

Time derivatives example

$$\mathbf{q} = \begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \end{bmatrix}$$

$$\phi(\mathbf{p}(t)) = 3x(t) + 2L \sin(\theta(t))$$

$$\frac{d}{dt} \mathbf{q} = \begin{bmatrix} 3 & 0 & 2L \cos(\theta(t)) \end{bmatrix}$$

$$\text{Then } \frac{d}{dt} \phi = \frac{d}{dt} \mathbf{q} \cdot \dot{\mathbf{q}} = \begin{bmatrix} 3 & 0 & 2L \cos(\theta(t)) \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

$$= 3\dot{x} + 2L\dot{\theta} \cdot \cos(\theta(t))$$

For $\frac{d}{dt} \phi(\mathbf{p}) = \begin{bmatrix} 3x(t) + 2L \sin \theta(t) \\ y(t) - 2L \cos \theta(t) \end{bmatrix}$, follow the same

steps as above to get,

$$\frac{d}{dt} \phi = \frac{d}{dt} \mathbf{q} \cdot \dot{\mathbf{q}} = \begin{bmatrix} 3 & 0 & 2L \cos(\theta(t)) \\ 0 & 1 & 2L \sin(\theta(t)) \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

$$= \begin{bmatrix} 3\dot{x} + 2L\dot{\theta} \cos(\theta(t)) \\ \dot{y} + 2L\dot{\theta} \sin(\theta(t)) \end{bmatrix}$$

