

Symbol Index

Symbol	Definition	Page
\vec{a}	Geometric vector	20
$a, \vec{a} $	Magnitude of vector \vec{a}	20
$\mathbf{a} = [a_x, a_y, a_z]^T$	Column vector of global components of \vec{a}	311
$\mathbf{a}' = [a_{x'}, a_{y'}, a_{z'}]^T$	Column vector of local components of \vec{a}	311
$\vec{a}^\perp, \mathbf{a}^\perp$	Vector orthogonal to \vec{a}, \mathbf{a}	23, 30
$\tilde{\mathbf{a}}$	3×3 vector product matrix associated with \mathbf{a}	312
$\dot{\mathbf{a}}, \ddot{\mathbf{a}}$	First and second time derivatives of \mathbf{a}	37
$a(\mathbf{q})$	Scalar valued function of vector variable	38
$a_{\mathbf{q}}$	partial derivative of $a(\mathbf{q})$ with respect to \mathbf{q}	38
\mathbf{A}	Rotation transformation matrix	33, 322
$\dot{\mathbf{A}}, \ddot{\mathbf{A}}$	First and second time derivative of \mathbf{A}	37
\mathbf{A}_i	Transformation from $x_i'-y_i'-z_i'$ to $x-y-z$	33, 327
\mathbf{A}_{ij}	Transformation from $x_j'-y_j'-z_j'$ to $x_i'-y_i'-z_i'$	34, 327
\mathbf{A}^T	Transpose of matrix \mathbf{A}	24
\mathbf{A}^{-1}	Inverse of matrix \mathbf{A}	29
$\vec{a} \cdot \vec{b}$	Scalar product (dot product)	22, 309
$\vec{a} \times \vec{b}$	Vector product (cross product)	310
\mathbf{B}	$d\mathbf{A}/d\phi$	41, 202
$\mathbf{B}'\dot{\mathbf{q}}(t_0) = \mathbf{v}'$	Initial velocity conditions	228, 442
c	Damping coefficient	216, 446
c_0	Torsional damping coefficient	218, 448
\mathbf{C}_i^P	Transformation from $x_i''-y_i''-z_i''$ to $x_i'-y_i'-z_i'$	348
\mathbf{d}_{ij}	Vector from P_i on body i to P_j on body j	65, 351
$dm(P)$	Differential mass at point P	200, 417
DOF	Degrees of freedom	51
\mathbf{E}	3×4 matrix that depends on Euler parameters	343
f	Spring-damper-actuator force	216, 447
F	General actuator force	216, 447
$\mathbf{f}, \mathbf{g}, \mathbf{h}$	Unit vectors along $x', y',$ and z' axes	315
\mathbf{F}	Resultant force on body	204, 420
\mathbf{F}^A	Applied force on body	219, 441
\mathbf{F}^C	Constraint force on body	219, 441
\mathbf{F}_i^{nk}	Joint reaction force	234, 450
\mathbf{F}^P	Force acting at point P	214

Symbol	Definition	Page
G	3×4 matrix that depends on Euler parameters	343
h	Time step	264
I, I_n	Identity matrix	25
J'	Polar moment of inertia of planar body	204
J'	Inertia matrix of body	420
ℓ	Length of spring-damper-actuator	215, 446
ℓ_0	Spring free length	216, 447
k	Spring constant	216, 447
k_b	Torsional spring constant	218, 448
m	Mass of rigid body	203, 420
M	Composite mass matrix	219, 440
n	Torque acting on planar body	204
\mathbf{n}'	Torque acting on body	420
N	General actuator torque	218
\mathbf{n}'^A	Applied torque on body	441
\mathbf{n}'^C	Constraint torque on body	441
nb	Number of bodies	49, 382
nc	Number of generalized coordinates	49
nh	Number of holonomic constraints	50
$\mathbf{p} = [e_0, \mathbf{e}^T]^T = [e_0, e_1, e_2, e_3]^T$	Euler parameter vector	338, 340
P_i, Q_i, R_i	Joint definition points	348
$\mathbf{q} = [q_1, q_2, \dots, q_{nc}]^T$	Vector of generalized coordinates	49, 219, 382
Q	Generalized force	213, 219
Q^A	Vector of generalized applied forces	219
Q^C	Vector of generalized constraint forces	219
\mathbf{r}	Vector to centroid of body	33, 323
\mathbf{r}^P	Vector to point P on body	33, 323
R	Orthogonal rotational matrix	31
R^n	n -dimensional real space	311
SE	Strain energy of compliant components	231
\mathbf{s}^P	Global representation of body fixed vector to point P	33, 323
\mathbf{s}'^P	Local representation of body fixed vector to point P	33, 323
\mathbf{T}_i^{jk}	Joint reaction torque	234, 450
t_n	Time grid point	264
TPE	Total potential energy	231
$\text{tr } \mathbf{A}$	Trace of matrix A	340
u	Vector of dependent generalized coordinates	249
v	Vector of independent generalized coordinates	249

Symbol	Definition	Page
$V(\mathbf{q})$	Potential energy	278
x - y , x - y - z	Global reference frame	50, 318
x' - y' , x' - y' - z'	Body fixed local reference frame	318
x'' - y'' - z''	Body fixed joint definition frame	50, 348
t_0	Initial time	227
δ	Variation operator	333
$\delta\phi$	Planar rotation variation	202
$\delta\pi$	Virtual rotation	333
$\delta\mathbf{r}$	Virtual displacement	202
$\delta\mathbf{r}^P$	Virtual displacement of point P	200, 332
$\delta\mathbf{q}$	Variation in generalized coordinate vector \mathbf{q}	201
δW	Virtual work	213
$\Delta\mathbf{q}^{(k)}$	Newton–Raphson correction	100, 146, 384
$\Phi(\mathbf{q}, t) = \mathbf{0}$	Combined constraints	38, 51, 383
$\Phi^D(\mathbf{q}, t) = \mathbf{0}$	Driving constraints	51, 383
$\Phi^K(\mathbf{q}, t) = \mathbf{0}$	Kinematic constraints	50, 383
$\Phi^{d1}(\mathbf{a}_i, \mathbf{a}_j) = 0$	Dot-1 constraint	350
$\Phi^{d2}(\mathbf{a}_i, \mathbf{d}_{ij}) = 0$	Dot-2 constraint	351
$\Phi^I(\mathbf{q}(t_0), t_0) = \mathbf{0}$	Initial position conditions	228, 442
$\Phi^P(\mathbf{p}) = \mathbf{0}$	Euler parameter normalization constraints	383, 440
$\Phi^{p1}(\mathbf{h}_i, \mathbf{h}_j) = \mathbf{0}$	Parallel-1 constraint	355
$\Phi^{p2}(\mathbf{h}_i, \mathbf{d}_{ij}) = \mathbf{0}$	Parallel-2 constraint	355
$\Phi_{\mathbf{q}}$	Constraint Jacobian	38, 53, 384
Φ_{π}	Constraint rotation Jacobian	357
$\Phi^{sj}(P_i, P_j) = \mathbf{0}$	Spherical joint constraint	353
$\Phi^{ss}(P_i, P_j, C) = 0$	Spherical–spherical joint constraint	353
$\Phi_{\mathbf{u}}$	Constraint \mathbf{u} -Jacobian	250
$\Phi_{\mathbf{v}}$	Constraint \mathbf{v} -Jacobian	250
γ	Right side of acceleration equation	53, 385
$\hat{\gamma}$	Modified right side of acceleration equation	257
λ	Lagrange multiplier vector	223, 441
ν	Right side of velocity equation	52, 384
Θ_{ij}	Angle from x_i' axis to x_j' axis	218
Θ_0	Free angle of rotational spring	218
$\Theta(\vec{a}, \vec{b})$	Angle between vector \vec{a} and \vec{b}	22, 308
ρ''	Location of centroid in the x'' - y'' - z'' frame	207, 425
ω	Angular velocity	330
$\mathbf{0}$	Zero matrix	25
$\vec{0}$	Zero vector	20

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