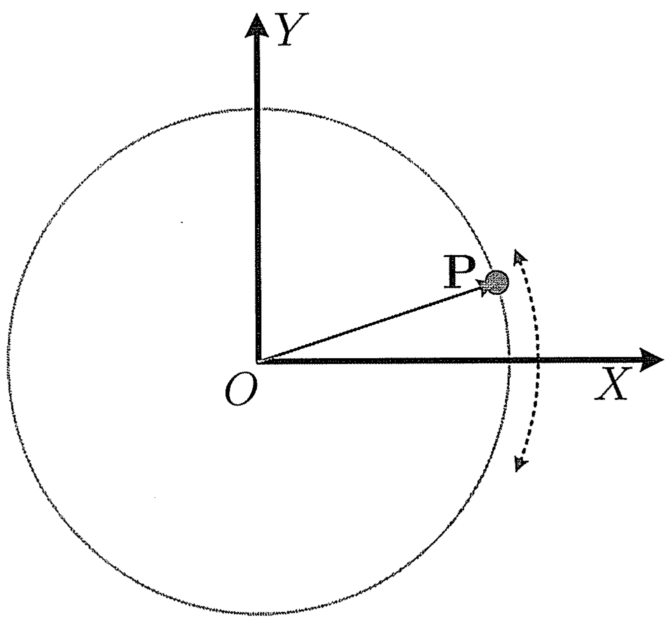
Example: Kinematic Analysis of Particle in circular movement



Particle P: use generalized coordinates $q = \begin{bmatrix} x \\ y \end{bmatrix}$ to capture the location.

* kinematic constraint:

$$x^2 + y^2 - 1 = 0$$

(captures the fact that particle is constrained to move on a circle of radius 1)

* Driving constraint:

$$y - 0.1 \sin(50\pi t) = 0.$$

Then,

$$\Phi(q,t) = \begin{bmatrix} x^2 + y^2 - 1 \\ y - 0.1 \sin(50\pi t) \end{bmatrix} = 0.$$

Note that $nc = 2$ & $m_K = 1 = m_D \Rightarrow m = 2 \Rightarrow NDOF = 0$

Therefore, one can do kinematics Analysis.

For position analysis, one needs to use Newton-Raphson to solve $\Phi(q,t) = 0$ at each time t_n on a grid

$$t_0 \; t_1 \; t_2 \; \ldots \; t_N = t_{END} \rightarrow t$$

For velocity analysis:

$$\nu = -\Phi_t = -\begin{bmatrix} 0 \\ -5\pi\cos(50\pi t) \end{bmatrix} = \begin{bmatrix} 0 \\ 5\pi\cos(50\pi t) \end{bmatrix}$$

$$\Phi_q = \begin{bmatrix} 2x & 2y \\ 0 & 1 \end{bmatrix}$$

Then, after performing position analysis, the velocity is obtained at t_n by solving

$$\begin{bmatrix} 2x_n & 2y_n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_n \\ \dot{y}_n \end{bmatrix} = \begin{bmatrix} 0 \\ 5\pi\cos(50\pi t_n) \end{bmatrix}$$

For acceleration analysis we need to evaluate $\ddot{\Phi}$:

$$\dot{\Phi} = \begin{bmatrix} 2\dot{x}x + 2y\dot{y} \\ \dot{y} - 5\pi\cos(50\pi t) \end{bmatrix} \Rightarrow \ddot{\Phi} = \begin{bmatrix} 2\ddot{x}x + 2\dot{x}^2 + 2\dot{y}^2 + 2y\ddot{y} \\ \ddot{y} + 250\pi^2\sin(50\pi t) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\ddot{x}x + 2\ddot{y}y \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} -2\dot{x}^2 - 2\dot{y}^2 \\ -250\pi^2\sin(50\pi t) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x_n & 2y_n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{x}_n \\ \ddot{y}_n \end{bmatrix} = \begin{bmatrix} -2\dot{x}_n^2 - 2\dot{y}_n^2 \\ -250\pi^2\sin(50\pi t_n) \end{bmatrix}$$

Thus, at each t_n, once the position and velocity are available $(x_n, y_n, \dot{x}_n, \dot{y}_n)$ you get the accelerations $\ddot{x}_n$ and $\ddot{y}_n$ by solving the linear system $\Phi_q \ddot{q} = \gamma$ above