Exercise, Negative Euler Parameter

Given to you:

* $\vec{u}$, a unit vector, serves as axis of rotation
* χ, angle of rotation, $\chi \in (0, 2\pi)$

Recall the way e_0, e_1, e_2, e_3 were defined:

$$e_0 = \cos\frac{\chi}{2} \qquad e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \cdot \sin\frac{\chi}{2} = u \cdot \sin\frac{\chi}{2}.$$

Now, if we have $p^{new} = -p \Rightarrow e_0^{new} = -e_0 = \cos\frac{\chi^{new}}{2}$

$$e^{new} = -e = u^{new} \cdot \sin\frac{\chi^{new}}{2}$$

$$\Rightarrow \begin{cases} \cos\frac{\chi^{new}}{2} = -\cos\frac{\chi}{2} \\ u^{new}\sin\frac{\chi^{new}}{2} = -u\sin\frac{\chi}{2} \end{cases} \qquad (1)$$

$$\Rightarrow \frac{\chi^{new}}{2} = \pi - \frac{\chi}{2} \Rightarrow \boxed{\chi^{new} = 2\pi - \chi}$$

Then $\sin\frac{\chi^{new}}{2} = \sin\left(\pi - \frac{\chi}{2}\right) = \sin\frac{\chi}{2}$

Therefore, using Eq. (1), $(u^{new} + u)\sin\frac{\chi}{2} = 0$

$\Rightarrow u^{new} = -u$. (since $\chi \in (0, 2\pi)$) ⊛