

$$\boxed{\frac{\partial(Bx)}{\partial x} = B}, \text{ where } B \in \mathbb{R}^{m \times n} \text{ and } B \text{ doesn't depend on } x \in \mathbb{R}^n.$$

$$\text{let } B = \begin{bmatrix} b_{11} & \dots & b_{1n} \\ \dots & \dots & \dots \\ b_{m1} & \dots & b_{mn} \end{bmatrix}$$

$$Bx = \begin{bmatrix} b_{11}x_1 + \dots + b_{1n}x_n \\ \dots \\ b_{m1}x_1 + \dots + b_{mn}x_n \end{bmatrix} = \begin{bmatrix} r_1 \\ \vdots \\ r_m \end{bmatrix},$$

$$\text{where } r_i = \sum_{j=1}^n b_{ij} \cdot x_j \in \mathbb{R}$$

Then for any r_i ,

$$\frac{\partial r_i}{\partial x} = [b_{i1} \quad b_{i2} \quad \dots \quad b_{in}]$$

As such,

$$\frac{\partial(Bx)}{\partial x} = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix} = B$$

