence of x on z and of y on z. That is, for any value of z, there is a unique and globally valid expression of $x(z)$ and of $y(z)$ induced by the specified relation. Is there any contradiction here, given that the Implicit Function Theorem only guarantees locally an expression for say $x(z)$ yet you are able to find a global one?

   NOTE: In the future, you'll see that the independent variable z actually will change as a function of time: $z=z(t)$. Consequently, since x and y depend on z, which now depends on time, the relation in Eq. (2) will implicitly dictate how x and y change as functions of time.