CHAPTER TEN

Spatial Kinematic Modeling and Analysis

10.1 MODELING AND ANALYSIS TECHNIQUES

The basic techniques for kinematic modeling of spatial mechanisms and machines are identical to those discussed in Section 5.1 for planar systems. The principal and very important difference between modeling planar and spatial kinematic systems concerns the redundancy of constraints. In spatial systems, it is easy to implement what appear to be proper constraints, only to find that redundancies exist.

Example 10.1.1: Consider modeling the slider–crank mechanism of Fig. 10.1.1 with three revolute joints and one translational joint. If Euler parameters are used for orientation, each body has seven generalized coordinates. There are, therefore, 28 generalized coordinates for the four bodies. The revolute and translational joints each have five constraint equations, yielding 20 constraint equations. In addition, the six constraints for fixing ground and four Euler parameter normalization constraints yield a total of 30 constraint equations. Since the actual mechanism is intended to have one degree of freedom, there must be three redundant constraints. To understand why this is the case, note that if the revolute joint axes in bodies 4, 1, and 2 at points A, B, and C in Fig. 10.1.1 are all parallel then the revolute axes in bodies 2 and 3 at point C are automatically parallel, yielding two redundant constraint equations. Finally, the revolute joints at points A and B require bodies 1 and 2 to move in the x-y plane, and the translational joint between body 3 and ground also requires that the origin of the body-fixed reference frame in body 3 lie in the x-y plane. Therefore, the constraint on the z coordinate of the revolute joint at point C is redundant. These conditions define the three degrees of redundancy in this overly constrained model.

As an alternative check on redundancy, the *manufacturing imperfection test* outlined in step 1(b) of the procedure suggested in Section 5.1 can be applied to this mechanism. The imperfect configuration of the slider–crank shown in Fig.

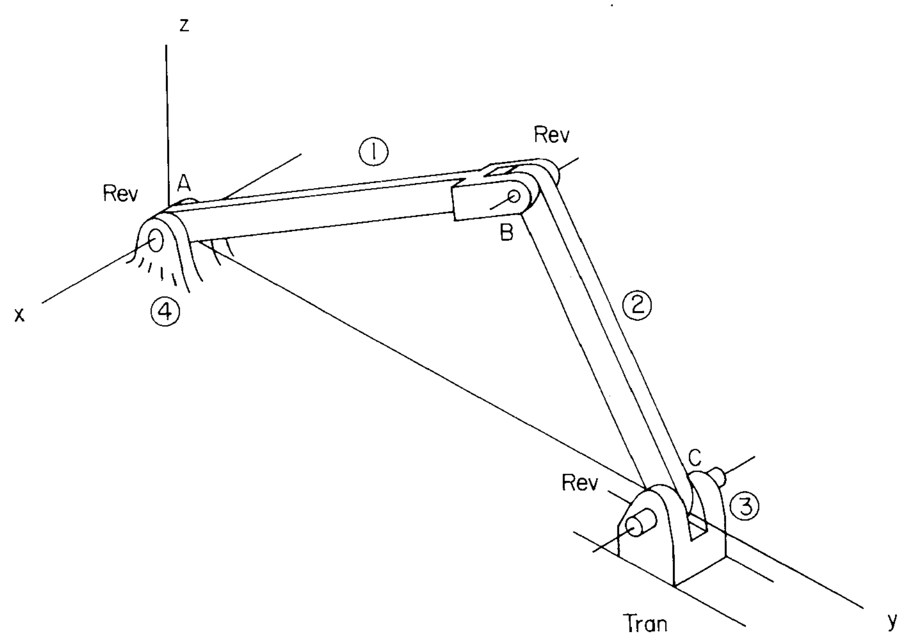

Figure 10.1.1 Spatial slider–crank.

10.1.2 has a misaligned revolute joint axis in ground at point A. If all other revolute joint axes are parallel, then a misalignment of the revolute joint axes between bodies 2 and 3 exists at point C, preventing assembly. This implies two degrees of redundancy, since two parameters must be adjusted to precisely align the axes of the revolute joint at point C. A third degree of redundancy exists, since the center point of the revolute joint on body 2 does not lie in the y-z plane; hence there is an offset, which defines one additional degree of redundancy.

An alternative model might be considered in which the translational joint between bodies 3 and 4 is replaced by absolute x and z constraints on point C in body 3. This reduces the number of constraint equations by 3 and creates a situation in which the number of generalized coordinates minus the number of constraints is one, which is desired. While the counting check of step 1(a) of Section 5.1 is thus satisfied, this is still not a good model. To see why, consider again the misalignment shown in Fig. 10.1.2. The offset inconsistency in the revolute joint between bodies 2 and 3 remains, so the system cannot be assembled. This apparent difficulty could be overcome by replacing the revolute joint between bodies 2 and 3 by a cylindrical joint, which has one fewer constraint equations. As a result, the mechanism can be assembled, as shown in Fig. 10.1.3. While the mechanism can be assembled, an extra degree of freedom has inadvertently been created that permits body 3 to rotate about the cylindrical

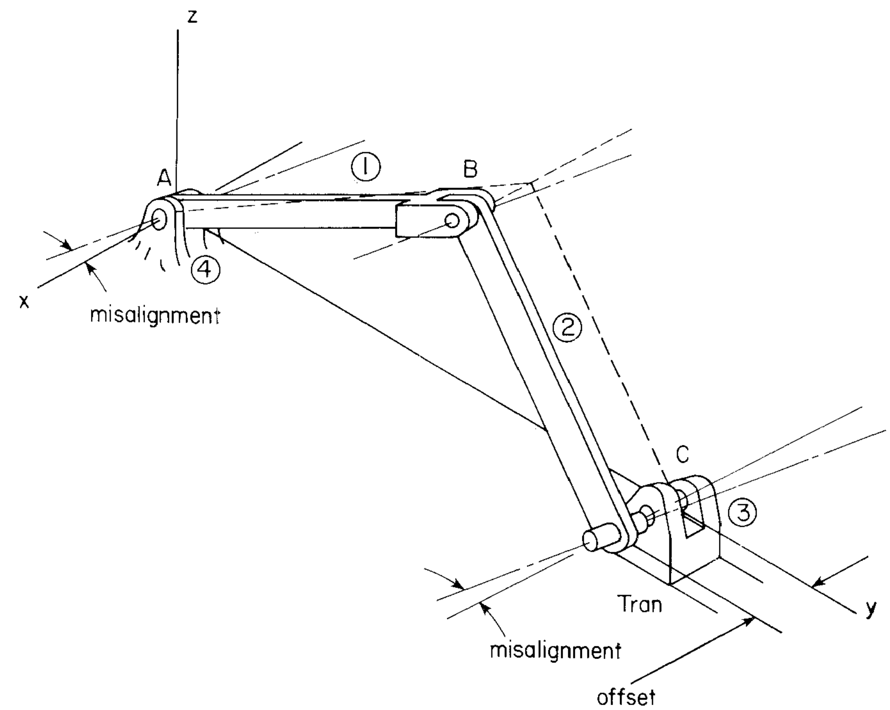

Figure 10.1.2 Imperfect spatial slider–crank.

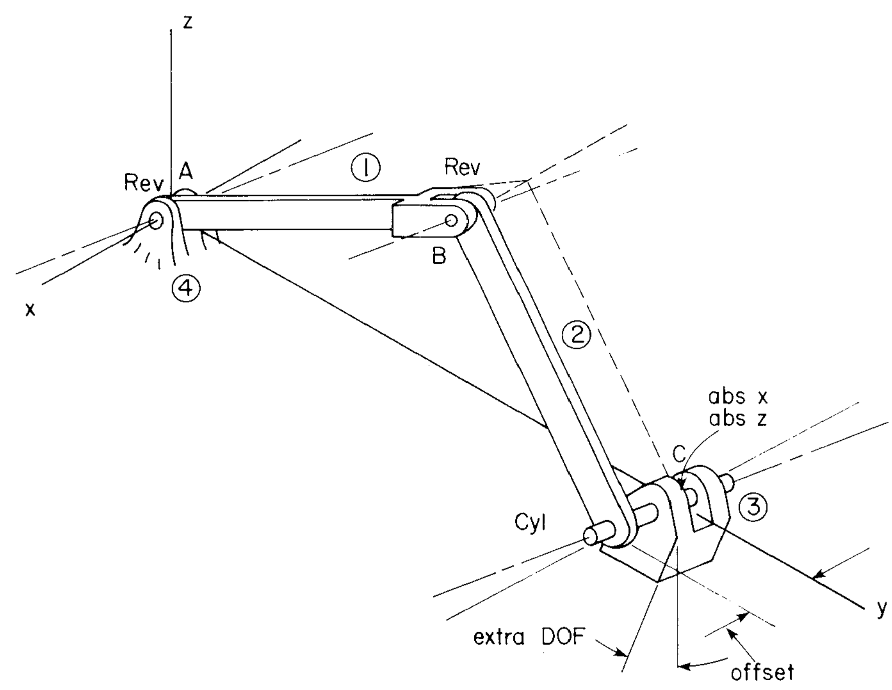

Figure 10.1.3 Modified spatial slider–crank.

joint axis, which is not desired in the actual mechanism. Thus, careless adjustment in constraint definition, based only on counting constraint equations, can lead to either or both redundant constraints and unwanted extra degrees of freedom.

Apart from the need for careful definition of independent kinematic constraints, the kinematic analysis of spatial systems is a direct extension of the kinematic analysis of planar systems. Kinematic analyses of realistic mechanisms are carried out in the remaining sections of this chapter using the DADS computer code [27]. The effects of design variations are studied to illustrate the use of the methods of Chapter 9 in support of the design of mechanical systems.

10.2 KINEMATIC ANALYSIS OF A SPATIAL SLIDER–CRANK MECHANISM

10.2.1 Model

The spatial slider–crank mechanism shown in Fig. 10.2.1 is modeled using four bodies. The model is defined as follows:

Model	
Bodies	
Four bodies	$nc = 28$
Constraints	
Revolute joint (crank, ground)	5
Spherical joint (crank, connecting rod)	3
Revolute–cylindrical joint (connecting rod, slider)	3
Translational joint (slider, ground)	5
Distance constraint (connecting rod, slider)	1
Ground constraint	6
Euler parameter normalization constraint	4
DOF = 28 − 27 = 1.	$nh = 27$

The motion of the system can be defined by requiring that the orientation of the crank (body 1) be some function of time. This is equivalent to imposing a driving constraint so that the remaining degrees of freedom are determined.

To define a kinematic joint, six points (three points on each body) are chosen, depending on the type of joint that is intended. These points, P_i, Q_i, R_i, P_j, Q_j, and R_j, defined in their respective centroidal body-fixed reference frames on bodies i and j, form joint reference triads.

For the revolute joint at point A in Fig. 10.2.1, the common point in the joint is defined by points P_1 and P_4 given in Table 10.2.1. Points Q_1 and Q_4 in Table 10.2.1 are chosen to define the axis of rotation in the bodies. Vectors P_1Q_1 and P_4Q_4 define the z'' axes of the joint reference triads on each body. Points R_1

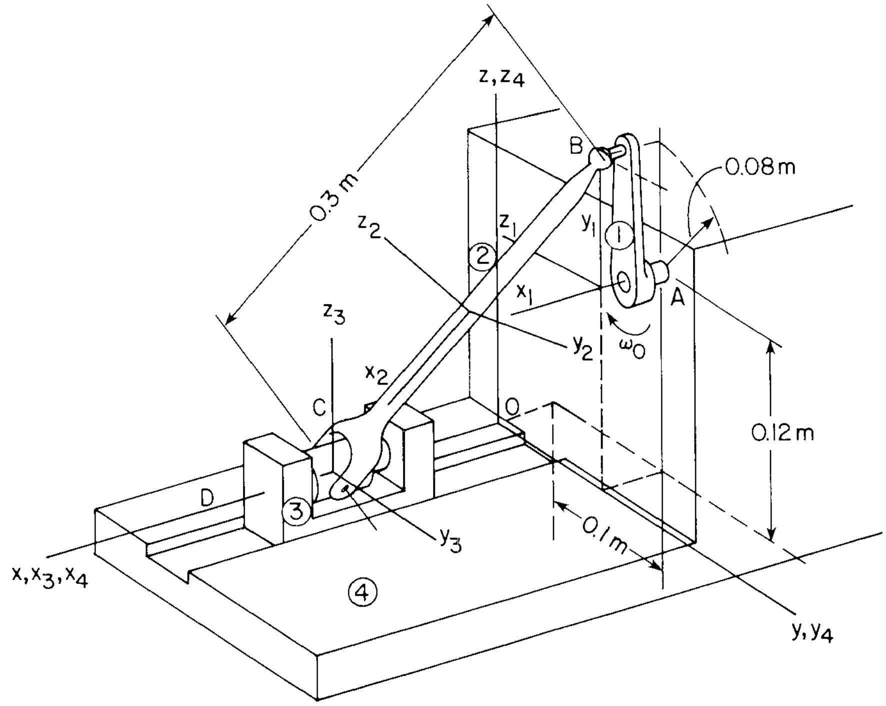

Figure 10.2.1 Spatial slider–crank.

and R_4 in Table 10.2.1 define the joint x'' axes. These six points define the revolute joint at point A in the model.

TABLE 10.2.1 Data for Revolute Joint

Body \ Point	P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Crank ①	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0
Ground ④	0.0	0.1	0.12	1.0	0.1	0.12	0.0	0.1	1.12

For the spherical joint at point B in Fig. 10.2.1, points P_1 and P_2 define the common point in the joint. Points Q_1, Q_2, R_1, and R_2 can be arbitrarily chosen to define the joint reference triads. These six points for the spherical joint at point B in the model are defined in Table 10.2.2.

For the revolute–cylindrical joint at point C in Fig. 10.2.1, points P_2, P_3, Q_2, and Q_3 are chosen to define the axis of rotation of the joint, and hence the z'' axes of joint reference triads. Points R_2 and R_3 then define the x'' axes of the

TABLE 10.2.2 Data for Spherical Joint

Body \ Point	P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Crank ①	0.0	0.08	0.0	0.0	0.08	1.0	1.0	0.08	0.0
Connecting rod ②	−0.15	0.0	0.0	−0.15	0.0	1.0	1.15	0.0	0.0

TABLE 10.2.3 Data for Revolute–Cylindrical Joint

Body \ Point	P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Connecting rod ②	0.15	0.0	0.0	0.15	1.0	0.0	1.15	0.0	0.0
Slider ③	0.2	0.0	0.0	1.2	0.0	0.0	0.2	1.0	0.0

joint reference triads. These six points for the revolute–cylindrical joint in the model are defined in Table 10.2.3.

For the translational joint at point D in Fig. 10.2.1, points P_3, P_4, Q_3, and Q_4 are chosen to define the common lines of translation of the joint, which are the z'' axes of the joint reference triads. Points R_3 and R_4 then define the x'' axes of the joint reference triads. These six points for the translational joint are defined in Table 10.2.4.

TABLE 10.2.4 Data for Translational Joint

Body \ Point	P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Slider ③	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0
Ground ④	0.2	0.0	0.0	1.2	0.0	0.0	0.2	1.0	0.0

For the distance constraint between the connecting rod and slider, P_2 and P_3 are points between which the distance is fixed. Points Q_2, R_2, Q_3, and R_3 can be arbitrarily chosen to define the joint reference triads. These six points for the distance constraint are defined in Table 10.2.5.

TABLE 10.2.5 Data for Distance Constraint

Body \ Point	P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'	Distance
Connecting rod ②	0.15	0.0	0.0	0.15	1.0	0.0	0.15	0.0	1.0	1.0
Slider ③	1.0	0.0	0.0	1.0	1.0	0.0	1.0	0.0	1.0	

10.2.2 Assembly

The position and orientation of each body reference frame in the global reference frame is estimated for initial assembly analysis. Table 10.2.6 provides estimates of generalized coordinates (abbreviated GC) for the model. Euler parameters are used to specify the orientation of each body. Table 10.2.7 shows the resulting assembled configuration.

TABLE 10.2.6 Position and Orientation Estimates

Body \ GC	x	y	z	e_1	e_2	e_3
Crank ①	0.0	0.1	0.12	0.71	0.0	0.0
Connecting rod ②	0.1	0.05	0.1	−0.21	0.40	−0.1
Slider ③	0.2	0.0	0.0	0.0	0.0	0.0
Ground ④	0.0	0.0	0.0	0.0	0.0	0.0

TABLE 10.2.7 Assembled Configuration

Body \ GC	x	y	z	e_0	e_1	e_2	e_3
Crank ①	0.00002	0.09982	0.12005	0.72090	0.69306	0.00004	−0.00004
Connecting rod ②	0.09993	0.05183	0.09998	0.88723	−0.21202	0.39833	−0.09569
Slider ③	0.19959	0.00057	−0.00008	1.0	0.0	−0.00012	−0.00025
Ground ④	0.0	0.0	0.0	1.0	0.0	0.0	0.0

10.2.3 Driver Specification

The crank can only rotate in the revolute joint. Taking the relative angle θ in the joint as the driving coordinate, it is specified that the crank rotate at $\omega_0 = 2\pi$ rad/s. The driver is thus specified by the condition

$$\theta = 2\pi t$$

10.2.4 Analysis

Three runs are made with varying connecting rod lengths of 0.3, 0.27, and 0.24 m. Lock-up occurs when the length of the connecting rod is less than 0.2362 m, as shown in Fig. 10.2.2. The position, velocity, and acceleration histories of point C on the slider are shown in Figs. 10.2.3, 10.2.4, and 10.2.5. Notice the high peak of acceleration in Fig. 10.2.5 when $\ell = 0.24$ m, which is a near singular design.

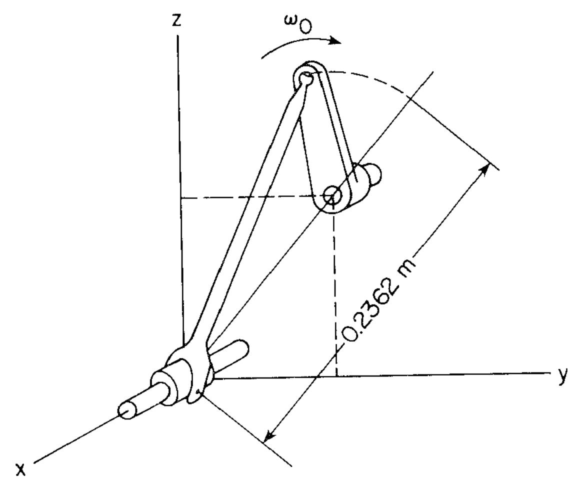

Figure 10.2.2 A lock-up configuration.

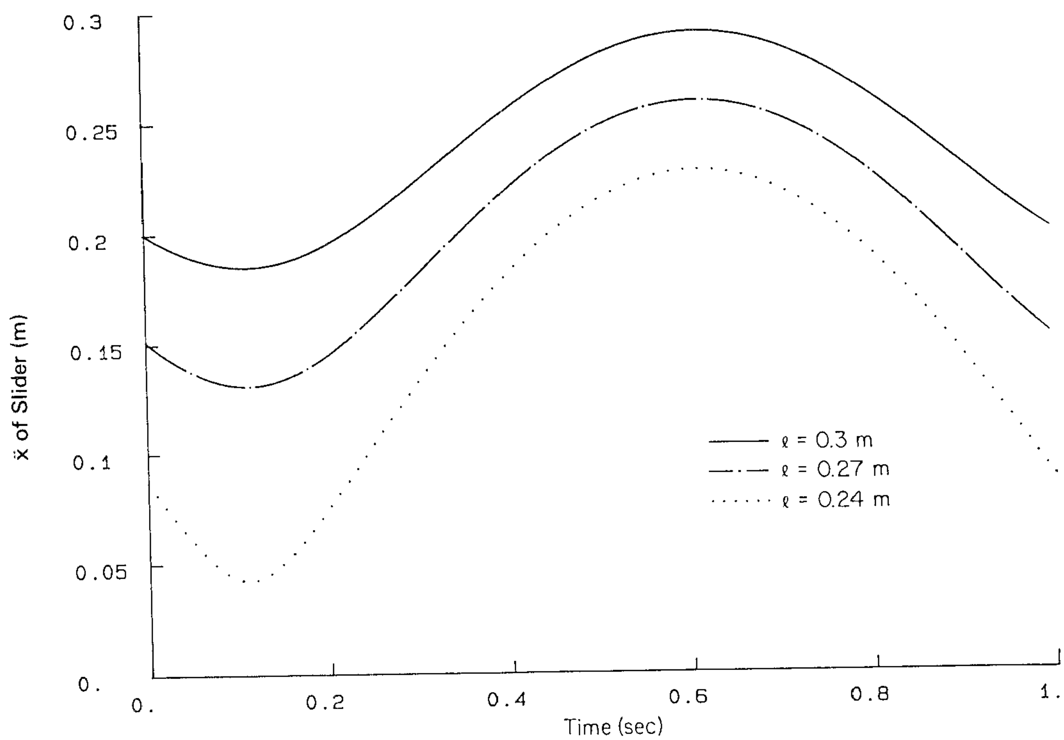

Figure 10.2.3 x of slider versus time.

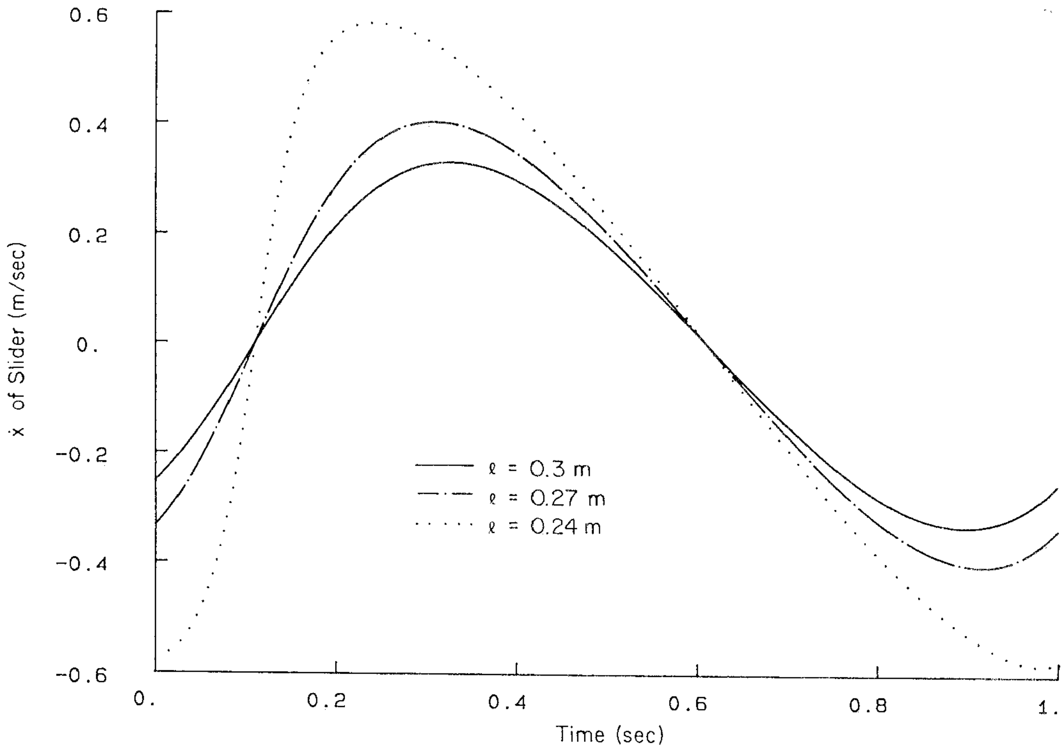

Figure 10.2.4 $\dot{x}$ of slider versus time.

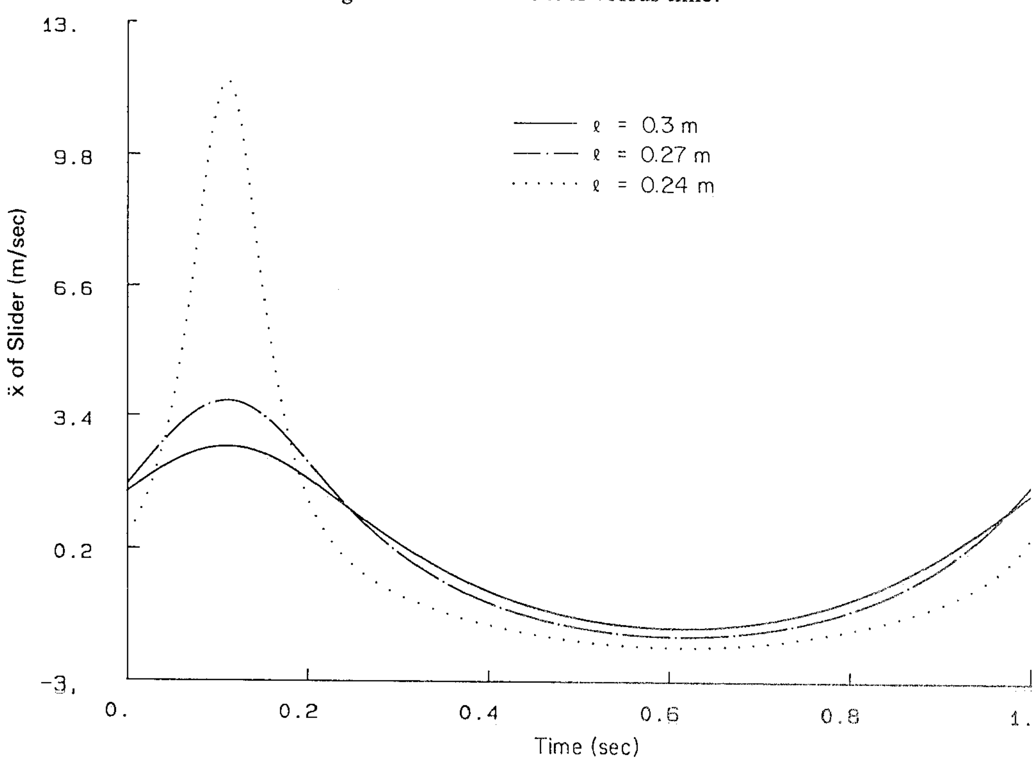

Figure 10.2.5 $\ddot{x}$ of slider versus time.

10.3 KINEMATIC ANALYSIS OF A SPATIAL FOUR-BAR MECHANISM

10.3.1 Alternative Models

A spatial four-bar mechanism can be modeled in many different ways. Two kinematically equivalent models are developed and analyzed here. In model 1 (Fig. 10.3.1), each link in the mechanism and ground is modeled as a body. Revolute, spherical, and universal joints are used to model this mechanism, as follows:

Model 1	
Bodies	
Four bodies	$nc = 28$
Constraints	
Revolute joints (links 1 and 3, ground): A	5
D	5
Universal joint (link 1, link 2): B	4
Spherical joint (link 2, link 3): C	3
Ground constraints (link 4):	6
Euler parameter normalization constraints	4
	$nh = 27$
DOF = 28 − 27 = 1.	

For kinematic analysis, the one remaining degree of freedom is eliminated by imposing a driving constraint.

To define each kinematic joint, six points are chosen according to the type of joint that is intended. For a revolute joint, points P_i and P_j are chosen to locate a common point in the joint. Points Q_i and Q_j are chosen to define the axis of rotation. Vectors P_iQ_i and P_jQ_j define the z'' axes of the joint reference triad on each body. Points R_i and R_j define the joint x'' axes. Revolute joint definition data for model 1 are given in Table 10.3.1.

TABLE 10.3.1 Revolute Joint Data, Model 1

Joint *A*

Body \ Point	P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Ground ④	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0
Link ①	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0

Joint *D*

Body \ Point	P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Link ③	0.0	3.7	0.0	1.0	3.7	0.0	0.0	2.7	0.0
Ground ④	−4.0	−8.5	0.0	−4.0	−9.5	0.0	−3.0	−8.5	0.0

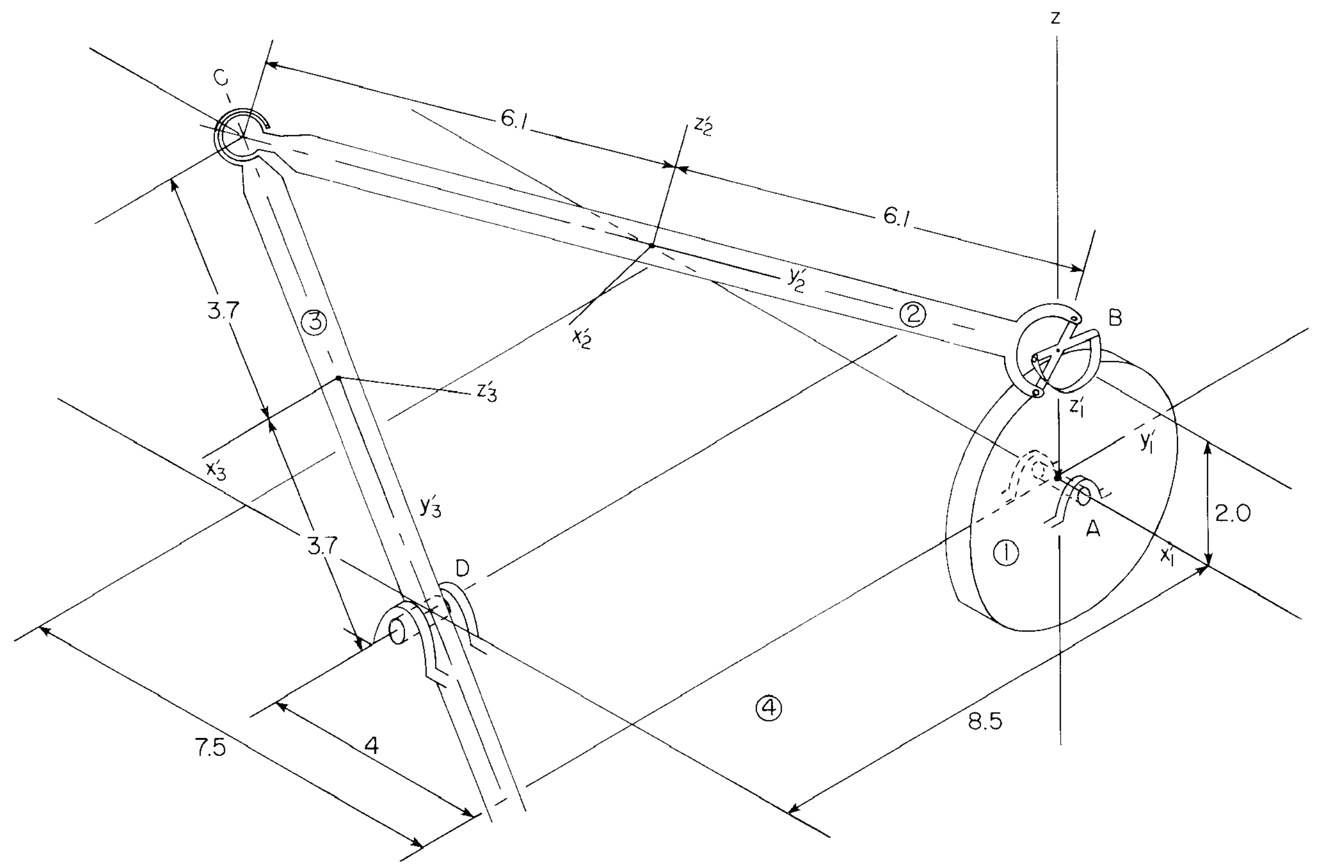

Figure 10.3.1 Spatial four-bar mechanism, model 1.

TABLE 10.3.2 Universal Joint Data, Model 1

Point / Body	P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Link ①	0.0	0.0	2.0	0.750	−0.662	2.0	0.244	0.277	2.929
Link ②	0.0	6.1	0.0	0.0	6.1	1.0	0.0	6.1	0.0

For a universal joint, the common points in the joint are P_i and P_j. Vectors P_iQ_i and P_jQ_j define the axes about which bodies are allowed to rotate. In addition, vectors P_iQ_i and P_jQ_j should be orthogonal to each other. Points R_i and R_j define the x'' axes of the joint reference triads. Universal joint definition data for joint B of model 1 are given in Table 10.3.2.

For spherical joints, P_i and P_j define the common point in the joint. Points Q_i, Q_j, R_i, and R_j are chosen to define the joint reference triads. These six points for the spherical joint at point C in model 1 are defined in Table 10.3.3.

TABLE 10.3.3 Spherical Joint Data, Model 1

Point / Body	P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Link ②	0.0	−6.1	0.0	0.0	−6.1	1.0	0.0	−5.1	0.0
Link ③	0.0	−3.7	0.0	0.0	−3.7	1.0	0.0	−2.7	0.0

In model 2 (Fig. 10.3.2), link BC is modeled as the coupler in a spherical–spherical composite joint. The other joints are the same as in model 1. This mechanism model is defined as follows:

Model 2	
Bodies	
Three bodies	$nc = 21$
Constraints	
Revolute joints 2 (links 1 and 2, ground): A	5
D	5
Spherical–spherical joint (link 1, link 2): BC	1
Ground constraints (link 3)	6
Euler parameter normalization constraints	3
	$nh = 20$
DOF = 21 − 20 = 1.	

To define the spherical–spherical joint between bodies 1 and 2, points P_1 and P_2 are used to locate the spherical joint on each body. Points Q_1, Q_2, R_1, and R_2

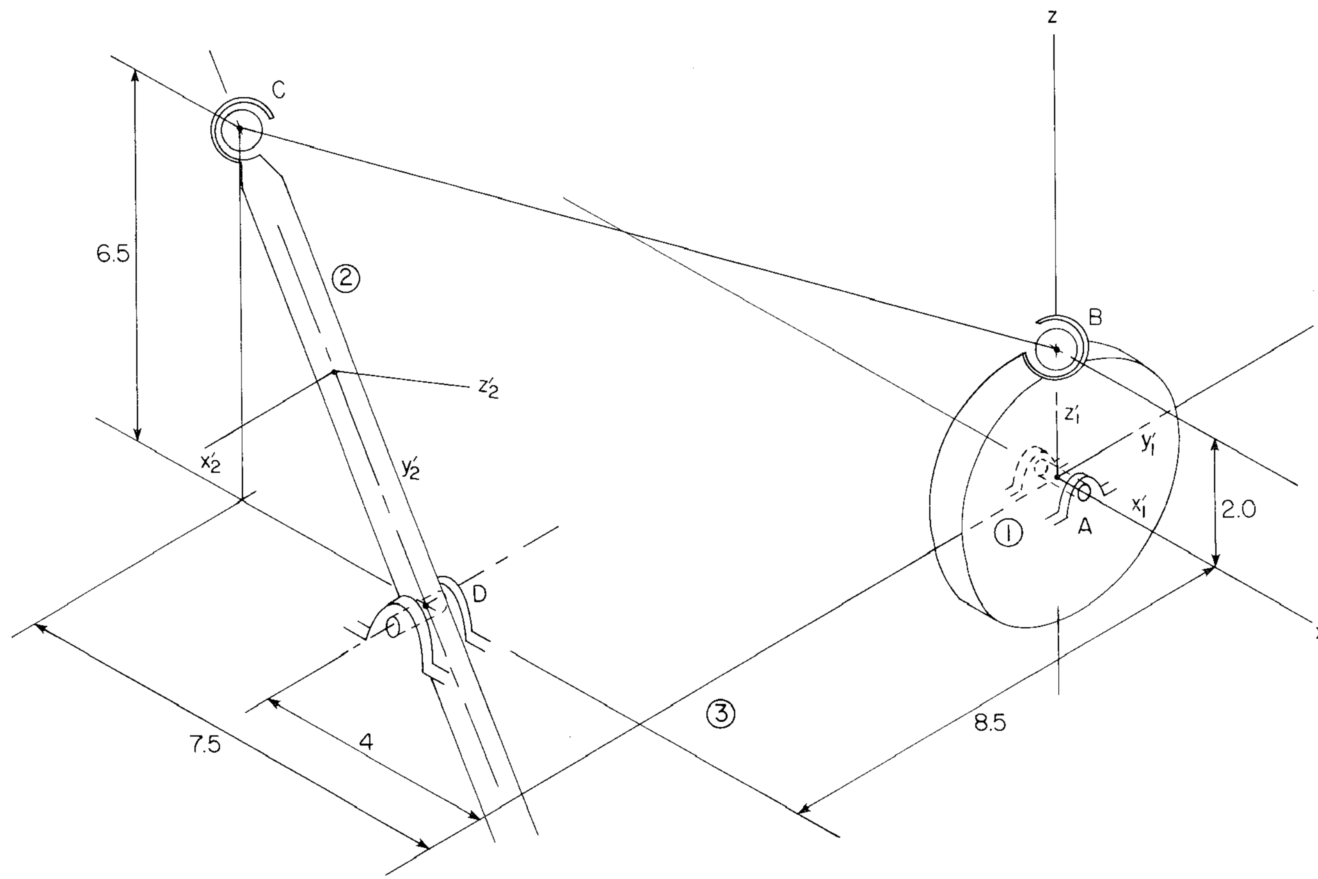

Figure 10.3.2 Spatial four-bar mechanism, model 2.

TABLE 10.3.4 Spherical–Spherical Joint Data, Model 2

Body \ Point	P			Q			R		
	x'	y'	z'	x'	y'	z'	x'	y'	z'
Link ①	0.0	0.0	2.0	0.0	1.0	2.0	1.0	0.0	2.0
Link ②	0.0	−3.7	0.0	1.0	−3.7	0.0	0.0	−2.7	0.0

are chosen to define the z'' and x'' axes of the joint reference triads. The length of the link is the distance between points P_1 and P_2, 12.19 m in this model. Spherical–spherical joint point definition data for model 2 are given in Table 10.3.4.

10.3.2 Assembly Analysis

Initial estimates for the position and orientation generalized coordinates (abbreviated GC) of each body reference frame with respect to the global reference frame are given in Table 10.3.5. Euler parameters are used to specify the

TABLE 10.3.5 Position and Orientation Estimates, Model 1

Body \ GC	x	y	z	e_1	e_2	e_3
Link ①	0.0	0.0	0.0	0.0	0.0	0.0
Link ②	−3.75	−4.25	4.25	−0.29	−0.27	−0.26
Link ③	−5.75	−8.5	3.25	−0.36	0.36	−0.61
Ground ④	0.0	0.0	0.0	0.0	0.0	0.0

orientation of each body. Table 10.3.6 shows the resulting assembled configuration.

For model 2, the position and orientation of link 2 are not required, because it is not modeled as a body. The remaining data in Tables 10.3.5 and 10.3.6 are valid for model 2.

TABLE 10.3.6 Assembled Configuration, Model 1

Body \ GC	x	y	z	e_0	e_1	e_2	e_3
Link ①	0.00016	0.00005	−0.00027	1.0	0.00001	−0.00004	0.00005
Link ②	−3.75300	−4.25020	4.25530	0.87944	−0.29098	−0.27410	−0.25910
Link ③	−5.75300	−8.50010	3.25530	0.60687	−0.36245	0.36247	−0.60684
Ground ④	0.0	0.0	0.0	1.0	0.0	0.0	0.0

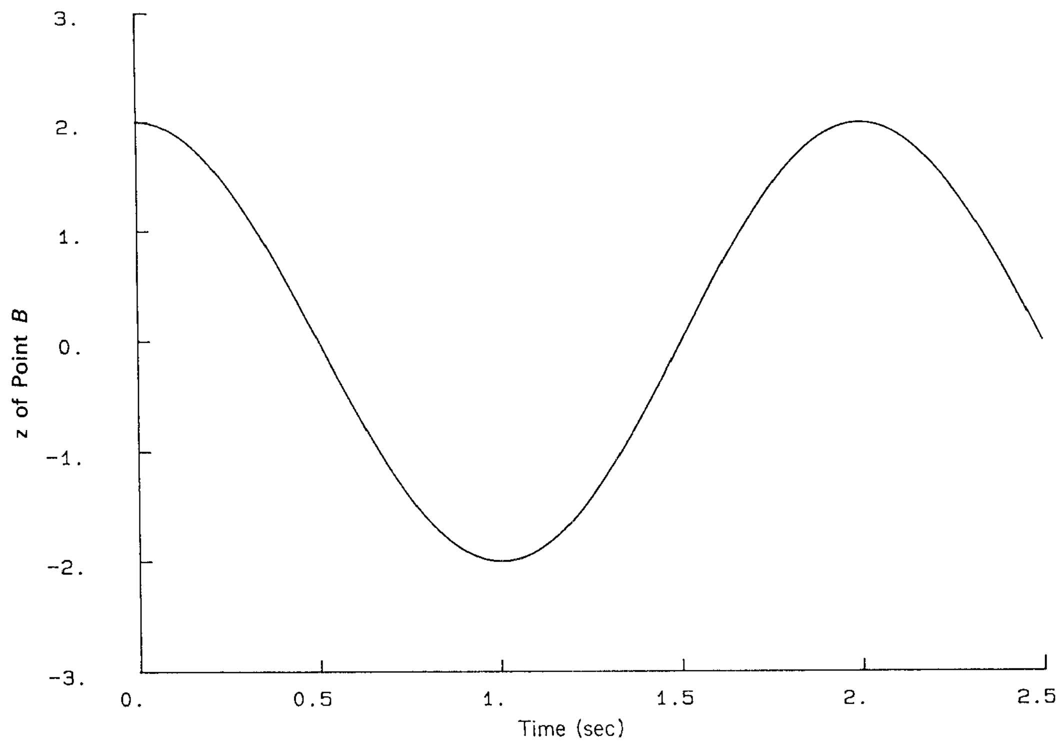

Figure 10.3.3 *z* coordinate of point *B* versus time, model 1.

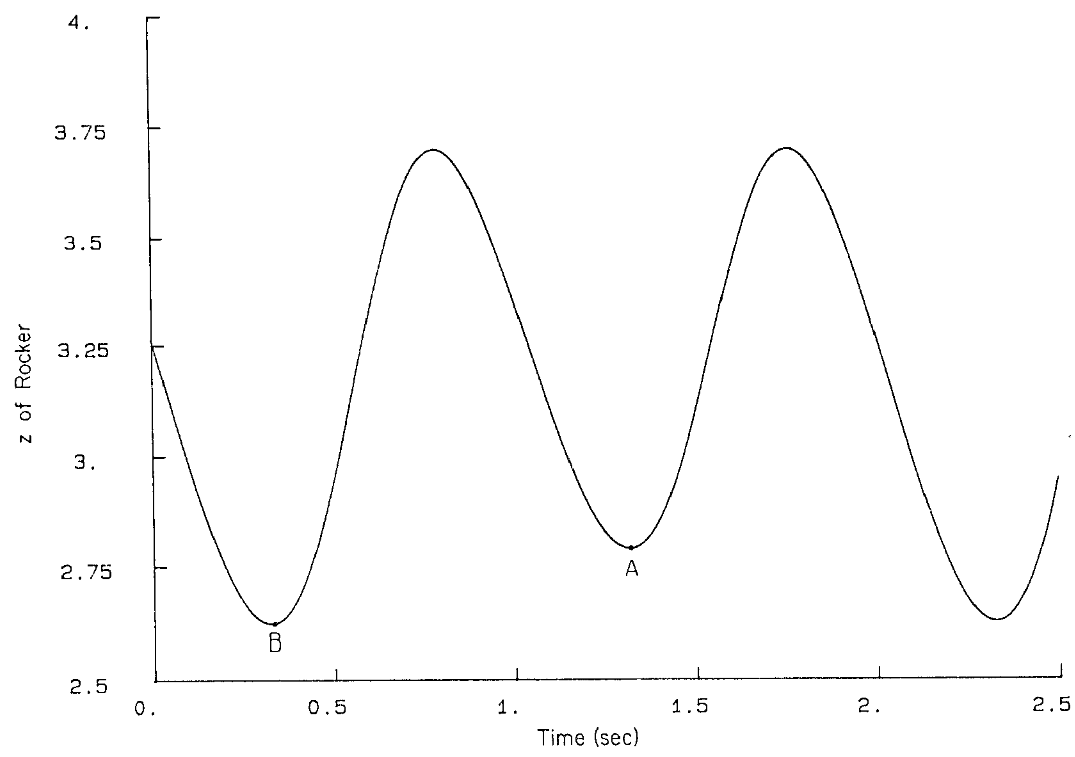

Figure 10.3.4 *z* coordinate of body 3 versus time, model 1.

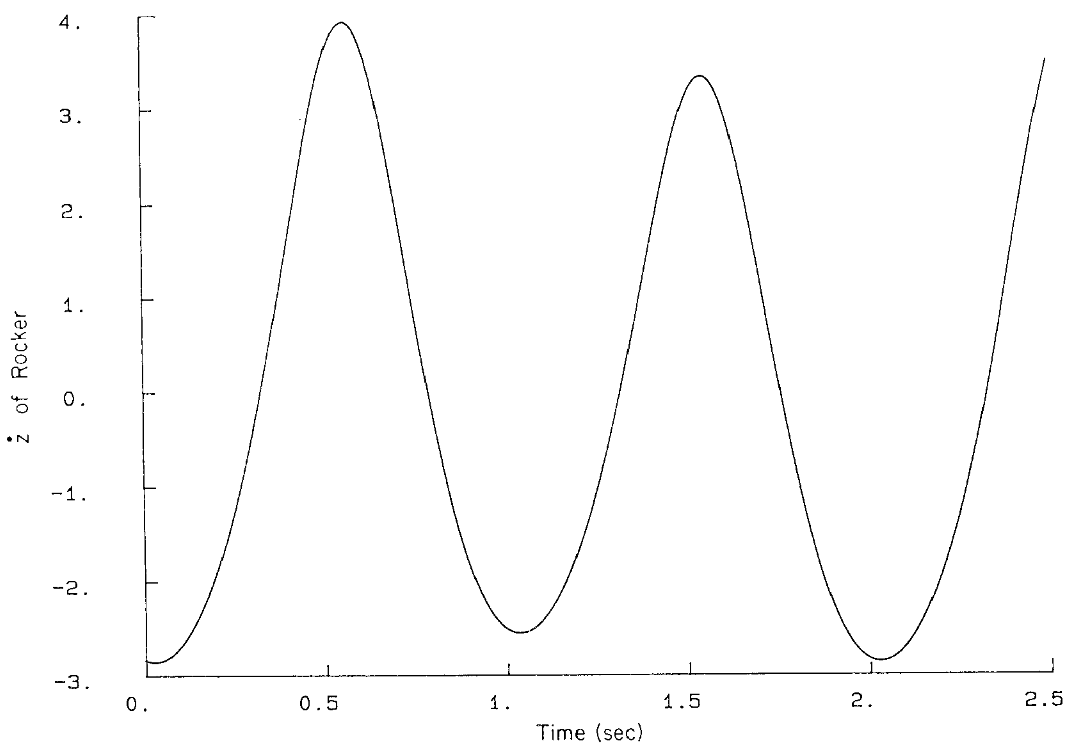

Figure 10.3.5 z velocity of body 3 versus time, model 1.

10.3.3 Driver Specification

Since each model has one kinematic degree of freedom, one driver is specified to rotate link AB about the global x axis. Taking the relative angle θ as the driven coordinate, it is specified that link AB rotate at π rad/s. The driver is thus

$$\theta = \pi t$$

10.3.4 Analysis

Some typical plots of the results for model 1 are shown in Figs. 10.3.3 to 10.3.5. Identical results are obtained for model 2.

10.4 KINEMATIC ANALYSIS OF AN AIR COMPRESSOR

10.4.1 Model

The air compressor shown in Fig. 10.4.1 has six pistons. The disk (body 3) has one end of each of the six connecting rods attached and evenly spaced (60°) on

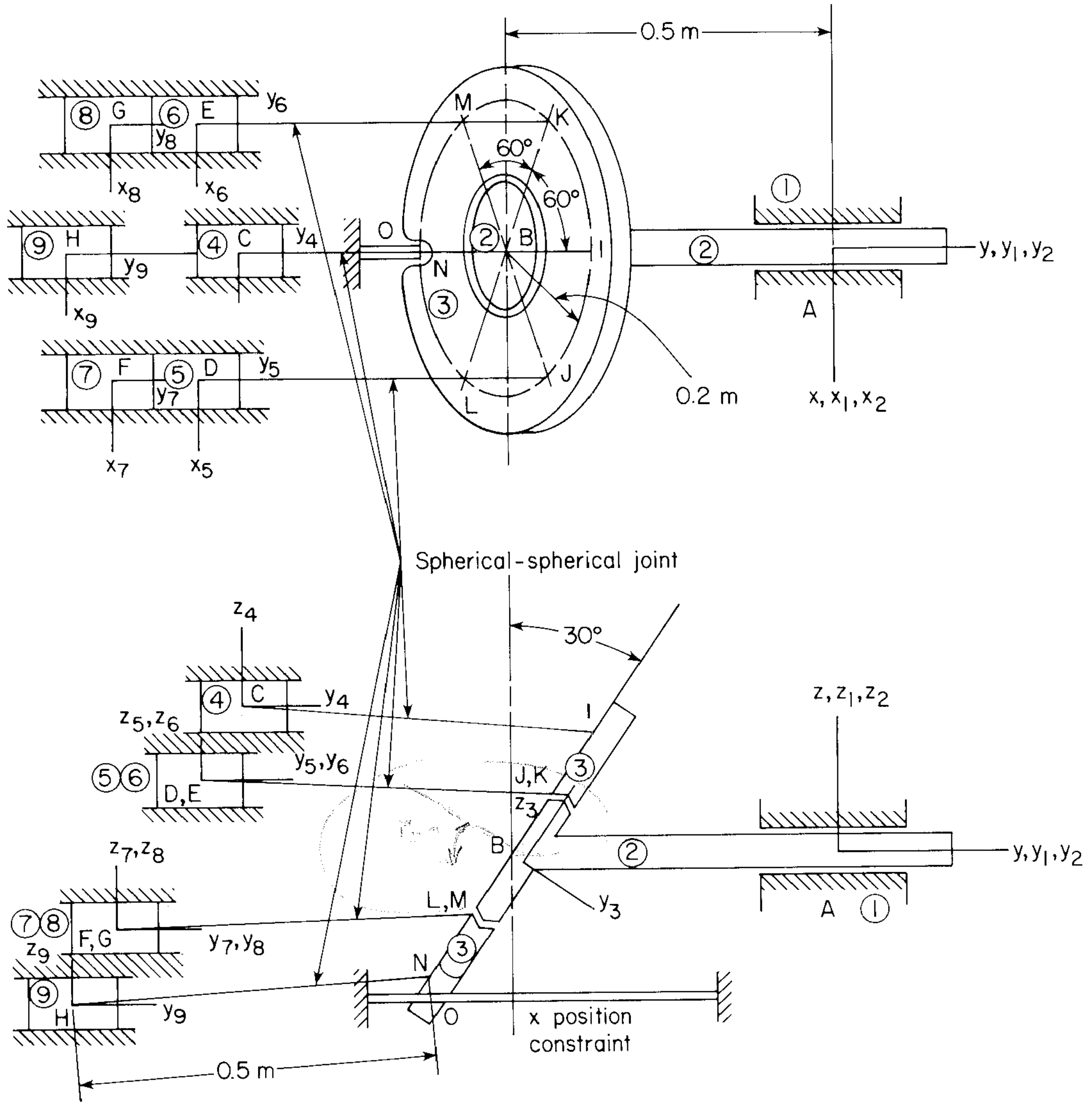

Figure 10.4.1 Air compressor with six pistons.

the circumference of a circle of radius 0.2 m. The disk is connected to the rotor (body 2) by a revolute joint whose axis of rotation is perpendicular to the disk and 30° from the axis of the rotor. As the rotor turns, the disk is prevented from rotating by an x position absolute constraint at point O, which models a slot in the disk through which a bar parallel to the global y axis passes. Canting of the disk generates reciprocating motion of the pistons as the rotor turns. Unlike the connecting rods of an automobile engine, the connecting rods of the compressor have spherical joints at each end. They are modeled here as spherical–spherical joints between the disk and pistons. The compressor model is defined as follows:

Model	
Bodies	
Nine bodies	$nc = 63$
Constraints	
Revolute joints (bodies 1 and 3, 2): *A*	5
B	5
Translational joints (bodies 4, 5, 6, 7, 8, and 9, 1) *C, D, E, F, G,* and *H*	6×5
Distance constraints (bodies 4, 5, 6, 7, 8, and 9, 3) *C–I, D–J, E–K, F–L, G–M,* and *H–N*	6×1
Position constraint (body 3): *O*	1
Ground constraints	6
Euler parameter normalization constraints	9
	$nh = 62$
$DOF = 63 - 62 = 1.$	

Revolute joint data, translational joint data, and spherical–spherical joint data are summarized in Tables 10.4.1 to 10.4.3.

TABLE 10.4.1 Revolute Joint Data

Joint	Body \ Point	P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
A	Ground ①	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0
	Rotor ②	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0
B	Rotor ②	0.0	−0.5	0.0	0.0	0.3660	−0.5	1.0	−0.5	0.0
	Disk ③	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0

10.4.2 Assembly

The position and orientation of each body reference frame in the global frame is estimated for initial assembly analysis. Table 10.4.4 provides position and orientation estimates for the generalized coordinates (abbreviated GC), using Euler parameters to specify the orientation of each body. Table 10.4.5 shows the resulting assembled configuration.

TABLE 10.4.2 Translational Joint Data

Joint	Body \ Point	P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
C	Ground ①	0.0	−1.0	0.2	0.0	0.0	0.2	1.0	−1.0	0.2
	Piston 1 ④	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0
D	Ground ①	0.1732	−1.0	0.1	0.1732	0.0	0.1	1.1732	−1.0	0.1
	Piston 2 ⑤	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0
E	Ground ①	−0.1732	−1.0	0.1	−0.1732	0.0	0.1	0.8268	−1.0	0.1
	Piston 3 ⑥	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0
F	Ground ①	0.1732	−1.0	−0.1	0.1732	0.0	−0.1	1.1732	−1.0	−0.1
	Piston 4 ⑦	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0
G	Ground ①	−0.1732	−1.0	−0.1	−0.1732	0.0	−0.1	0.8268	−1.0	−0.1
	Piston 5 ⑧	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0
H	Ground ①	0.0	−1.0	−0.2	0.0	0.0	−0.2	1.0	−1.0	−0.2
	Piston 6 ⑨	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0

TABLE 10.4.3 Spherical–Spherical Joint Data

Joint	Body \ Point	P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'	Distance
C-I	Disk ③	0.0	0.0	0.2	0.0	1.0	0.2	1.0	0.0	0.2	0.5
	Piston 1 ④	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	
D-J	Disk ③	0.1732	0.0	0.1	0.1732	1.0	0.1	1.1732	0.0	0.1	0.5
	Piston 2 ⑤	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	
E-K	Disk ③	−0.1732	0.0	0.1	−0.1732	1.0	0.1	0.8268	0.0	0.1	0.5
	Piston 3 ⑥	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	
F-L	Disk ③	0.1732	0.0	−0.1	0.1732	1.0	−0.1	1.1732	0.0	−0.1	0.5
	Piston 4 ⑦	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	
G-M	Disk ③	−0.1732	0.0	−0.1	−0.1732	1.0	−0.1	0.8268	0.0	−0.1	0.5
	Piston 5 ⑧	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	
H-N	Disk ③	0.0	0.0	−0.2	0.0	1.0	−0.2	1.0	0.0	−0.2	0.5
	Piston 6 ⑨	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	

TABLE 10.4.4 Position and Orientation Estimates

Body \ GC		x	y	z	e_1	e_2	e_3
Ground	①	0.0	0.0	0.0	0.0	0.0	0.0
Rotor	②	0.0	0.0	0.0	0.0	0.0	0.0
Disk	③	0.0	−0.5	0.0	−0.26	0.0	0.0
Piston 1	④	0.0	−1.0	0.2	0.0	0.0	0.0
Piston 2	⑤	0.17	−1.0	0.1	0.0	0.0	0.0
Piston 3	⑥	−0.17	−1.0	0.1	0.0	0.0	0.0
Piston 4	⑦	0.17	−1.0	−0.1	0.0	0.0	0.0
Piston 5	⑧	−0.17	−1.0	−0.1	0.0	0.0	0.0
Piston 6	⑨	0.0	−1.0	−0.2	0.0	0.0	0.0

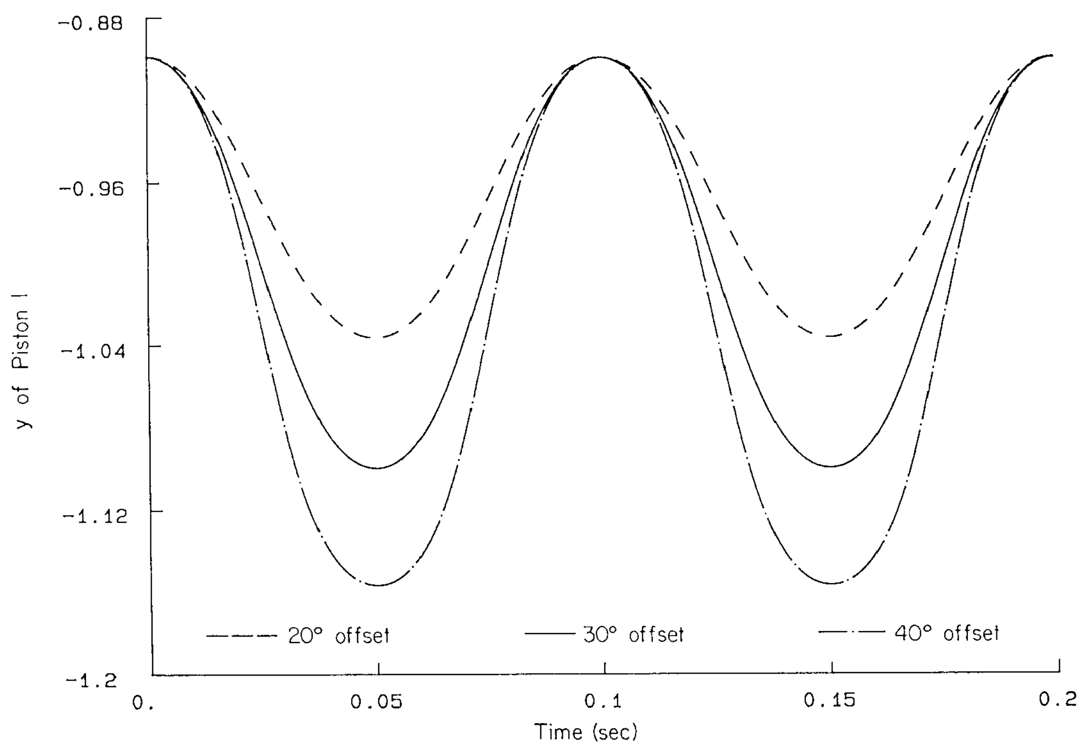

Figure 10.4.2 y position of piston 1.

TABLE 10.4.5 Assembled Configuration

Body \ GC		x	y	z	e_0	e_1	e_2	e_3
Ground	①	0.0	0.0	0.0	1.0	0.0	0.0	0.0
Rotor	②	0.0	−0.00090	0.00026	1.00020	0.00019	0.0	0.0
Disk	③	0.0	−0.50203	0.00020	1.96597	−0.25844	0.0	0.0
Piston 1	④	0.0	−0.90244	0.19992	1.0	0.0	0.0	0.0
Piston 2	⑤	0.17317	−0.95243	0.09999	1.0	0.0	0.0	0.0
Piston 3	⑥	−0.17317	−0.95243	0.09999	1.0	0.0	0.0	0.0
Piston 4	⑦	0.17317	−1.05130	−0.10002	1.0	0.0	0.0	0.0
Piston 5	⑧	−0.17317	−1.05130	−0.10002	1.0	0.0	0.0	0.0
Piston 6	⑨	0.0	−1.10110	−0.20000	1.0	0.0	0.0	0.0

10.4.3 Driver Specification

The rotor speed is chosen as 600 rpm (revolutions per minute), so the relative angle driven in the revolute joint between ground and the rotor is

$$\theta = 62.832t$$

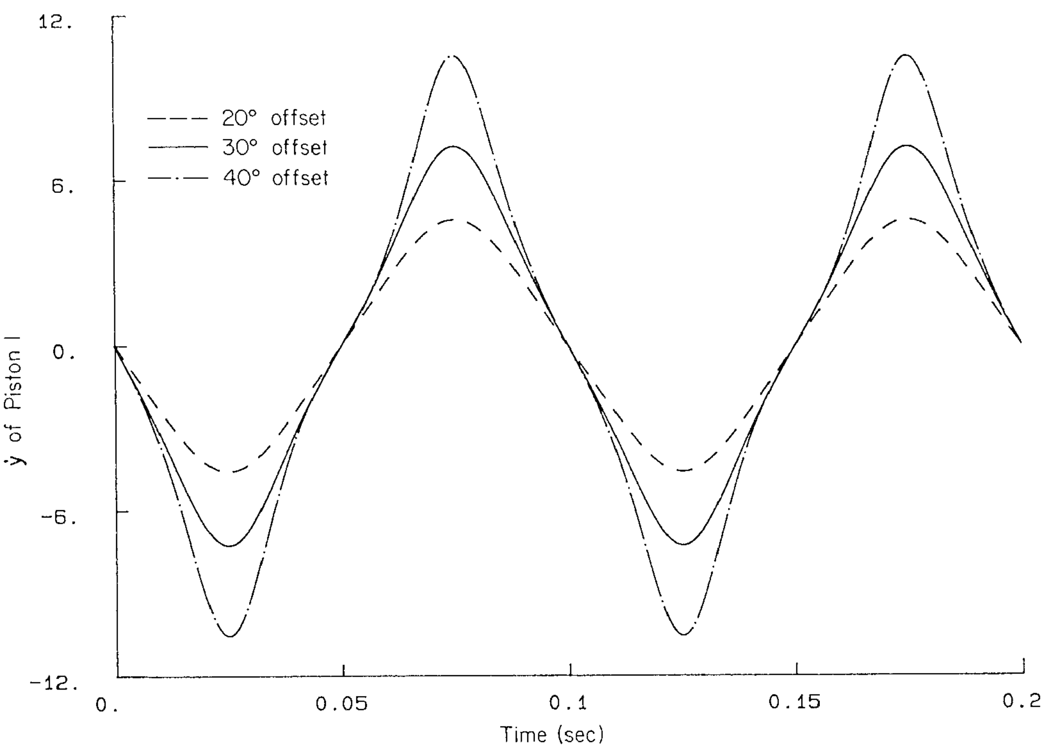

Figure 10.4.3 y velocity of piston 1.

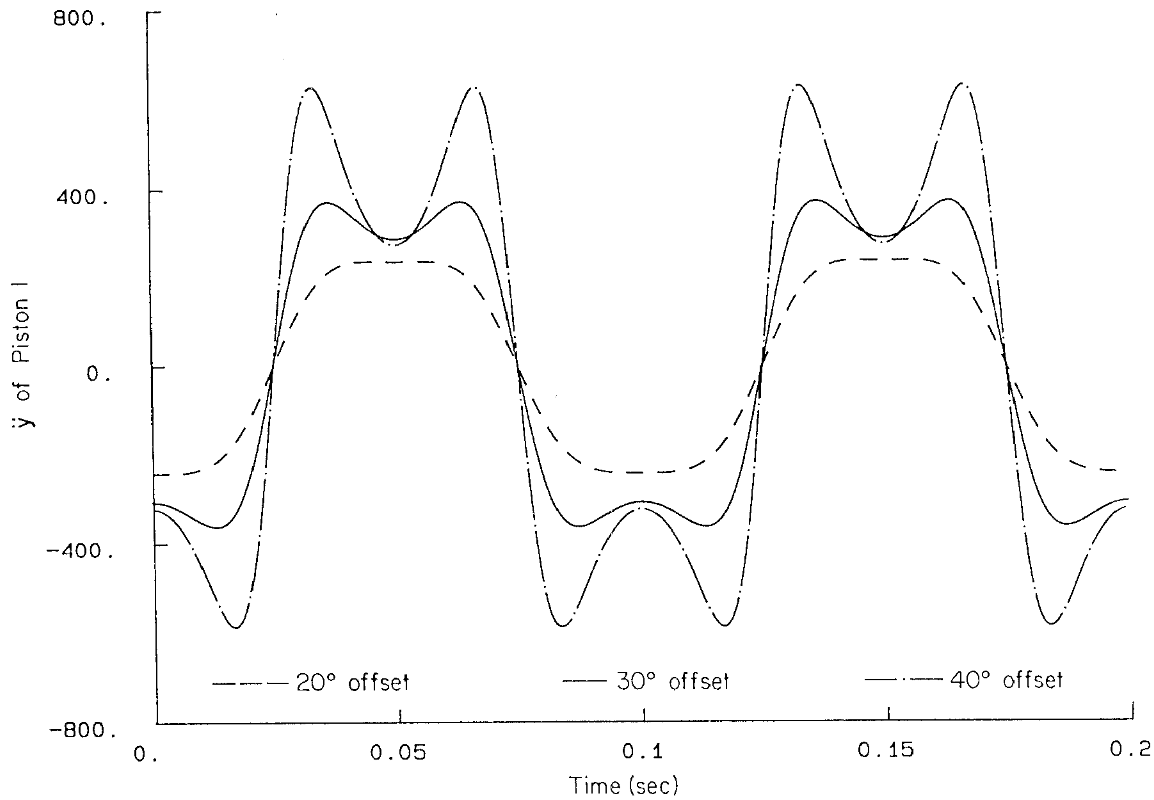

Figure 10.4.4 y acceleration of piston 1.

10.4.4 Analysis

Three different runs are made with the offset angle between the axis of the rotor and the axis of revolution equal to 20°, 30°, and 40°.

As shown in Fig. 10.4.2, the model with 40° offset angle generates the longest stroke of the piston. It also has the most extreme variations in velocity and acceleration of the pistons, as shown in Figs. 10.4.3 and 10.4.4.

PROBLEMS

DADS Projects

10.1. Set up a three-body DADS model of the spatial slider–crank mechanism in Fig. 10.2.1. The bodies are the crank, slider, and ground. Use revolute and translational joints between the crank and ground and the slider and ground, respectively, as in Section 10.2. Model the connecting rod as a spherical–spherical constraint between the crank and slider.

Using the estimates given in Section 10.2.2, carry out assembly and show that identical results are obtained with the new model. Use the driver of Section 10.2.3 and repeat the analysis carried out in Section 10.2.4. Verify that identical results are obtained.

10.2. Set up a fifteen-body DADS model of the air compressor in Fig. 10.4.1. Replace the spherical–spherical joints that model the connecting rods with six bodies. Use a spherical joint to connect each connecting rod to the disk (body 3) and a universal joint to connect it to the associated piston. The purpose of the universal joint is to control rotation of the connecting rod about its own axis.

Repeat the analysis of Sections 10.4.2 to 10.4.4 and show that identical results are obtained.