EQUATION 2.5.13

Assume $x \in \mathbb{R}^K$ and $q = q(x) \in \mathbb{R}^n$; $p = p(x) \in \mathbb{R}^n$

Show that

$$\frac{d(q^T p)}{dx} = q^T p_x + p^T q_x$$

Proof:

Let $F : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be $F \triangleq F(q, p) = q^T p$

and let $\phi : \mathbb{R}^k \to \mathbb{R}$ be $\phi(x) \triangleq F(q(x), p(x))$

Then $$\phi_x = \frac{\partial F}{\partial q} q_x + \frac{\partial F}{\partial p} p_x$$

$$= \frac{\partial (q^T p)}{\partial q} q_x + \frac{\partial (q^T p)}{\partial p} p_x$$

$$= \frac{\partial (p^T q)}{\partial q} q_x + \frac{\partial (q^T p)}{\partial p} p_x$$

$$= p^T q_x + q^T p_x \qquad \square$$