COMPUTER AIDED KINEMATICS AND DYNAMICS OF MECHANICAL SYSTEMS

VOLUME I: BASIC METHODS

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Preface

OBJECTIVES

The field of kinematics and dynamics of mechanical systems has progressed from a manual graphics art to a highly developed discipline in analytical geometry and dynamics. Large displacements and rotations that occur in the kinematic and dynamic performance of mechanical systems lead to nonlinear mathematical models that must be formulated and analyzed. The analytical complexity of nonlinear algebraic equations of kinematics and nonlinear differential equations of dynamics makes it impossible to obtain closed-form solutions in most applications. The literature in this field, therefore, contains a large number of specialized techniques and elegant analytical methods for analysis of special-purpose mechanisms and machines. This situation contrasts sharply with the literature on finite element structural analysis and electronic circuit analysis, both of which are much more systematically developed and oriented toward both computer formulation and solution of their governing equations.

This text presents basic methods for the analysis of the kinematics and dynamics of planar and spatial systems, using a Cartesian coordinate approach that is applicable to broad classes of mechanisms and machines. The approach taken places emphasis on systematic methods that are used in computer formulation and solution of the governing equations of kinematics and dynamics, thus removing the burden of routine derivation and computation from the engineer. Basic theoretical considerations that must be understood by the engineer in preparing models for computer simulation and in interpreting results, to assure that reality has indeed been modeled, are emphasized throughout the text. The objective is to prepare the reader to use a large-scale computer code that implements tedious computation, but requires its user to make modeling judgments and to critically evaluate reasonableness of results.

The text is intended for use in junior or senior undergraduate and beginning graduate engineering instruction and for self-study by practicing engineers who have a batchelor's degree. Matrix methods for vector analysis and multivariable calculus are introduced and used throughout the text. Equations of kinematics are developed using only basic methods of vector analysis and calculus.

Equations of dynamics are derived, beginning with two of Newton's basic laws of motion for particles: (1) force equals mass times acceleration, and (2) reaction forces are equal in magnitude and opposite in direction. Numerical methods are presented in adequate detail for an understanding of all the calculations that are required for computer solution of the equations of kinematics and dynamics.

The focus of the text is on the kinematic and dynamic analysis of moderate- and large-scale mechanical systems that are encountered in diverse fields of application. Considerable emphasis is therefore placed on modeling mechanical systems and interpreting the results obtained from computer simulations. Examples are presented to illustrate the physical difficulties associated with redundant constraints, lock-up of mechanisms, and singular configurations that prevent a mechanism or machine from performing the intended function. Since poorly designed mechanisms and machines may fail physically, it is clear that the mathematics that represents their performance must get into trouble. The mathematical consequences of mechanism and machine physical failure are identified and illustrated to assist the reader in interpreting the numerical results and determining the physical implications of the performance of the system.

This text serves as the theoretical foundation for a large-scale computer code called the Dynamic Analysis and Design System (DADS), which is commercially available from Computer Aided Design Software Incorporated, P.O. Box 203, Oakdale, Iowa 52319. This computer code is available, at minimal cost, for use in academic instruction. Examples and exercises are presented in the text, using the DADS computer code to present formulations and results for a variety of applications, to interpret results, and to investigate design alternatives that may improve performance. While the reader can gain insights with analytical examples and exercises in the text, it is very difficult to develop a practical capability in kinematics and dynamics of machines without substantial experience with real mechanical systems. It is therefore recommended that the DADS computer code be used in conjunction with the text, so that the reader can create models and gain experience in actual kinematic and dynamic analysis.

ORGANIZATION OF TEXT

Chapter 1 discusses the scope of the kinematics and dynamics of mechanical systems and considerations that guide the development of computer-aided engineering methods in kinematics and dynamics.

Chapters 2 through 8 comprise Part One of the text, which is concerned with planar kinematics and dynamics of mechanical systems. Chapter 2 presents a self-contained development of the matrix algebra and multivariable calculus notation and methods that are used throughout the text. Cartesian coordinates that are used to position and orient bodies in the plane and a library of kinematic constraints between bodies are presented and analyzed in Chapter 3. Methods of controlling, or driving, the motion of mechanisms and for carrying out position, velocity, and acceleration analysis are presented and illustrated with analytical examples. Numerical methods for solving equations of kinematics are presented in Chapter 4. Matrix methods for solving linear equations and the Newton–Raphson method for solving nonlinear equations are developed and illustrated. Planar kinematic systems are modeled and analyzed in Chapter 5 to illustrate the use of methods developed to evaluate performance and consider the effects of design changes.

The equations of dynamics for planar systems are developed in Chapter 6 to include internal forces due to springs and dampers, calculation of reaction forces in joints and drivers, and the determination of equilibrium configurations. Numerical methods for solving the equations of dynamics and statics are presented in Chapter 7. A brief, but self-contained, development of numerical integration methods that are suitable for large-scale system dynamic analysis is presented. Dynamic analyses of realistic examples, including those whose kinematics are studied in Chapter 5, are presented in Chapter 8.

Chapters 9 through 12 comprise Part Two of the text, which treats the kinematics and dynamics of mechanical systems that move in three-dimensional space. Euler parameters are introduced for orientation of bodies in space in Chapter 9. A library of kinematic constraints is derived and the governing kinematic equations for determining position, velocity, and acceleration are developed and analyzed. Kinematic analyses of spatial systems are presented in Chapter 10. Dynamic equations for spatial systems are developed in Chapter 11, as a direct extension of the derivation for planar systems in Chapter 6. Finally, dynamic analyses of spatial systems are carried out in Chapter 12.

USE OF TEXT IN TEACHING AND SELF-STUDY

The contents of this text have been used in four different courses and course sequences at the University of Iowa during the past eight years. One option is to teach a one-semester course on planar kinematics and dynamics of mechanical systems, at the undergraduate level, based on Part One of the text. The second one-semester course taught, at a slightly higher level, reviews basic ideas of planar systems and progresses to Part Two of the text, plus Chapters 4 and 7, for the study of spatial kinematics and dynamics of mechanical systems. A third single-semester course has been offered in planar and spatial kinematics alone. Finally, the entire content of the text has been used as the basis for a two-semester sequence that is taught at the senior undergraduate and beginning graduate level on the kinematics and dynamics of both planar and spatial systems.

In virtually all course offerings that have used this text, it has been found that project assignments using the DADS code have been a valuable tool in permitting students to formulate realistic models and obtain numerical feedback to gain analysis experience. One major challenge in teaching kinematic and dynamic analysis of mechanical systems is to develop a physical understanding of mechanisms and machines and the techniques for modeling and interpreting results. While special-purpose computer programs can be written to implement the computational methods presented in the text, it is far more efficient to use an available and thoroughly debugged computer program such as DADS to permit students to develop facility in the modeling and analysis of realistic mechanical systems. Studies of the effect of design variations on the kinematic and dynamic performance of machines, as in the examples treated in Chapters 5, 8, 10, and 12, are remarkably effective in motivating students and in gaining insights into use of the tools of kinematics and dynamics of mechanical systems.

CONTENTS OF VOLUME II: ADVANCED AND INTERDISCIPLINARY METHODS

Advanced and interdisciplinary topics in computer aided kinematics and dynamics of mechanical systems will be treated in a forthcoming companion volume. The foundations for the second volume include the basic formulation and numerical methods contained in this text, fundamental concepts of dynamics that are contained in a related text, *Intermediate Dynamics* [35], and recently developed fundamental theory and numerical methods.

The contents of Volume II will include methods of computer graphics for model formulation and animated graphics for interpretation of computer predictions. Feedback control and hydraulic subsystems, which are integral to dynamic performance of modern mechanical systems, will be modeled using formulations similar to those presented in this text.

Advanced relative coordinate kinematic and dynamic formulations that offer potential for high-speed kinematic and dynamic simulation will be developed and computational methods that are suitable for their implementation will be presented. Emerging multiprocessor parallel computer methods that provide a unique opportunity for high-speed computation, to include real-time man-in-the-loop simulation, will be developed with relative coordinate formulations. This body of emerging technology represents a major extension of the basic methods presented in the current text.

Special problems and interdisciplinary considerations in modern machine dynamics will be developed, including friction, stiction, impact, and intermittent motion; analysis of singular kinematic configurations and dynamic stability; definition of boundaries of robot and manipulator workspaces; and analysis of flexible multibody dynamics. Methods developed in these areas will be applicable with the basic formulations presented in this text and with advanced relative coordinate formulations.

Advanced numerical methods for the analysis and solution of differential-algebraic equations of motion, using emerging manifold theoretic and differential geometric methods of mathematics and numerical analysis, will be developed.

These methods provide powerful new tools for solving the equations of motion of broad classes of mechanical systems. Finally, design sensitivity analysis and optimization methods that predict the effect of design variations on dynamic performance and that optimize designs will be presented and illustrated.

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