CHAPTER ONE

Elements of Computer-Aided Kinematics and Dynamics

The impact of the digital computer on all fields of science and engineering is already significant, to the point of becoming dominant in many disciplines. Well-developed computer software has already revolutionized the analysis of structures and electronic circuits. The situation, however, is quite different in the kinematics and dynamics of mechanical systems. While the potential for computational techniques in this field is at least as great as for structures and circuits, development has lagged behind. The objective of this text is to present basic methods for the computer formulation and solution of the equations of kinematics and dynamics of mechanical systems, to permit mechanical engineers to enjoy some of the benefits of modern computer methods that have served structural and electrical engineers for some time. The purpose of this chapter is to define the scope of the subject, to introduce prototype applications, and to outline methods that may be brought to bear for computer formulation and solution of the governing equations of kinematics and dynamics.

1.1 SCOPE OF MECHANICAL SYSTEM KINEMATICS AND DYNAMICS

For the purposes of this text, a *mechanical system* is defined as a collection of interconnected rigid bodies that can move relative to one another, consistent with joints that limit relative motion of pairs of bodies. The motion of a mechanical system may be prescribed by defining the time history of the position or relative position of some of its bodies. The motion of the system is then determined by algebraic kinematic relations or from differential equations of motion and externally applied forces, in which case the motion of the system is determined by laws of physics. Kinematics and dynamics of mechanical systems are characterized by large amplitude motion, which leads to geometric nonlinearity that is reflected in the algebraic equations of constraint and differential equations of motion.

Three basically different kinds of analysis are employed in the design of mechanical systems:

Kinematic analysis of a mechanical system concerns the motion of the system independent of forces that produce the motion. Typically, the time history of position or relative position of one or more bodies in the system is prescribed. Time histories of position, velocity, and acceleration of the remaining bodies are then determined by solving systems of nonlinear algebraic equations for position and linear algebraic equations for velocity and acceleration.

Dynamic analysis of a mechanical system concerns the motion of the system that is due to the action of applied forces. A special case of dynamic analysis is the determination of an equilibrium position of the system under the action of forces that are independent of time. The motion of the system, under the action of applied forces, is required to be consistent with kinematic relations that are imposed on the system by joints that connect bodies in the system. The equations of dynamics are differential equations or a combination of differential and algebraic equations.

Inverse dynamic analysis is a hybrid form of kinematic and dynamic analysis in which the time history of positions or relative positions of one or more bodies in the system is prescribed, leading to complete determination of position, velocity, and acceleration of the system from the equations of kinematics. The equations of motion of the system are then solved, with known position, velocity, and acceleration, as algebraic equations to determine the forces that are required to generate the prescribed motion.

An important consideration that serves to classify mechanical systems concerns the source of forces that act on the system. This is particularly important in modern mechanical systems in which some form of control is exerted. Force effects due to electrical and hydraulic feedback control subsystems play a crucial role in the dynamics of modern mechanical systems. The scope of mechanical system dynamics is, therefore, heavily dependent on the classes of force systems that act on the system.

The most elementary form of force that acts on a mechanical system is *gravitational force,* which is normally taken as constant and acting perpendicular to the surface of the earth. Other relatively simple forces that act on bodies in a system, due to interaction with their environment, include aerodynamic forces and friction and damping forces that act due to the relative motion of the components of the system. An important class of forces that act in a mechanical system is associated with *compliant elements,* such as coil springs, leaf springs, tires, shock absorbers, and a multitude of other deformable components that have reaction forces and moments associated with them. Forces due to compliant elements act between bodies in the system and are functions of their relative position and velocity.

To be more concrete regarding classes of mechanical system kinematic and

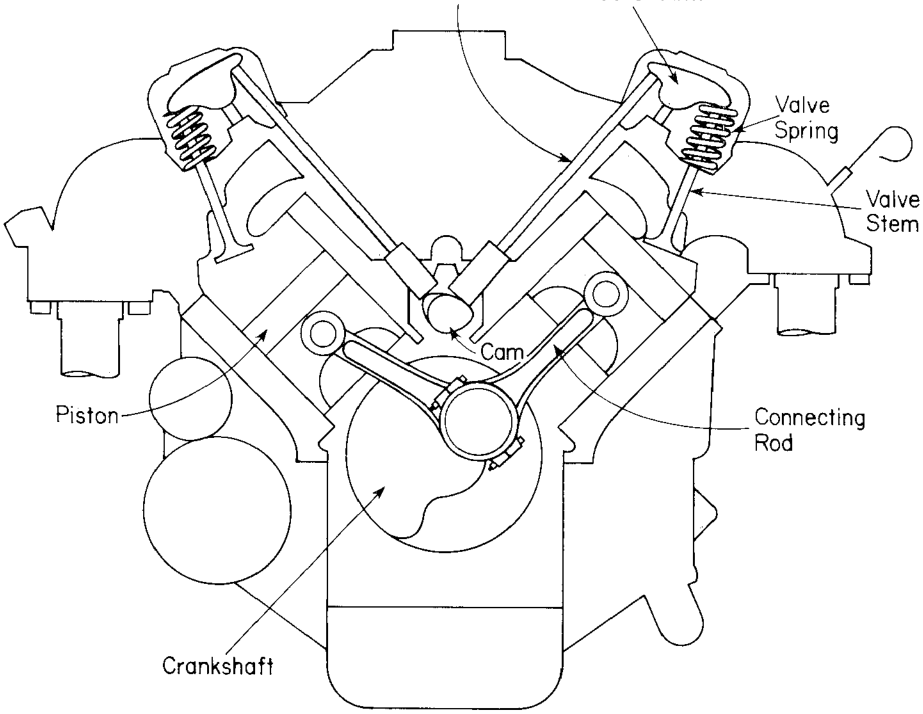

Figure 1.1.1 Cross section of a V-8 engine.

dynamic applications to be considered in the text, it is helpful to review a few typical engineering applications. The V-8 engine shown schematically in Fig. 1.1.1 contains many moving parts and illustrates a number of the most common mechanisms employed in machine design. The crankshaft of the engine rotates in lubricated bearings and contains eccentric rotational bearings with connecting rods, which are subsequently coupled through rotational bearings to translating pistons that move in combustion cylinders. The crankshaft–connecting rod–piston assembly comprises what is commonly called a *slider–crank mechanism,* which is used in this application and in many other machine components. The basic purpose of this mechanism is to transfer forces that are induced by combustion of fuel on the pistons into torques that act about the axis of rotation of the crankshaft, hence inducing the rotational motion that is used to propel a vehicle or to drive rotating machinery. Cams are typically used to induce the precisely timed motion of the cam–follower, which controls the position of the valve stem through a rocker arm, to open and close the intake and exhaust valves during engine operation. To close a valve and to maintain contact between the cam and follower, valve springs are used, as shown. While there are numerous

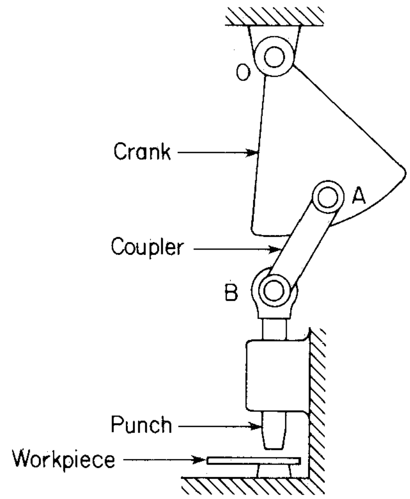

Figure 1.1.2 Punch mechanism.

other mechanisms within an engine, these basic components provide examples of typical machine elements.

A second application of the slider–crank mechanism is the punch mechanism shown schematically in Fig. 1.1.2. The crank is rotated about pivot point O through some angle of oscillation. The coupler, which is pivoted with the crank at point A and with the punch at point B, transmits load to the punch that translates in the machine housing, to provide reciprocating motion of the punch. Careful control of dimensions permits a modest torque that acts through a substantial driving angle of the crank to be transformed into a very large force that acts through a short range of motion of the punch, to deform or cut the workpiece.

A third example of a slider–crank mechanism is the fly-ball governer shown in Fig. 1.1.3. Relatively massive balls are attached to arms that are pivoted to a rotating shaft, so that they rotate with the shaft. Coupler arms are pivoted in the ball arms and a collar that is constrained to translate along the shaft. The ball arms, couplers, collar, and shaft form two slider–crank mechanisms that operate in spatial motion. This entire mechanism rotates with angular velocity ω of the shaft. As the shaft speeds up, centrifugal forces act on the balls to throw them out, causing the collar to move upward, hence increasing the distance s shown in Fig. 1.1.3.

The purpose of the fly-ball governor is to control the operating speed of an engine. A mechanism couples the position s of the collar to the fuel feed of an internal combustion engine that drives the shaft. The mechanism is designed so that, at the desired speed of the engine, centrifugal forces on the balls and gravitational and spring forces that act on the mechanism reach an equilibrium

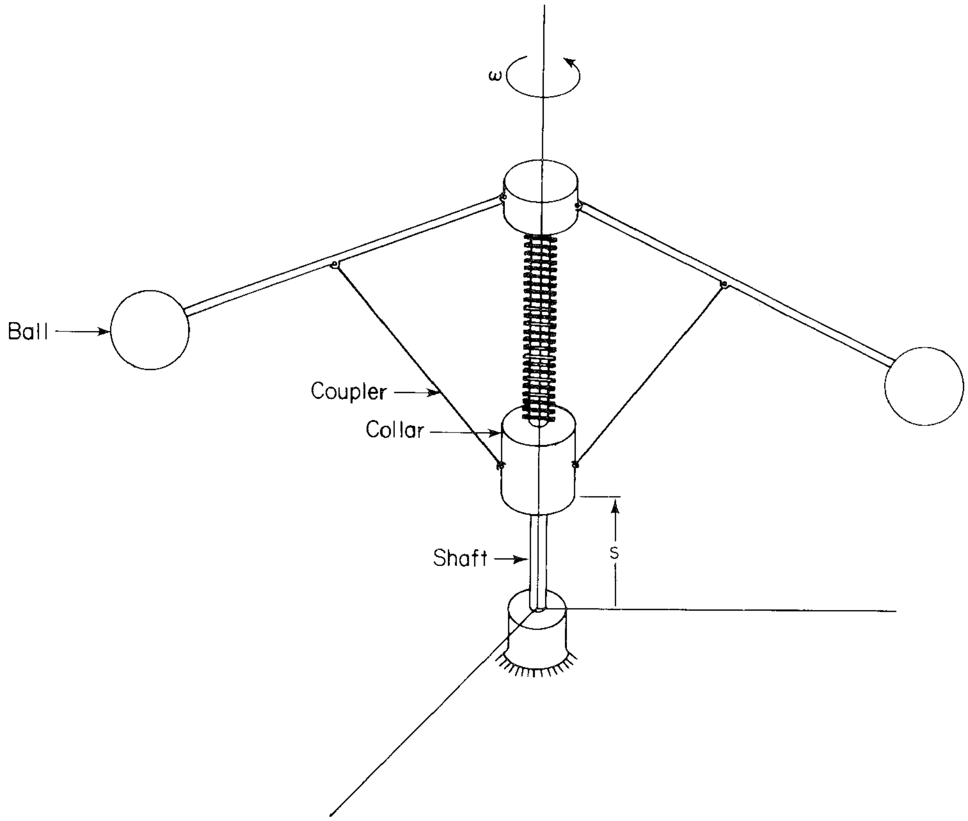

Figure 1.1.3 Fly-ball governor.

state with the collar at a given height. If an increased load is applied to the engine that reduces the angular velocity ω of the shaft (e.g., a vehicle encountering a hill or a lawn mower encountering tall grass), the balls will drop and the collar will move downward. The mechanism that couples the position of the collar with the fuel intake provides additional fuel, which in turn speeds the engine and causes centrifugal forces on the balls to increase, raising the balls toward their nominal height and reducing fuel feed to its nominal value. This is a typical application of a slider–crank mechanism.

Another mechanism that is commonly encountered in mechanical design is the *four-bar linkage,* such as that shown in the vehicle-suspension application of Fig. 1.1.4. The suspension linkage on each side of the vehicle is made up of upper and lower control arms that are pivoted in rotational joints in the frame and in the wheel assembly. This mechanism permits motion of the wheel assembly relative to the frame and transmission of road forces to the frame through a coil suspension spring and shock absorber, as indicated in Fig. 1.1.4.

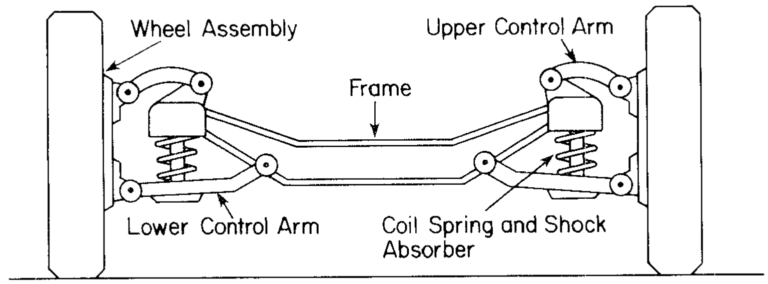

Figure 1.1.4 Suspension linkage.

The dimensions of the arms and attachments are carefully designed to cause the wheels to remain in as nearly a vertical position as possible during roll motion of the vehicle. Suspension springs and dampers are designed to provide vehicle stability and to transmit loads with small variation to the frame of the vehicle, even though extreme variations in force occur between the tire and road surface.

The windshield wiper mechanism shown in Fig. 1.1.5 is another application of four-bar linkages that transmits motor-driven rotation of the crank to the reciprocating motion of windshield wipers. The crank and left rocker arm are pivoted in the vehicle frame at points A and B. The crank coupler is pivoted in the crank at point C and in the left rocker arm at point D. The crank, crank coupler, left rocker, and frame of the vehicle constitute a four-bar linkage. Since the distance from B to D is greater than the distance from A to C, a full rotation of the crank causes only a partial rotation of the left rocker arm, leading to the

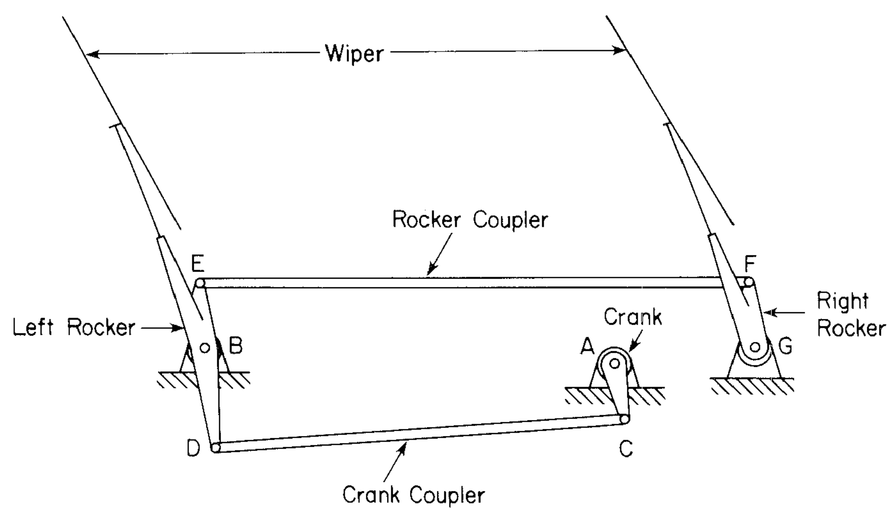

Figure 1.1.5 Windshield wiper mechanism.

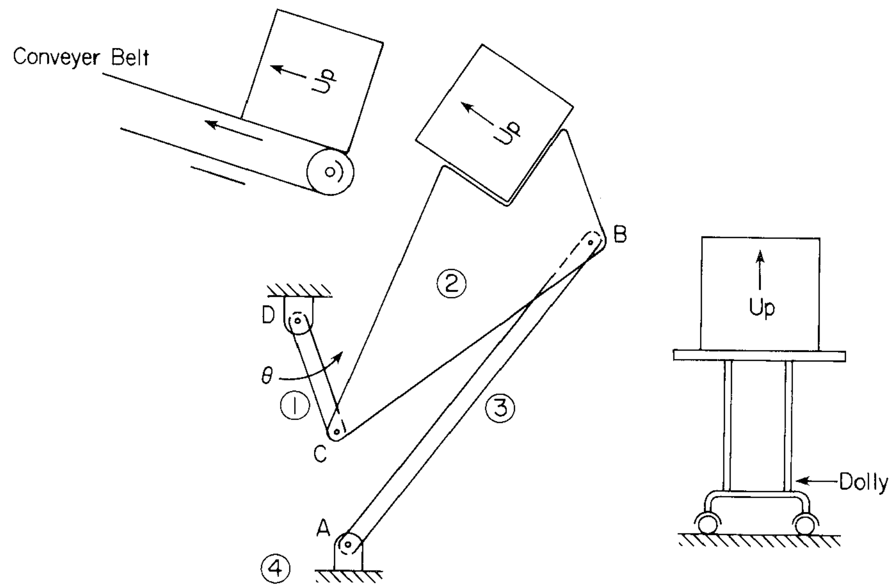

Figure 1.1.6 Material-handling mechanism.

desired reciprocating motion of the left windshield wiper. The dimensions of the various links are carefully selected to generate the desired range of motion. A second four-bar linkage is formed by the right rocker arm that is pivoted in the frame of the vehicle at point G and the rocker coupler that is pivoted in the left and right rocker arms at points E and F. This second linkage transmits reciprocating motion from the left rocker arm to the right rocker arm, hence driving the right windshield wiper.

Still another example of a four-bar linkage is the material-handling mechanism shown in Fig. 1.1.6. The crank (body 1) is pivoted in ground (body 4) at point D and with the material handler (body 2) at point C. The material handler is in turn connected to the follower arm (body 3) at point B and the follower arm is pivoted in ground at point A. The purpose of the mechanism is to permit counterclockwise rotation θ of the crank to lower the material-handling arm to a position that permits loading of cargo from a dolly located on the floor. Subsequent clockwise rotation of the crank raises the cargo so that it can be transmitted to a conveyer belt and moved to another station within a fabrication or storage facility. It is geometrically clear from the schematic diagram of Fig. 1.1.6 that the dimensions of components of the mechanism must be carefully selected so that the material handler is in the proper position and orientation for both pickup and deposit of the cargo onto the conveyer belt.

Gears, typified by the circular gear pair shown in Fig. 1.1.7, are commonly used in mechanical equipment to transmit rotation and torque at varying speeds and magnitudes, respectively, for both control of motion and transmission of

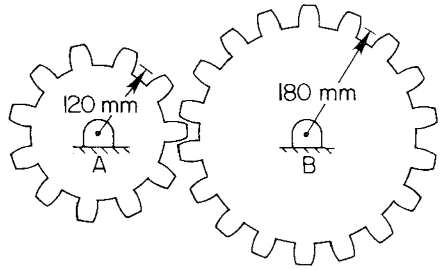

Figure 1.1.7 Gear pair.

power. While the details of the design of gear teeth are not treated in this text, it is presumed that the geometry of the gears is designed so that continuous contact is maintained at the gear pitch circles, shown in Fig. 1.1.7 with radii of 120 and 180 mm. If the smaller gear is driven, the larger gear follows. One full revolution of the larger gear requires 1.5 revolutions of the smaller gear. However, one unit of torque applied to the smaller gear is transmitted as 1.5 units of torque to the shaft of the larger gear. Gearing mechanisms, therefore, permit great latitude in the adjustment of speeds of shafts and torques transmitted.

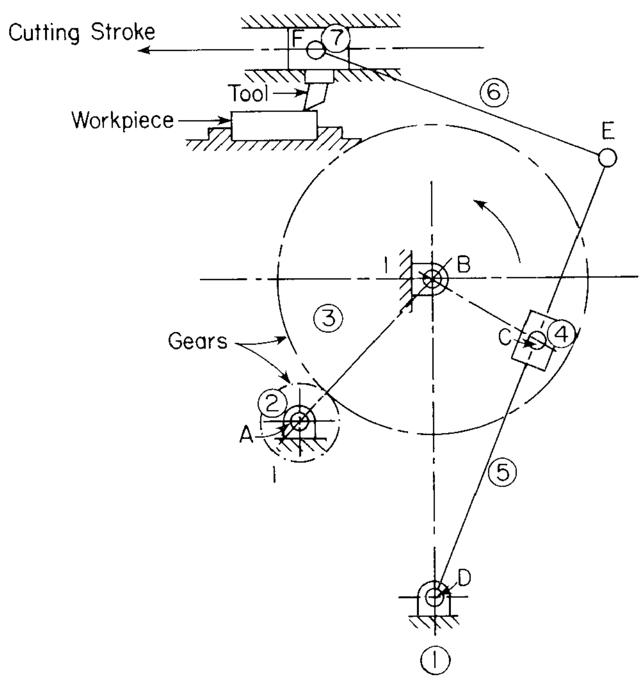

Figure 1.1.8 Quick-return shaper mechanism.

Compound mechanisms, such as the quick-return shaper mechanism shown schematically in Fig. 1.1.8, are made up from combinations of many of the basic kinematic couplings that have been encountered in the preceding examples. In this application, body 1 is designated as ground. Bodies 2 and 3 are gears of substantially different pitch diameter, with gear 2 pivoted in ground at point *A* and gear 3 pivoted in ground at point *B*. A motor drives the shaft of gear 1, resulting in smaller angular velocity of the larger gear. A coupler, body 4, is attached to the larger gear with a rotational joint at point *C*. A rocker arm, body 5, is pivoted in ground at point *D* and slides freely through the coupler of body 4. Thus, as body 3 rotates, body 5 undergoes an oscillating motion. Another coupler, body 6, is connected by rotational joints with body 5 at point *E* and with the slider, body 7, at point *F*. Body 7 translates relative to ground and carries a cutting tool that contacts a workpiece and removes material. By carefully selecting the dimensions of the components of the machine, the cutting tool can be made to move to the left (the cutting stroke) at relatively low speed and to return more quickly to the beginning of the cutting stroke.

The robot, or manipulator, of Fig. 1.1.9 is made up of nine bodies, including ground (body 1). The first degree of freedom is the rotation angle q_1 of the base (body 2) about a vertical axis fixed in ground. The second degree of freedom is the rotation q_2 of the pivot arm (body 3) about the horizontal axis fixed in body 2. The third degree of freedom is translation q_3 of the boom (body 4) in a guide that is fixed in body 3. The fourth degree of freedom is rotation q_4

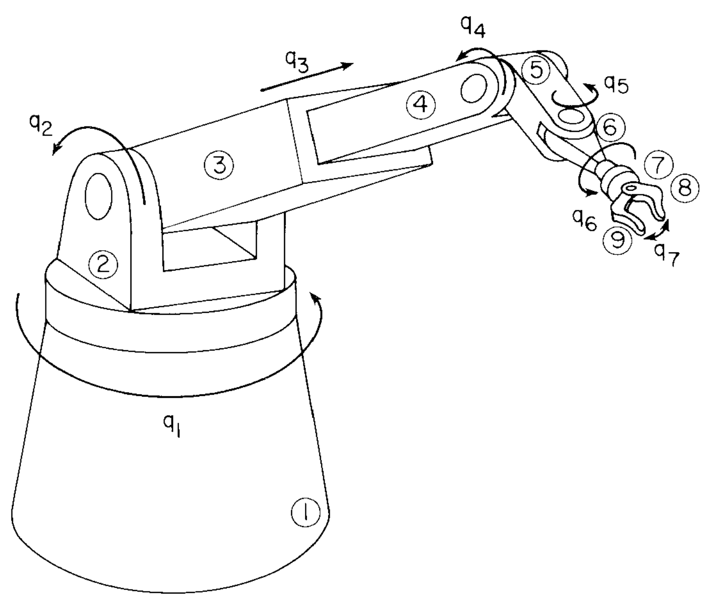

Figure 1.1.9 Robotic manipulator.

of the first wrist pivot (body 5) relative to body 4. The fifth degree of freedom is rotation q_5 of the second wrist pivot (body 6) relative to body 5. The sixth degree of freedom is rotation q_6 of the hand mechanism (body 7) relative to body 6. The final degree of freedom is relative rotation q_7 of the robot fingers (bodies 8 and 9). Such a mechanism permits the end-effector to grasp and manipulate workpieces.

The final compound mechanism example that illustrates the scope of study to follow is the vehicle of Fig. 1.1.10, whose suspension system is shown schematically in Figs. 1.1.11 and 1.1.12. This commonly employed high-performance vehicle suspension consists of a McPherson strut front suspension and a trailing arm rear suspension. Each front wheel assembly is attached to the chassis of the vehicle through a lower control arm and a telescoping strut assembly, as shown in Fig. 1.1.11. Concentric with the strut assembly are suspension spring and damping components. The spherical joints at the top and bottom of the strut assembly permit steering rotation of the wheel assembly about the strut. The more elementary rear suspension shown in Fig. 1.1.12 is simply a control arm that is pivoted in the chassis to permit the rear wheel assembly to move relative to the chassis. Spring and damping components attached between the rear control arm and chassis provide for support of the chassis and cushioning of extreme tire–road forces.

These examples represent typical machines that are encountered in mechanical system kinematic and dynamic analysis and design. The breadth of such

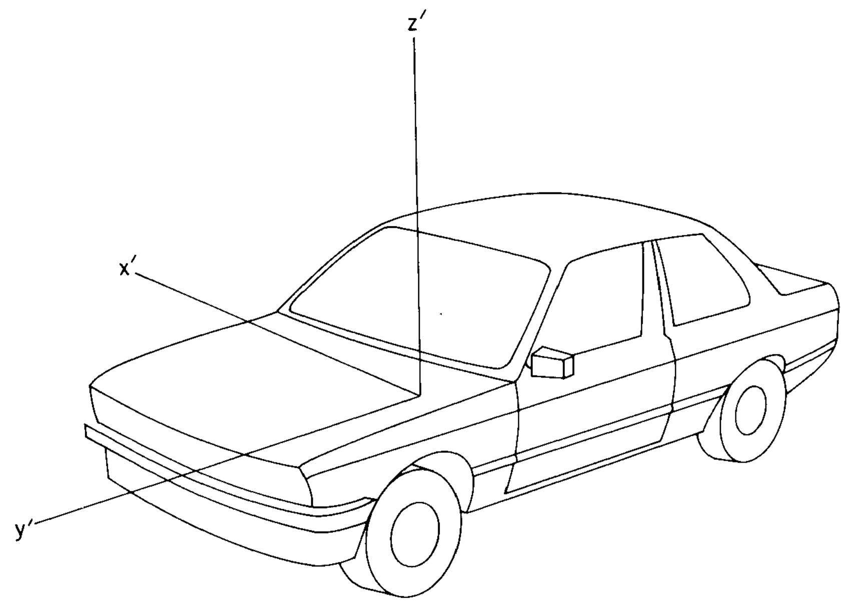

Figure 1.1.10 Automobile.

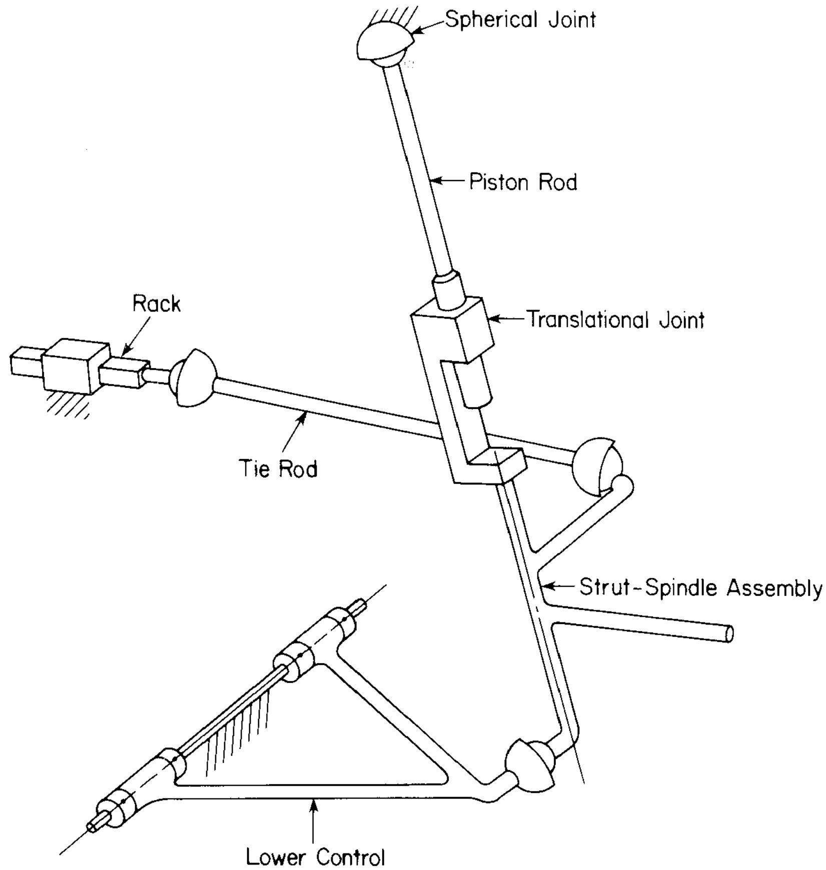

Figure 1.1.11 McPherson strut front suspension.

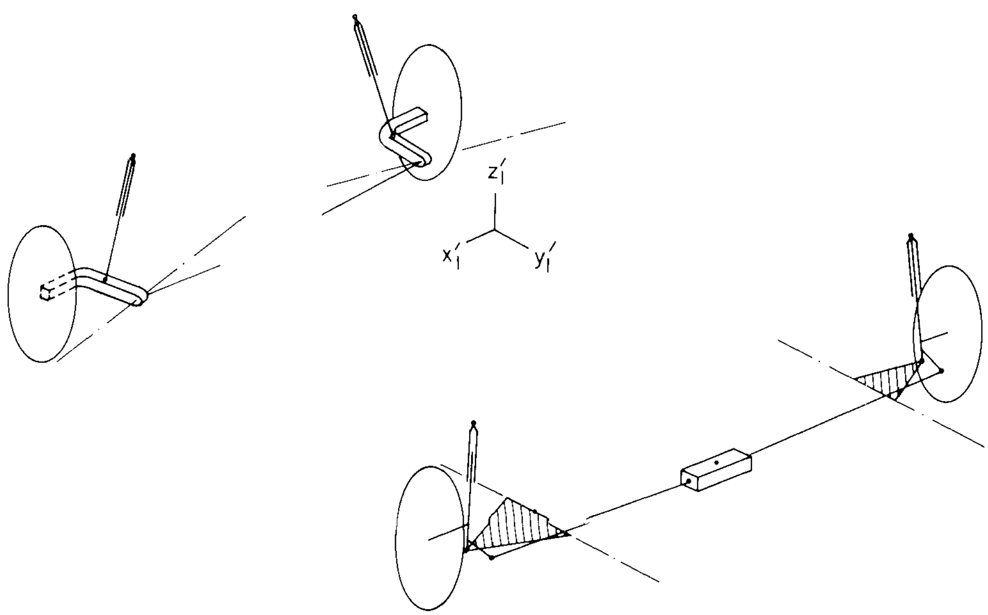

Figure 1.1.12 Automobile suspension schematic.

applications is extensive. While applications and environments differ greatly, many technical similarities permit the development of a uniform approach to computer-aided kinematic and dynamic analysis.

1.2 CONVENTIONAL METHODS OF KINEMATIC AND DYNAMIC ANALYSIS

Due to the nonlinear nature of large displacement kinematics, the mechanism designer has traditionally resorted to graphical techniques and physical models for the kinematic analysis of mechanical systems [1, 2]. As might be expected, such methods are limited in generality and rely heavily on the designer's intuition. For more contemporary treatments of mechanism and machine dynamics, References 3 through 6 may be consulted. The conventional approach to the dynamic analysis of mechanical systems is to use Lagrangian methods of formulating the system equations of motion in terms of a minimal set of variables that define absolute or relative position and orientation. Numerous texts on dynamics [4, 7–9, 35] provide the fundamentals that are needed for mechanical system dynamic analysis.

Mathematical models of kinematic and dynamic systems with several degrees of freedom have traditionally been characterized by "clever formulations" that take advantage of the properties of a specific system to obtain simplified forms of the equations of kinematics and dynamics. The ingenious selection of position and orientation variables can occasionally lead to a formulation with independent variables that allows manual derivation, but rarely analytical solution of the equations of motion. More often, analysis of systems with even three or four degrees of freedom leads to massive algebraic manipulation in constructing equations of motion. The "clever formulation" approach is, therefore, limited to relatively simple mechanical systems. Some extension has been achieved using symbolic computation [10], in which the computer is used to carry out differentiation and algebraic manipulation, creating terms that are required in the equations of motion.

After the governing equations of motion have been derived by manual or symbolic computation methods, the engineer or analyst is still faced with the problem of obtaining a solution of the differential equations and initial conditions. Since these equations are highly nonlinear, the prospect of obtaining closed-form solutions is remote, except in very simple cases. With the advent of digital computers, engineers began to use the computer and available numerical integration methods to solve their equations of motion. This, however, still involved a substantial amount of time and personnel for deriving equations of motion and writing ad hoc digital computer programs to carry out numerical integration.

In contrast to the traditional ad hoc approach that has been employed in mechanical system kinematics and dynamics, a massive literature has evolved in finite element structural analysis [11, 12] and the analysis of electronic circuits [13, 14]. Developments in these fields are characterized by the same technical approach. Rather than relying on "clever formulations," a "systematic approach" is taken and digital computers are used for both the formulation and solution of the governing equations. Through systematic formulation and selection of numerical techniques, user-oriented computer codes have been developed that are capable of simulating a broad range of structures and circuits. The overwhelming success of finite element structural and electronic circuit analysis computer codes suggests that such a formulation be adopted for mechanical system kinematics and dynamics.

1.3 OBJECTIVE OF COMPUTATIONAL KINEMATICS AND DYNAMICS

The objective of computational methods in kinematics and dynamics is to create a formulation and digital computer software that allow the engineer to (1) input data that define mechanical systems of interest and automatically formulate governing equations of kinematics and dynamics, (2) automatically solve nonlinear equations for kinematic and dynamic response, and (3) provide computer graphics output of results of simulations to communicate results to the designer or analyst. The essence of this objective is to make maximum use of digital computer power for rapid and accurate data manipulation and numerical computation, hence relieving the engineer of tedious and error-prone calculations that heretofore have been carried out manually or with ad hoc computer programs.

As suggested by advances in computer-aided finite element structural and electronic circuit analysis, a systematic approach to the formulation and solution of the equations of kinematics and dynamics of mechanical systems is required to implement computations in a user-oriented computer program. Great care must be taken to consider the numerous alternatives that are available in selecting a formulation and numerical methods to achieve this objective.

Several computer programs for kinematic and dynamic analysis were developed in the late 1960s and early 1970s [15–17] using relative coordinates between bodies. These programs are satisfactory for many applications. An alternative method of formulating system constraints and equations of motion, in terms of global Cartesian coordinates, was introduced in the late 1970s [18–20], bypassing topological analysis and making it easier for the user to supply constraints and forcing functions. This approach leads to a general-purpose computer program, with practically no limitation on the type of mechanism or machine that can be analyzed. The penalty, however, is a larger system of equations to be solved.

To be specific concerning some of the alternatives and trade-offs that exist in the field of computational kinematics and dynamics, an elementary example is

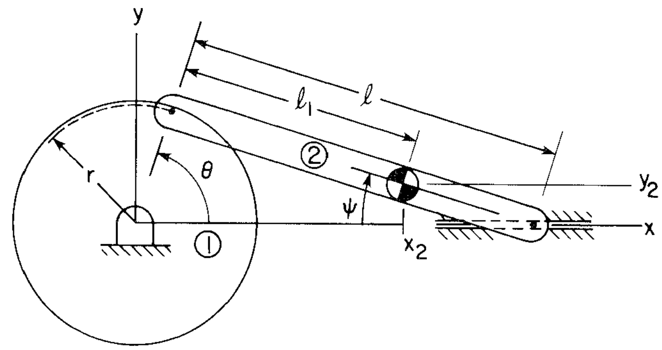

Figure 1.3.1 Elementary slider–crank model.

employed to discuss approaches to system dynamics. Consider a simplified model of the slider–crank mechanisms of Figs. 1.1.1 through 1.1.3 idealized to include the slider mass at the right end of the connecting rod (body 2), as shown in Fig. 1.3.1. The center of mass of the connecting rod has been adjusted to reflect the incorporation of the mass of the slider as a point mass at the right end, which must slide along the x axis. The center of mass of the crank is at its pivot with ground.

It is clear from simple trigonometry that once the crank angle θ (called an independent *Lagrangian generalized coordinate*) is fixed, as long as $\ell > r$, the angle ψ and positions x_2 and y_2 can be determined, with some analytical complexity. A single, highly nonlinear, and complicated second-order differential equation of motion in the independent variable θ can then be derived. It must, of course, be solved numerically.

A more systematic approach to deriving equations of motion for the simplified slider–crank of Fig. 1.3.1 is to first consider the bodies as being disconnected, as shown in Fig. 1.3.2. In this formulation, the angular orientation ϕ_1 of the crank and coordinates x_2 and y_2 of the center of mass of body 2 and its angular orientation ϕ_2 are taken as position and orientation coordinates called *Cartesian generalized coordinates*. To assemble the linkage, however, these four variables must satisfy three kinematic relations. Specifically, points A_1 and A_2 must concide in order to have a rotational joint between the crank and connecting rod, leading to two algebraic constraint equations. Similarly, for point B_2 on the connecting rod to slide along the x axis, it is necessary that its y coordinate be zero, leading to an additional constraint equation. These three algebraic equations comprise three constraints among the four generalized coordinates ϕ_1, x_2, y_2, and ϕ_2. Since these equations are nonlinear, a closed-form solution for three of the variables in terms of the remaining variable is difficult to obtain.

The Lagrange multiplier form of the equations of motion [4, 7–9, 35, or Chapter 6, herein] may be written for this system as four second-order differential equations and three algebraic equations of constraint, for four Cartesian

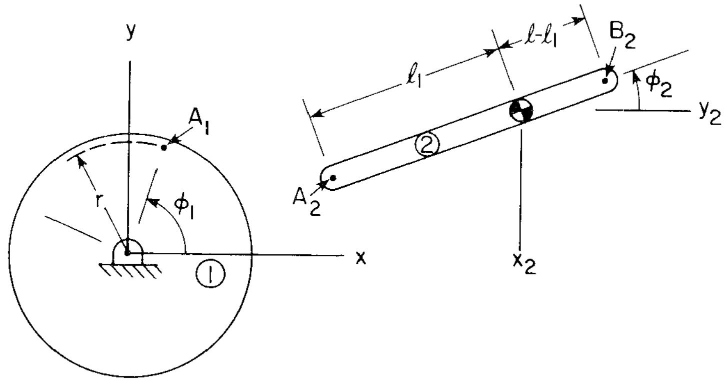

Figure 1.3.2 Cartesian coordinates for slider–crank.

generalized coordinates and three Lagrange multipliers. This is a mixed system of *differential–algebraic equations* that must be solved to determine the motion of the system. The form of this large number of equations is simple, however, permitting computer generation and solution.

The systematic Cartesian coordinate approach is adopted in this text, since it is well suited for both computer formulation and solution of the governing equations of kinematics and dynamics. The general-purpose Dynamic Analysis and Design System (DADS) computer code [27] has been developed, based on the methods presented in this text, and is available for use with the text as an instructional tool. To enhance the student's experience with realistic kinematic and dynamic applications, examples and exercises using the DADS computer code are presented in Chapters 5, 8, 10, and 12.

1.4 GUIDE TO THE TEXT CONTENTS

The text begins with Part One (Chapters 2 through 8) on planar kinematics and dynamics and associated numerical analysis and modeling methods. Spatial kinematic and dynamic analysis formulations are presented in Part Two (Chapters 9 through 12), and their use in the analysis of spatial mechanical systems is illustrated. This organization has been selected to permit the beginning reader to master basic concepts and to gain experience in the kinematic and dynamic analysis of planar systems for which the level of analytical complexity is minimal. Having developed confidence in basic methods of formulation and numerical analysis, the reader may proceed to the study of spatial systems, which is characterized by a higher degree of technical complexity, but follows the same basic approach and utilizes identical numerical methods. The reader with some experience in kinematics and dynamics who is primarily interested in spatial systems can proceed directly to Part Two of the text without any loss of continuity.

Part One of the text begins with Chapter 2, which presents planar vector analysis methods and matrix and multivariable calculus notations that are used throughout the text. Of key importance in Chapter 2 is the formulation of vector analysis techniques in terms of the matrix operations that are needed for digital computer implementation. Differentiation of vectors, multivariable differential calculus, and virtual displacement concepts that are needed in subsequent kinematic and dynamic analysis are also developed.

The planar Cartesian kinematics formulation is developed in Chapter 3. Standard constraints are formulated and governing equations for position, velocity, and acceleration analysis are derived and illustrated. Numerical methods for kinematic analysis (applicable to both planar and spatial systems) are presented in Chapter 4. Computational methods of assembling the required equations and matrices for kinematic analysis are discussed and numerical methods for solving linear and nonlinear equations are presented. Chapter 5 presents and illustrates methods for planar kinematic modeling and analysis. Modeling and analysis methods are illustrated using the DADS computer code. Technical difficulties that are encountered due to poorly formulated system models are illustrated to guide the reader in interpreting the results of numerical analysis and to demonstrate that it is easy to define a mechanical system that will not perform the intended kinematic functions.

The governing equations of planar system dynamics are derived in Chapter 6. This development is self-contained, beginning with Newton's laws of motion for a particle and developing the equations of motion of planar rigid bodies. The virtual work method is introduced for use throughout the text. The Lagrange multiplier form of equations of motion for constrained dynamic systems is developed. Methods for inverse dynamic and equilibrium analysis are presented. Numerical methods for solving equations of motion (applicable to both planar and spatial systems) are presented in Chapter 7. The process of formulating mixed differential–algebraic equations of motion is discussed, and algorithms that may be employed to reduce such systems to an integrable form are presented. Numerical integration methods are outlined, and their use in algorithms for numerical solution of the differential–algebraic equations of motion is presented. Chapter 8 presents methods for planar dynamic analysis. Examples used in Chapter 5 for kinematic analysis are employed to illustrate methods for the dynamic analysis of planar systems and to study the effect of design variations on dynamic performance.

Part Two of the text is devoted to the kinematics and dynamics of spatial systems. Chapter 9 develops the theory of spatial position and orientation of rigid bodies. Euler parameter orientation variables are defined and their properties are developed as required for analysis and applications. Spatial kinematic equations for constrained multibody systems are developed for a library of joints, and the computation of needed derivatives is presented. Spatial kinematic modeling and analysis methods are discussed in Chapter 10, where examples are analyzed using the DADS code, to illustrate modeling methods and their use in kinematic analysis. The importance of careful definition of kinematic joints is emphasized, and pitfalls that arise due to improper formulation of mechanical system models are illustrated.

Chapter 11 presents a self-contained derivation of the equations of motion of spatial multibody systems. The virtual work approach introduced in Chapter 6 is employed. Finally, methods of spatial dynamic modeling and analysis are presented in Chapter 12 and illustrated through the study of examples using the DADS code.

Part One

PLANAR SYSTEMS

Chapters 2 through 8 are devoted to the kinematics and dynamics of systems in which all bodies move in a plane or in parallel planes. Initial focus on this class of systems is motivated by the following considerations: (1) analytical representation of the position and orientation of bodies in a plane is much less complex than for bodies in space, (2) analytical and modeling concepts are most easily learned in the context of planar systems, and (3) once the engineer has a clear understanding of the concepts and modeling methods, the study of spatial systems involves only analytical and algebraic extensions. Consistent with the objective of restricting attention in Part I to planar systems, only planar vector analysis and planar position and orientation generalized coordinates are introduced and used. The matrix and differential calculus methods presented in Chapter 2 are, however, adequate for both planar and spatial applications.

The reader who is new to the field of the kinematics and dynamics of machines is encouraged to thoroughly master concepts in the planar setting. To develop a sound understanding and facility with mathematical and numerical analysis methods, the reader is encouraged to build on a physical foundation of mechanics and machine applications, in order to achieve a practical capability in the kinematics and dynamics of machines. The insidious presence of nonlinearity in virtually all aspects of the kinematics and dynamics of machines leads to intricacies in mathematical and numerical analysis that are not as easily solved as is the case in linear structural mechanics and associated finite element methods. Pathological forms of behavior, such as lock-up and branching of kinematic solutions, can best be understood and overcome by an engineer who develops both a firm mathematical foundation and a clear physical understanding of the behavior of mechanisms and machines. Examples and exercises are presented throughout the text to assist the engineer in developing mathematical skills and in relating them to the mechanics of machines. The reader is encouraged to study the examples thoroughly and to carry out independent analysis of mechanisms of his or her own choice.