ME451
Kinematics and Dynamics of Machine Systems

Post Dynamics
Inverse Analysis
Equilibrium Analysis
December 11, 2014

Quote of the day: “My mother said to me, 'If you are a soldier, you will become a general. If you are a monk, you will become the Pope.' Instead, I was a painter, and became Picasso.”

-- Pablo Picasso
Before we get started…

- Last time
  - Solving the constrained equations of motion using the Newmark integration formulas

- Today
  - Inverse Dynamics Analysis
  - Equilibrium Analysis

- Project 2 due on 12/16 at 11:59 PM

- End of semester here, all scores in Learn@UW
  - Please let me know now if you notice any mistakes

- I will travel next week, Dr. Pazouki will proctor the Final Exam
  - Tuesday 12/16, 2:45PM - 4:45PM
  - Room : 2109ME (computer lab)
Final Exam Structure/Content

- Comprehensive exam

- Same format as the Kinematics and Dynamics midterm exams
  
  - First problem made up of several short questions
  
  - Second problem – you have to do some busy work
  
  - In last problem you will be asked to generate an acf/adm file
    - This will require you to have access to a tablet/laptop/workstation

- Bonus: Running simEngine2D with your acf/adm combo and producing the right plots for a reaction force in a joint
Inverse Dynamics: The idea

- First of all, what does (forward) dynamic analysis mean?
  - Apply some forces and/or torques on a mechanical system and look at how the configuration of the mechanism changes in time
  - The mechanism evolution also depends on the specified initial conditions
  - This is a forward process: forces $\Rightarrow$ motion

- In inverse dynamics, the situation is quite the opposite:
  - Specify driving constraints on the mechanical system (that is specify the desired motion) and find the set of forces and/or torques that should have been applied to the mechanical system to lead to this motion
  - Note that the ICs are implicitly defined by the specified desired motion
  - Note that we need to include as many driver constraints as kinematic degrees of freedom there are (exactly like in Kinematics, we require NDOF = 0)
  - This is a reverse (inverse) process: forces $\Leftarrow$ motion

- Inverse dynamics is useful in controls
  - Example: controlling the motion of a robot – we know how we want this robot to move and we need to figure out what joint torques we should apply to make it move that way
When can one talk about Inverse Dynamics?
- Given a mechanical system, a prerequisite for Inverse Dynamics is that the number of degrees of freedom associated with the system is zero.
  - We must have as many generalized coordinates as constraints (THIS IS KEY)
- This effectively makes the problem a Kinematics problem

The two stages of the Inverse Dynamics analysis
- First solve for accelerations (recall the acceleration equation):
  \[ \Phi_q \ddot{q} = \gamma \]
- Next solve for the Lagrange multipliers and then the reaction forces:
  \[ \bar{\Phi}_q^T \lambda = Q^A - M\dot{q} \]
Inverse Dynamics: Conclusion

- Are we done once we computed the reaction forces?

  - Yes, because among the reaction forces we computed, we get all the forces/torques that are necessary to enforce the specified driving constraints $\Phi^D$ which were used to specify the desired motion

  \[
  F^D_i = - (\Phi^D_{r_i})^T \lambda^D \\
  T^D_i = \left( \Phi^D_{r_i} B_i s^P_i - \Phi^D_{\phi_i} \right)^T \lambda^D
  \]

  - Here, the driving constraint $\Phi^D$ acts between body $i$ and some other body. Reaction forces (induced by the corresponding Lagrange multiplier $\lambda^D$) are computed as “experienced” by body $i$

  - This gives us the forces/torques that need to be applied to get the prescribed motion
Example: Inverse Dynamics

- Compute the torque that the electrical motor must produce to open the door for 2 seconds following the prescribed motion:

\[ \phi = \frac{\pi}{2} \sin \left( \frac{\pi}{4} t \right) \]

- Half-length: \( L = 0.5 \)
- Mass: \( m = 30 \)
- Polar moment of inertia: \( J' = 2.5 \)
- RSDA spring coefficient: \( k = 8 \)
- RSDA free angle: \( \theta_0 = 0 \)
- RSDA damping coefficient: \( c = 1 \)
- All units are SI
Equilibrium Analysis
Equilibrium Analysis: The Idea

- A mechanical system is in equilibrium if the following conditions hold:

\[ \dot{q} = 0 \quad \& \quad \ddot{q} = 0 \]

- Equivalently, the system is at rest with zero acceleration.

- So what does it take to be in this state of equilibrium?
  - The system must be in a certain configuration \( q \).
  - The reaction forces (in other words, the Lagrange Multipliers \( \lambda \)) should assume certain values.
  - What does “certain” mean?
Equilibrium Analysis: The Math

- Equations of Motion:
  \[ M \ddot{q} + \Phi_q^T \lambda = Q^A \Rightarrow \Phi_q^T \lambda = Q^A \]

- Position Constraint Equations:
  \[ \Phi(q, t) = 0 \]

- Velocity Constraint Equations:
  \[ \Phi_q \dot{q} = -\Phi_t \triangleq \nu \Rightarrow \Phi_t = 0 \]

- Acceleration Constraint Equations:
  \[ \Phi_q \ddot{q} = - (\Phi_q \dot{q})_q \dot{q} - 2\Phi_q \dot{q} \dot{q} - \Phi_{tt} \triangleq \gamma \Rightarrow \Phi_{tt} = 0 \]
Equilibrium Analysis: The Math

- **Approach 1**
  - Simply solve the nonlinear system to find $q$ and $\lambda$
    \[
    \Phi_q^T \lambda = Q^A \\
    \Phi(q) = 0
    \]
  - One pretty big issue: we need a good initial guess

- **Approach 2**
  - Add damping in the system and perform dynamic analysis until it stops
  - Brute force, but effective

- **Approach 3**
  - Cast it as an optimization problem (minimize potential energy)
  - One big issue: works for conservative systems only
Example: Equilibrium Analysis

- Consider a pendulum connected to ground through a revolute joint and a rotational spring-damper.

- Length: $L = 1$
- Mass: $m = 10$

- RSDA spring coefficient: $k = 25$
- RSDA free angle: $\theta_0 = 0$
- RSDA damping coefficient: $c$

- Gravitational acceleration: $g = 9.81$

- All units are SI
ME451: Putting Things in Perspective
Summary of ME451

- Pick a set of Cartesian generalized coordinates

- Formulate the following sets of equations
  
  - Equations of Motion: \[ M \ddot{q} + \Phi_q^T \lambda = Q^A \]
  
  - Position Constraint Equations: \[ \Phi(q, t) = 0 \]
  
  - Velocity Constraint Equations: \[ \Phi_q \dot{q} = -\Phi_t \triangleq \nu \]
  
  - Acceleration Constraint Equations: \[ \Phi_q \ddot{q} = - (\Phi_q \dot{q})_q \dot{q} - 2 \Phi_q \ddot{q} - \Phi_{tt} \triangleq \gamma \]

- Solve the appropriate set of equations for Kinematics or Dynamics
Purpose of ME451

- Learn how to characterize the motion of any 2D mechanism:
  - Learn how to pose a Kinematics/Dynamics problem (modeling)
  - Learn how to solve a Kinematics/Dynamics problem (simulation)

- Improve your MATLAB programming skills
The End.