ME451
Kinematics and Dynamics of Machine Systems

Cam-Followers and Point-Follower

3.4.1, 3.4.2

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- Quote of the day: “Innovation distinguishes between a leader and a follower.”
-- Steve Jobs

Before we get started…

- Last time
  - Translational, composite relative constraints,
  - Started discussion about cam-follower
  - Two more lectures covered
    - ADAMS intro
    - Examples, formulating the constraints for mechanisms

- Today
  - Wrap up cam-follower
  - Start talking about motions (Rheonomic constraints)

- HW: Due on Th, *10/23*, at 9:30 am except ADAMS component
  - ADAMS component due this Thursday at 9:30
  - MATLAB and pen-and-paper components postponed one week

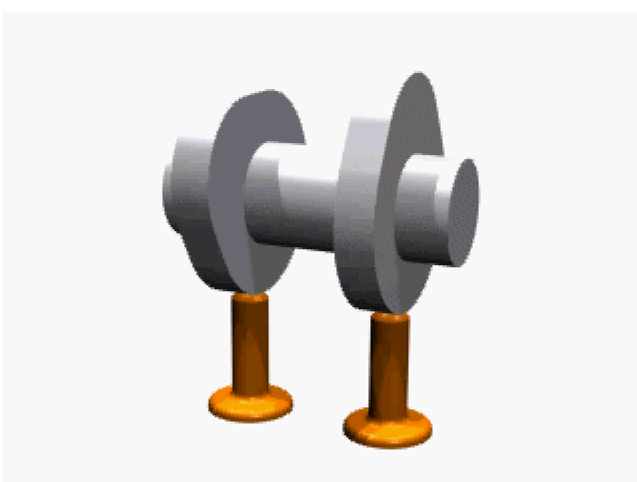
- I will start posting solutions of the MATLAB component
  - Posting what the grader recommends as a solid solution – "hall of fame" of assignment solutions

3.4.2

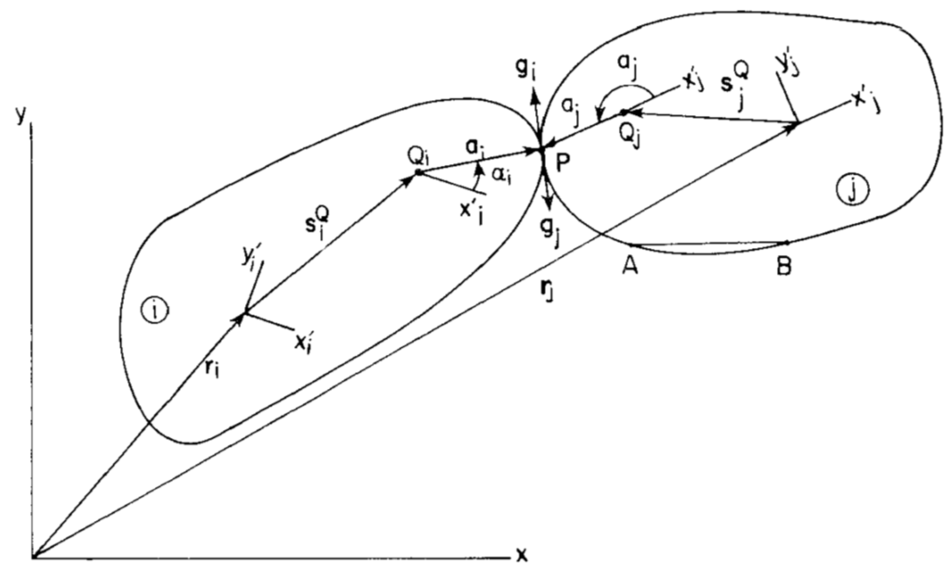
CAM – FOLLOWERS

Cam – Follower Pair

- Setup:
  - Two shapes (one on each body) that are always in contact (no chattering)
  - Contact surfaces are convex shapes (or one is flat)
  - Sliding is permitted (unlike the case of gear sets)
- Modeling basic idea:
  - The two bodies share a common point
  - The tangents to their boundaries are collinear

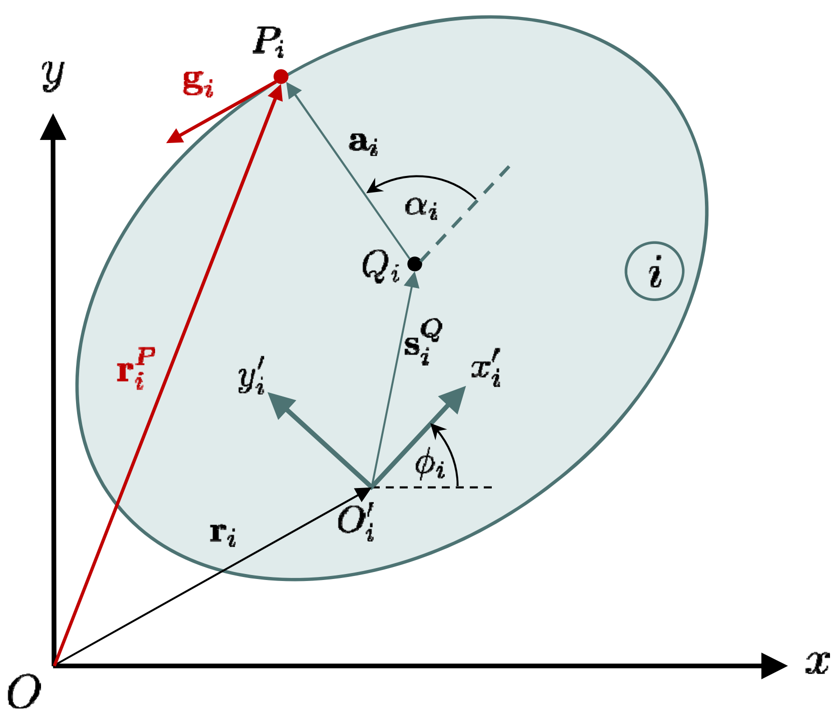


Source: Wikipedia.org



Interlude: Boundary of a Convex Shape (1)

- Convex shape assumption ⇒ any point on the boundary is defined by a unique value of the angle α.



- The distance from the reference point Q_i to any point P_i on the convex boundary is a function of α:

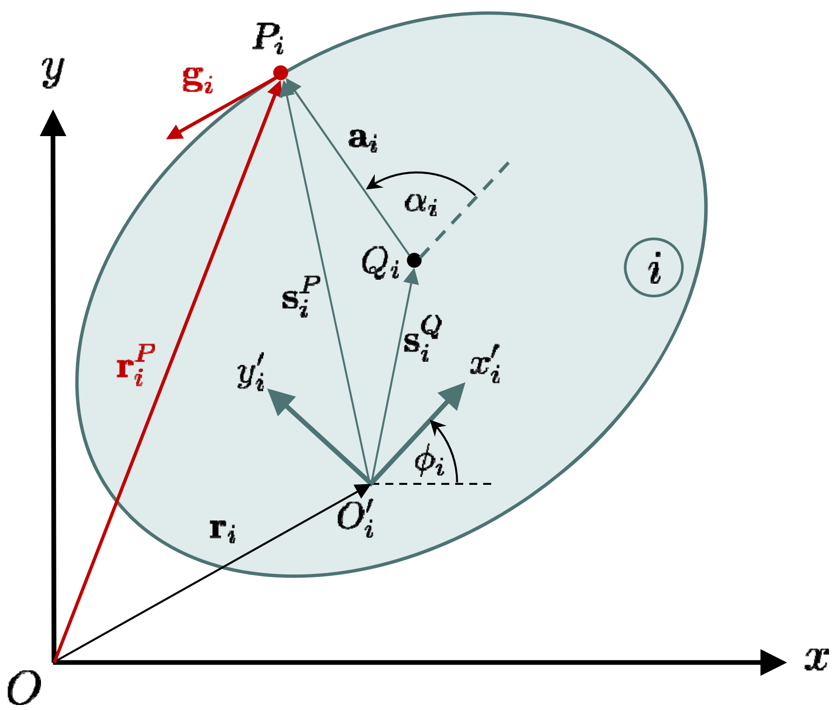
$$\|\overrightarrow{Q_iP_i}\| = \rho(\alpha_i)$$

- We need to express **two** quantities as functions of α:
  - The position of P_i, that is $\mathbf{r}_i^P$
  - The tangent at P_i, that is $\mathbf{g}$

- What are we doing here?
  - We are parameterizing the boundary; i.e., shape of a 2D body

[handout available online]

Interlude Boundary of a Convex Shape (2)



- In the LRF:

$$\mathbf{s}_i'^P = \mathbf{s}_i'^Q + \mathbf{a}_i'$$

$$\mathbf{a}_i' = \rho(\alpha_i)\mathbf{u}'(\alpha_i)$$

where

$$\rho(\alpha_i) = ||\mathbf{a}_i(\alpha_i)|| \triangleq \rho_i$$

$$\mathbf{u}'(\alpha_i) = \begin{bmatrix} \cos\alpha_i \\ \sin\alpha_i \end{bmatrix} \triangleq \mathbf{u}_i'$$

and therefore

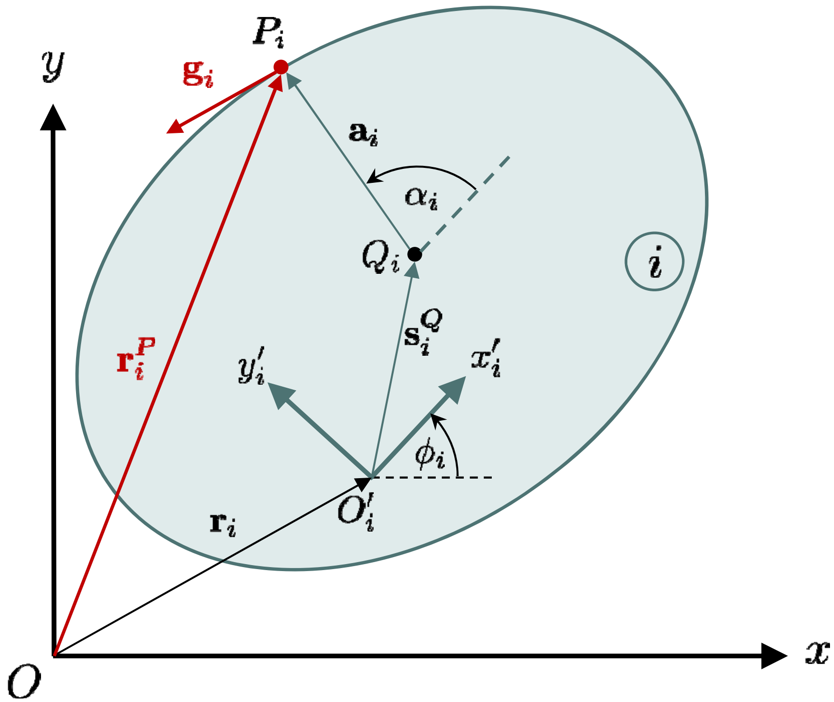
$$\mathbf{g}_i' = \frac{d\rho_i}{d\alpha_i}\mathbf{u}_i' + \rho_i \mathbf{u}_i'^{\perp}$$

- In the GRF:

$$\mathbf{r}_i{}^P = \mathbf{r}_i + \mathbf{s}_i^P = \mathbf{r}_i + \mathbf{A}_i \mathbf{s}_i'^P = \mathbf{r}_i + \mathbf{A}_i\left(\mathbf{s}_i'^Q + \mathbf{a}_i'\right) = \mathbf{r}_i + \mathbf{A}_i\left(\mathbf{s}_i'^Q + \rho_i \mathbf{u}_i'\right)$$

$$\mathbf{g}_i = \mathbf{A}_i \mathbf{g}_i' = \mathbf{A}_i\left(\frac{d\rho_i}{d\alpha_i}\mathbf{u}_i' + \rho \mathbf{u}_i'^{\perp}\right)$$

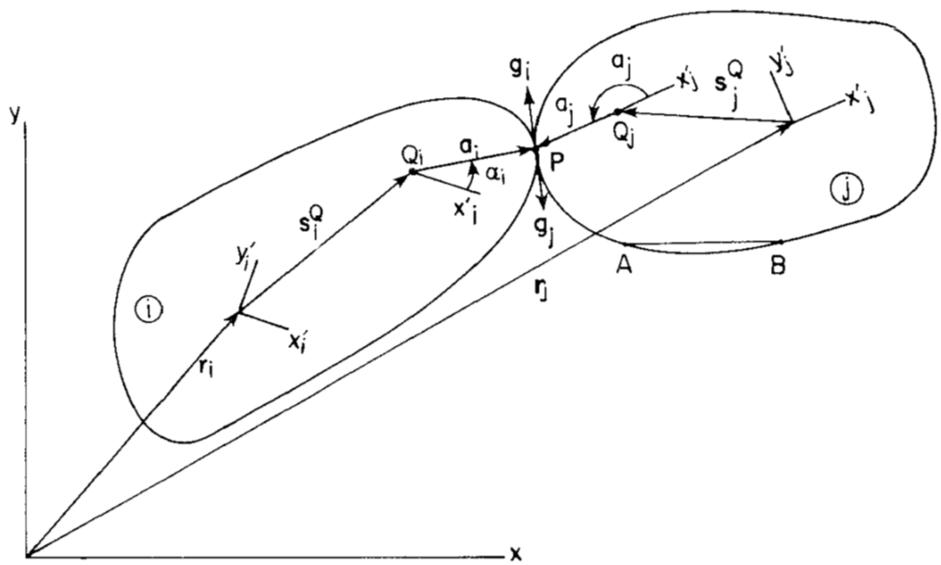
Interlude: Boundary of a Convex Shape (3)



- Bottom line: What are we doing here?
  - We are parameterizing the boundary; i.e., shape, of a 2D body
  - Shape of bodies comes into play here
  - It will be up to you where you slap the point Q_i on body i to produce the parameterization of the boundary (shape) of the body
  - Where should I place point Q_i on body i ?
    - Many possibilities, maybe choose it at the origin of the LRF attached to that body
    - Sometimes the shape of the body might recommend a different selection of Q_i

Cam – Follower Pair

- Step 1
  - The two bodies share the contact point: $\mathbf{r}_i^P - \mathbf{r}_j^P = \mathbf{0}$ (2 constraints)
  - The two tangents are collinear: ${\mathbf{g}_i^{\perp}}^T \mathbf{g_j} = 0$ (1 constraint)



- Recall that points P_i and P_j are located by the angles α_i and α_j, respectively.
- Therefore, in addition to the $[x, y, \phi]^T$ coordinates for each body, one needs to include one additional generalized coordinate, namely the angle α:

$$\textbf{Body } i: \quad [\mathbf{q}_i^T, \alpha_i] = [x_i, y_i, \phi_i, \alpha_i]$$

$$\textbf{Body } j: \quad [\mathbf{q}_j^T, \alpha_j] = [x_j, y_j, \phi_j, \alpha_j]$$

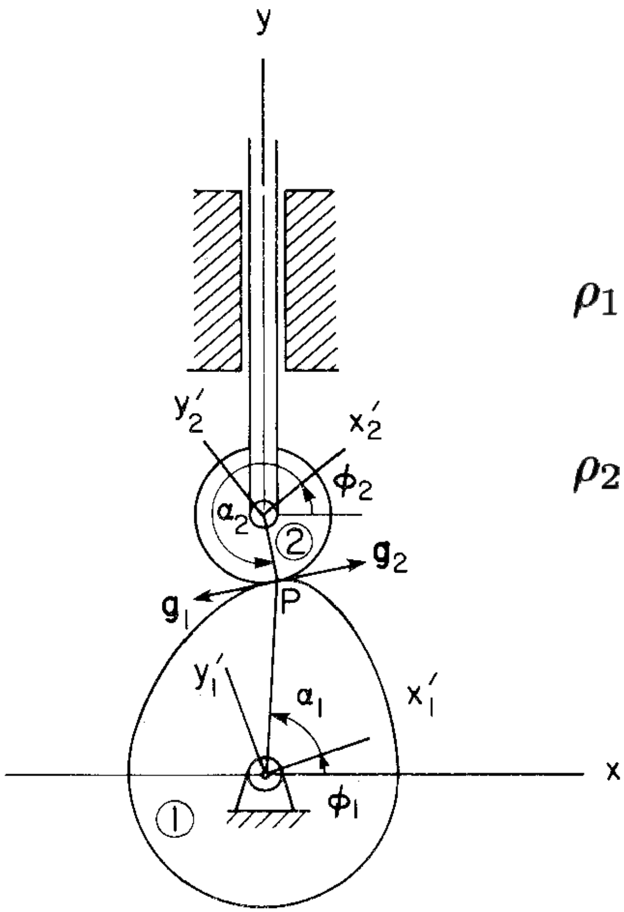
Cam – Follower Constraints

- Step 1: Understand the physical joint
- Step 2: $\mathbf{\Phi}^{cf(i,j)} = \begin{bmatrix} \mathbf{r}_i + \mathbf{A}_i \left(\mathbf{s'}_i^Q + \rho_i \mathbf{u}'_i \right) - \mathbf{r}_j - \mathbf{A}_j \left(\mathbf{s'}_j^Q + \rho_j \mathbf{u}'_j \right) \\ -\mathbf{g}'^T_i \mathbf{B}_{ij} \mathbf{g}'_j \end{bmatrix} = \mathbf{0}$
- Step 3: $\mathbf{\Phi}_{\mathbf{q}}^{cf(i,j)} = ?$
- Step 4: $\nu^{cf(i,j)} = ?$
- Step 5: $\gamma^{cf(i,j)} = ?$

$$\mathbf{q} = [\ldots, x_i, y_i, \phi_i, \alpha_i, \ldots, x_j, y_j, \phi_j, \alpha_j, \ldots]^T$$

Example 3.4.3

- Determine the expression of the tangents $\mathbf{g}_1$ and $\mathbf{g}_2$



$$\rho_1(\alpha_1) = \begin{cases} -\frac{1}{4}\cos 3\alpha_1 + \frac{5}{4} & \text{if} \quad 0 \le \alpha_1 \le \frac{2\pi}{3} \\ 1 & \text{if} \quad \frac{2\pi}{3} \le \alpha_1 \le 2\pi \end{cases}$$

$$\rho_2(\alpha_2) = \frac{1}{4}$$

Cam – Flat-Faced Follower Pair

- A particular case of the general cam-follower pair
  - Cam stays just like before
  - Flat follower
  - A particular instance of the general cam-follower setup just discussed
    - Good exam problem: go through the five stages associated with the modeling of a cam – flat-follower pair
  - Typical application: internal combustion engine

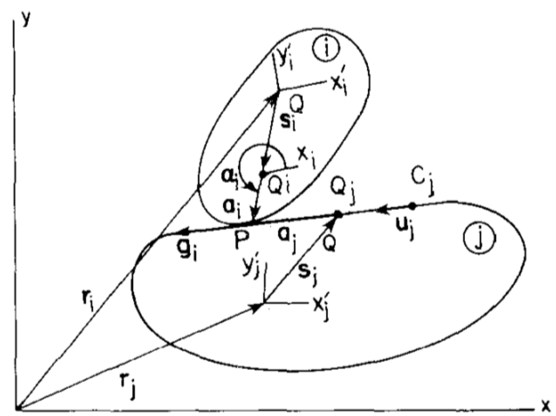


Figure 3.4.10 Cam–flat-faced follower pair.

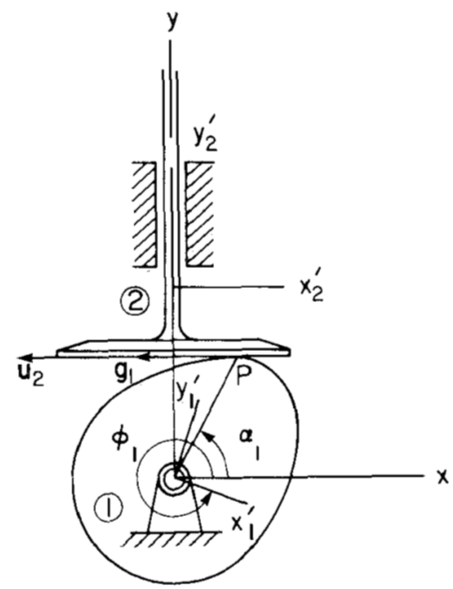


Figure 3.4.11 Cam–flat-faced follower in an internal combustion engine.

Errata:

- Page 80
  (subscript ‘j’ instead of ‘i’)

$$\mathbf{\Phi}_{\mathbf{q}_i}^{cf(i,j)} = \begin{bmatrix} \mathbf{I} & \mathbf{B}_i(\mathbf{s}_i'^Q + \rho_i \mathbf{u}_i') \\ \mathbf{0} & -\mathbf{g}_i'^T \mathbf{A}_{ij} \mathbf{g}_j' \end{bmatrix} \qquad \mathbf{\Phi}_{\alpha_i}^{cf(i,j)} = \begin{bmatrix} \mathbf{A}_i \mathbf{g}_i' \\ -(\mathbf{g}_i')_{\alpha_i}^T \mathbf{B}_{ij} \mathbf{g}_j' \end{bmatrix} \qquad (3.4.21)$$

$$\mathbf{\Phi}_{\mathbf{q}_j}^{cf(i,j)} = \begin{bmatrix} -\mathbf{I} & -\mathbf{B}_j(\mathbf{s}_j'^Q + \rho_j \mathbf{u}_j') \\ \mathbf{0} & \mathbf{g}_i'^T \mathbf{A}_{ij} \mathbf{g}_i' \end{bmatrix} \qquad \mathbf{\Phi}_{\alpha_j}^{cf(i,j)} = \begin{bmatrix} -\mathbf{A}_j \mathbf{g}_j' \\ -\mathbf{g}_i'^T \mathbf{B}_{ij} (\mathbf{g}_j')_{\alpha_j} \end{bmatrix}$$

- Page 83
  (Q instead of P)

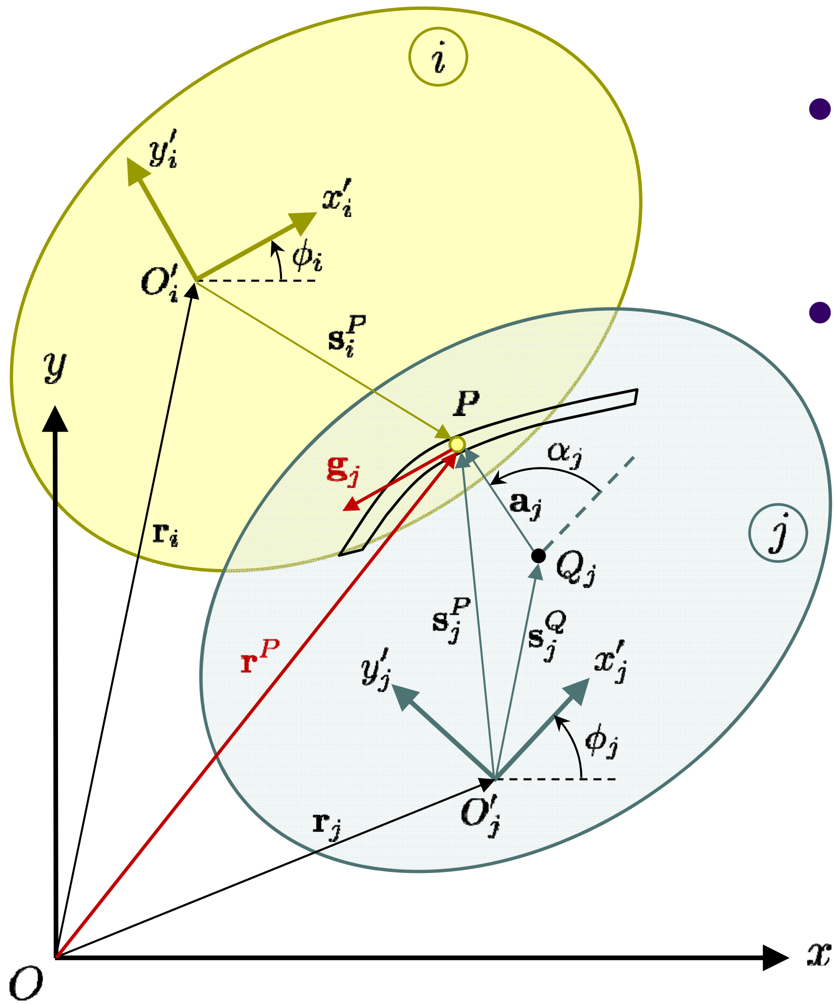
$$\mathbf{\Phi}_{\mathbf{q}_i}^{cff(i,j)} = \begin{bmatrix} \mathbf{u}_j'^T \mathbf{B}_j^T & (\mathbf{s}_i'^P + \rho_i \mathbf{u}_i')^T \mathbf{A}_{ij} \mathbf{u}_j' \\ \mathbf{0} & \mathbf{g}_i'^T \mathbf{A}_{ij} \mathbf{u}_j' \end{bmatrix}$$

$$\mathbf{\Phi}_{\alpha_i}^{cff(i,j)} = \begin{bmatrix} \mathbf{g}_i'^T \mathbf{B}_{ij} \mathbf{u}_j' \\ \ldots \end{bmatrix} \qquad (3.4.30)$$

3.4.3

POINT – FOLLOWER

Point – Follower Pair



- Setup:
  - Pin P, attached to body i can move (slide and rotate) in a slot attached to body j
- Modeling basic idea:
  - Very similar to a revolute joint, except…
  - …point P moves on body j
  - Location of point P on body j is parameterized by the angle α_j
  - Therefore, in addition to the $[x_j, y_j, \phi_j]^T$ coordinates for body j, one needs to include one additional generalized coordinate, namely the angle α_j:

$$\textbf{Body } i: \quad \mathbf{q}_i^T = [x_i\,,\, y_i\,,\, \phi_i]$$

$$\textbf{Body } j: \quad [\mathbf{q}_j^T\,,\, \alpha_j] = [x_j\,,\, y_j\,,\, \phi_j\,,\, \alpha_j]$$

- Note: this diagram is more general than the one in the textbook (includes point Q_j)

Point – Follower Constraints

- Step 1: Understand the physical joint
- Step 2: $\mathbf{\Phi}^{pf(i,j)} = \mathbf{r}_i + \mathbf{A}_i \mathbf{s}'^P_i - \mathbf{r}_j - \mathbf{A}_j \left(\mathbf{s}'^Q_j + \rho_j \mathbf{u}'_j \right) = \mathbf{0}$
- Step 3: $\mathbf{\Phi}^{pf(i,j)}_{\mathbf{q}} = ?$
- Step 4: $\nu^{pf(i,j)} = ?$
- Step 5: $\gamma^{pf(i,j)} = ?$

$$\mathbf{q} = \left[\ldots , x_i , y_i , \phi_i , \ldots , x_j , y_j , \phi_j , \alpha_j , \ldots \right]^T$$

3.5

DRIVING CONSTRAINTS

Motivating Slide:
How Time t Factors In

- Consider this system of equations

$$\mathbf{\Phi}(\mathbf{q},t) = \begin{bmatrix} x(t)+y(t) & - & 2 \\ x(t)-y(t) & - & \sin(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- We are in the business of computng $\mathbf{q}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$
- Recall that Scleronomic constraints do not depend on time
- If we don't have Rheonomic constraints; i.e., constraint equations that depend on time, or motions, then the system would have one solution only
- Since the RHS changes in time, the solution changes in time. That is, x and y depend on time. As the time passes, at each instance of t we have a solution $x(t)$ and $y(t)$