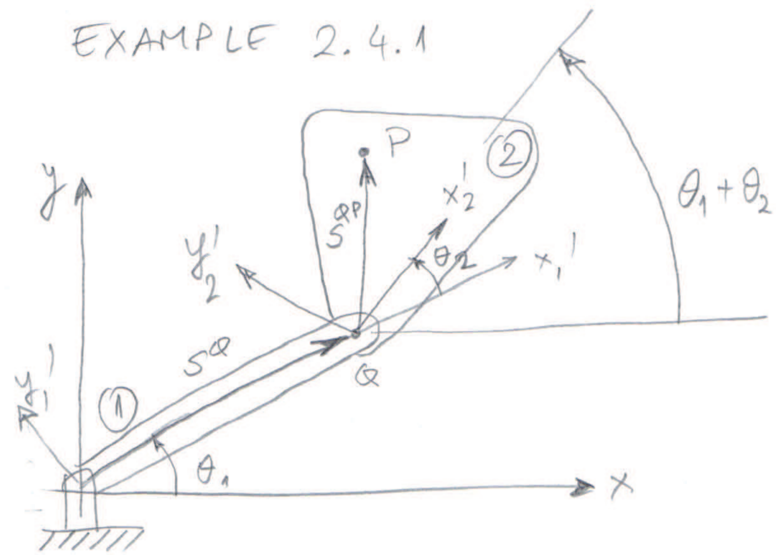
EXAMPLE 2.4.1



$$\varphi_1 \equiv \theta_1 \qquad \Rightarrow r^Q = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + A(\theta_1)\cdot s_1'^Q$$

$$\varphi_2 \equiv \theta_1 + \theta_2$$

$$r^P = r^Q + A(\theta_1+\theta_2)\cdot s_2'^P$$

$$\Rightarrow r^P = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x_1'^Q \\ 0 \end{bmatrix} + \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix} \begin{bmatrix} x_2'^P \\ y_2'^P \end{bmatrix}$$

$$= \begin{bmatrix} x_1'^Q \cos\theta_1 + x_2'^P \cos(\theta_1+\theta_2) - y_2'^P \sin(\theta_1+\theta_2) \\ x_1'^Q \sin\theta_1 + x_2'^P \sin(\theta_1+\theta_2) + y_2'^P \cos(\theta_1+\theta_2) \end{bmatrix}$$

Calculate the partial derivatives of the position of P with respect to the array of generalized coordinates $q^T = [\theta_1, \theta_2]^T$.

$$r^P \in \mathbb{R}^2 \; ; \; q \in \mathbb{R}^2 \Rightarrow r^P_q \in \mathbb{R}^{2\times 2}$$

$$r^P_q \triangleq \frac{\partial r^P}{\partial q} = \begin{bmatrix} \frac{\partial r^P}{\partial \theta_1} & \frac{\partial r^P}{\partial \theta_2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial x^P}{\partial q} \\ \frac{\partial y^P}{\partial q} \end{bmatrix} = \begin{bmatrix} \frac{\partial x^P}{\partial \theta_1} & \frac{\partial x^P}{\partial \theta_2} \\ \frac{\partial y^P}{\partial \theta_1} & \frac{\partial y^P}{\partial \theta_2} \end{bmatrix}$$

$$\frac{\partial r^P}{\partial \theta_1} = \begin{bmatrix} -x_1'^{Q} \sin\theta_1 - x_2'^{P} \sin(\theta_1+\theta_2) - y_2'^{P} \cos(\theta_1+\theta_2) \\ x_1'^{Q} \cos\theta_1 + x_2'^{P} \cos(\theta_1+\theta_2) - y_2'^{P} \sin(\theta_1+\theta_2) \end{bmatrix}$$

$$\frac{\partial r^P}{\partial \theta_2} = \begin{bmatrix} -x_2'^{P} \sin(\theta_1+\theta_2) - y_2'^{P} \cos(\theta_1+\theta_2) \\ x_2'^{P} \cos(\theta_1+\theta_2) - y_2'^{P} \sin(\theta_1+\theta_2) \end{bmatrix}$$