Example chain Rule

$$f(q) = \cos(q_1 + q_2^2)$$

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 4\alpha + \ln\beta + \sin\gamma \\ e^{\alpha} + \beta \end{bmatrix}$$

Then,

$$\phi = \cos\left[(4\alpha + \ln\beta + \sin\gamma) + (e^{\alpha} + \beta)^2 \right].$$

Note that

$$f : \mathbb{R}^2 \to \mathbb{R}$$

$$q : \mathbb{R}^3 \to \mathbb{R}^2$$

$$\phi = f \circ q : \mathbb{R}^3 \to \mathbb{R}.$$

Then

$$\frac{\partial \phi}{\partial x} = \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial x} \quad , \quad x = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}.$$

You can compute ϕ_x in two ways (see next page):

Approach 1:

We already determined that

$$\phi(x) = \cos\left[(4\alpha + \ln\beta + \sin\gamma) + (e^{\alpha} + \beta)^2\right]$$

Then

$$x = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \qquad \phi_x = \begin{bmatrix} \phi_\alpha & \phi_\beta & \phi_\gamma \end{bmatrix}$$

$$\phi_\alpha = -\sin\left[(4\alpha + \ln\beta + \sin\gamma) + (e^{\alpha} + \beta)^2\right] \cdot \left[4 + 2(e^{\alpha} + \beta) \cdot e^{\alpha}\right]$$

$$\phi_\beta = -\sin\left[(4\alpha + \ln\beta + \sin\gamma) + (e^{\alpha} + \beta)^2\right] \cdot \left[\frac{1}{\beta} + 2(e^{\alpha} + \beta)\right]$$

$$\phi_\gamma = -\sin\left[(4\alpha + \ln\beta + \sin\gamma) + (e^{\alpha} + \beta)^2\right] \cdot \cos\gamma.$$

Approach 2:

$$\frac{\partial\phi}{\partial x} = \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial x} = \left[-\sin(q_1 + q_2^2) \quad -\sin(q_1 + q_2^2) \cdot 2q_2\right] \cdot$$

$$\cdot \begin{bmatrix} 4 & \frac{1}{\beta} & \cos\gamma \\ e^{\alpha} & 1 & 0 \end{bmatrix} = -\sin(q_1 + q_2^2) \begin{bmatrix} 1 & 2q_2 \end{bmatrix} \begin{bmatrix} 4 & \frac{1}{\beta} & \cos\gamma \\ e^{\alpha} & 1 & 0 \end{bmatrix}$$

$$= -\sin(q_1 + q_2^2) \left[4 + 2e^{\alpha} q_2 \quad \frac{1}{\beta} + 2q_2 \quad \cos\gamma\right]$$

Since $q_1 = 4\alpha + \ln\beta + \sin\gamma$ & $q_2 = e^{\alpha} + \beta$ we get,

$$\phi_x = -\sin\left[(4\alpha + \ln\beta + \sin\gamma) + (e^\alpha + \beta)^2\right]\left[4 + 2e^\alpha(e^\alpha + \beta) \quad \frac{1}{\beta} + 2(e^\alpha + \beta) \quad \cos\gamma\right],$$

which is the same result as before.