Before we get started...

- **Last time**
  - Translational, composite relative constraints,
  - Started discussion about cam-follower
  - Two more lectures covered
    - ADAMS intro
    - Examples, formulating the constraints for mechanisms

- **Today**
  - Wrap up cam-follower
  - Start talking about motions (Rheonomic constraints)

- **HW:** Due on Th, *10/23*, at 9:30 am except ADAMS component
  - ADAMS component due this Thursday at 9:30
  - MATLAB and pen-and-paper components postponed one week

- I will start posting solutions of the MATLAB component
  - Posting what the grader recommends as a solid solution – “hall of fame” of assignment solutions
3.4.2

CAM – FOLLOWERS
Cam – Follower Pair

- **Setup:**
  - Two shapes (one on each body) that are *always* in contact (no chattering)
  - Contact surfaces are *convex* shapes (or one is flat)
  - Sliding is *permitted* (unlike the case of gear sets)

- **Modeling basic idea:**
  - The two bodies share a common point
  - The tangents to their boundaries are collinear

Source: Wikipedia.org
Interlude: Boundary of a Convex Shape (1)

- Convex shape assumption ⇒ any point on the boundary is defined by a unique value of the angle $\alpha$.

- The distance from the reference point $Q_i$ to any point $P_i$ on the convex boundary is a function of $\alpha$:
  \[ \|Q_iP_i\| = \rho(\alpha_i) \]

- We need to express two quantities as functions of $\alpha$:
  - The position of $P_i$, that is $r_i^P$
  - The tangent at $P_i$, that is $g$

- What are we doing here?
  - We are parameterizing the boundary; i.e., shape of a 2D body
In the LRF:

\[ s_i'P = s_i'Q + a_i' \]

\[ a_i' = \rho(\alpha_i)u'(\alpha_i) \]

where

\[ \rho(\alpha_i) = ||a_i(\alpha_i)|| \triangleq \rho_i \]

\[ u'(\alpha_i) = \begin{bmatrix} \cos \alpha_i \\ \sin \alpha_i \end{bmatrix} \triangleq u'_i \]

and therefore

\[ g_i' = \frac{d\rho_i}{d\alpha_i} u'_i + \rho_i u'_i^\perp \]

In the GRF:

\[ r_i'P = r_i + s_i'P = r_i + A_i s_i'P = r_i + A_i \left( s_i'Q + a_i' \right) = r_i + A_i \left( s_i'Q + \rho_i u'_i \right) \]

\[ g_i = A_i g_i' = A_i \left( \frac{d\rho_i}{d\alpha_i} u'_i + \rho_i u'_i^\perp \right) \]
Interlude: Boundary of a Convex Shape (3)

- Bottom line: What are we doing here?
  - We are parameterizing the boundary; i.e., shape, of a 2D body
  - Shape of bodies comes into play here
  
- It will be up to you where you slap the point $Q_i$ on body $i$ to produce the parameterization of the boundary (shape) of the body
  
- Where should I place point $Q_i$ on body $i$?
  - Many possibilities, maybe choose it at the origin of the LRF attached to that body
  - Sometimes the shape of the body might recommend a different selection of $Q_i$
Cam – Follower Pair

- Step 1
  - The two bodies share the contact point: \( \mathbf{r}_i^P - \mathbf{r}_j^P = 0 \) (2 constraints)
  - The two tangents are collinear: \( \mathbf{g}_i^T \mathbf{g}_j = 0 \) (1 constraint)

- Recall that points \( P_i \) and \( P_j \) are located by the angles \( \alpha_i \) and \( \alpha_j \), respectively.

- Therefore, in addition to the \([x, y, \phi]^T\) coordinates for each body, one needs to include one additional generalized coordinate, namely the angle \( \alpha \):

  \[
  \text{Body } i : \quad [q_{i}^T, \alpha_i] = [x_i, y_i, \phi_i, \alpha_i] \\
  \text{Body } j : \quad [q_{j}^T, \alpha_j] = [x_j, y_j, \phi_j, \alpha_j]
  \]
Cam – Follower Constraints

- Step 1: Understand the physical joint

- Step 2: $\Phi^{cf(i,j)} = \begin{bmatrix} r_i + A_i \left( s_i^Q + \rho_i u_i' \right) - r_j - A_j \left( s_j^Q + \rho_j u_j' \right) \\ -g_i' B_{ij} g_j' \end{bmatrix} = 0$

- Step 3: $\Phi^{cf(i,j)}_q = ?$

- Step 4: $\nu^{cf(i,j)} = ?$

- Step 5: $\gamma^{cf(i,j)} = ?$

$q = [\ldots, x_i, y_i, \phi_i, \alpha_i, \ldots, x_j, y_j, \phi_j, \alpha_j, \ldots]^T$
Example 3.4.3

- Determine the expression of the tangents $g_1$ and $g_2$

\[
\rho_1(\alpha_1) = \begin{cases} 
-\frac{1}{4} \cos 3\alpha_1 + \frac{5}{4} & \text{if } 0 \leq \alpha_1 \leq \frac{2\pi}{3} \\
1 & \text{if } \frac{2\pi}{3} \leq \alpha_1 \leq 2\pi
\end{cases}
\]

\[
\rho_2(\alpha_2) = \frac{1}{4}
\]
Cam – Flat-Faced Follower Pair

- A particular case of the general cam-follower pair
  - Cam stays just like before
  - Flat follower
  - A particular instance of the general cam-follower setup just discussed
    - Good exam problem: go through the five stages associated with the modeling of a cam – flat-follower pair
- Typical application: internal combustion engine

Figure 3.4.10 Cam–flat-faced follower pair.

Figure 3.4.11 Cam–flat-faced follower in an internal combustion engine.
Errata:

- Page 80 (subscript ‘j’ instead of ‘i’)

\[
\Phi_{\mathbf{q}^{(i,j)}} = \begin{bmatrix}
I & B_j(s_j^Q + \rho_i u_i^j) \\
0 & -g_i^j A_i g_j^i
\end{bmatrix}, \quad \Phi_{\mathbf{\alpha}^{(i,j)}} = \begin{bmatrix}
A_i g_j^i \\
-(g_i^j)^T B_j g_j^i
\end{bmatrix} \tag{3.4.21}
\]

- Page 83 (Q instead of P)

\[
\Phi_{\mathbf{q}^{\text{eff}(i,j)}} = \begin{bmatrix}
\mathbf{u}_i^T B_j^T (s_i^P + \rho_i u_i^j)^T A_i u_i^j \\
0 & g_i^j A_i u_i^j
\end{bmatrix}, \quad \Phi_{\mathbf{\alpha}^{\text{eff}(i,j)}} = \begin{bmatrix}
\mathbf{g}_i^j B_j u_i^j \\
(\mathbf{\alpha}^i \setminus T \mathbf{\alpha}^j, \ldots)
\end{bmatrix} \tag{3.4.30}
\]
3.4.3

POINT – FOLLOWER
Point – Follower Pair

- **Setup:**
  - Pin $P$, attached to body $i$ can move (slide and rotate) in a slot attached to body $j$.

- **Modeling basic idea:**
  - Very similar to a revolute joint, except...
  - …point $P$ moves on body $j$.
  - Location of point $P$ on body $j$ is parameterized by the angle $\alpha_j$.
  - Therefore, in addition to the $[x_j, y_j, \phi_j]^T$ coordinates for body $j$, one needs to include **one additional** generalized coordinate, namely the angle $\alpha_j$:

  **Body $i$:** \( q_i^T = [x_i, y_i, \phi_i] \)
  **Body $j$:** \( [q_j^T, \alpha_j] = [x_j, y_j, \phi_j, \alpha_j] \)

- Note: this diagram is more general than the one in the textbook (includes point $Q_j$).
Point – Follower Constraints

- Step 1: Understand the physical joint
- Step 2: \( \Phi^{pf(i,j)} = \mathbf{r}_i + \mathbf{A}_i \mathbf{s}'_i - \mathbf{r}_j - \mathbf{A}_j \left( \mathbf{s}'_j + \rho_j \mathbf{u}'_j \right) = 0 \)
- Step 3: \( \Phi^q_{q} = ? \)
- Step 4: \( \nu^{pf(i,j)} = ? \)
- Step 5: \( \gamma^{pf(i,j)} = ? \)

\[ q = [\ldots, x_i, y_i, \phi_i, \ldots, x_j, y_j, \phi_j, \alpha_j, \ldots]^T \]
3.5 DRIVING CONSTRAINTS
Motivating Slide:
How Time \( t \) Factors In

- Consider this system of equations

\[
\Phi(q,t) = \begin{bmatrix}
x(t) + y(t) - 2 \\
x(t) - y(t) - \sin(t)
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

- We are in the business of computing \( q(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \)

- Recall that scleronomic constraints do not depend on time

- If we don’t have rheonomic constraints; i.e., constraint equations that depend on time, or motions, then the system would have one solution only

- Since the RHS changes in time, the solution changes in time. That is, \( x \) and \( y \) depend on time. As the time passes, at each instance of \( t \) we have a solution \( x(t) \) and \( y(t) \)