HW 2

September 11, 2014

Due: September 18, 2014

Turning in your assignment: save your document as a PDF (scanned handwritten notes are OK) named "lastName_HW_02.pdf" and upload it in the appropriate Dropbox Folder (HW_02) at Learn@UW.

Problem 1. Assume that $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and consider a function $\phi : \mathbb{R}^2 \to \mathbb{R}$ defined as $\phi(\mathbf{y}) = 3y_1^2 + \sin y_2$.

Assume further that $\mathbf{y}$ depends on a variable $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ as follows:

$$\mathbf{y} \triangleq \mathbf{y}(\mathbf{x}) \equiv \begin{bmatrix} y_1(\mathbf{x}) \\ y_2(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} 2x_1 + \log_{10} x_2 + \sqrt{x_3} \\ (x_1 - x_2)^2 \end{bmatrix}$$

It follows that ϕ depends on $\mathbf{x}$, implicitly through $\mathbf{y}$. Apply the *chain rule of differentiation* to find the derivative of ϕ with respect to $\mathbf{x}$, that is:

$$\phi_{\mathbf{x}} \triangleq \begin{bmatrix} \frac{\partial \phi}{\partial x_1} & \frac{\partial \phi}{\partial x_2} & \frac{\partial \phi}{\partial x_3} \end{bmatrix} = ?$$

What is the dimension of the result $\phi_{\mathbf{x}}$?

Problem 2. Assume that $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and consider a function $\mathbf{\Phi} : \mathbb{R}^2 \to \mathbb{R}^2$ defined as $\mathbf{\Phi}(\mathbf{y}) = \begin{bmatrix} 2y_1 + y_2^2 \\ y_1 y_2 \end{bmatrix}$.

Assume further that $\mathbf{y}$ depends on a variable $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ as follows:

$$\mathbf{y} \triangleq \mathbf{y}(\mathbf{x}) \equiv \begin{bmatrix} y_1(\mathbf{x}) \\ y_2(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} 2x_1 + \log_{10} x_2 + \sqrt{x_3} \\ (x_1 - x_2)^2 \end{bmatrix}$$

It follows that $\mathbf{\Phi}$ depends on $\mathbf{x}$, implicitly through $\mathbf{y}$. Apply the *chain rule of differentiation* to find the derivative of $\mathbf{\Phi}$ with respect to $\mathbf{x}$, that is:

$$\mathbf{\Phi}_{\mathbf{x}} \triangleq \begin{bmatrix} \frac{\partial \mathbf{\Phi}}{\partial x_1} & \frac{\partial \mathbf{\Phi}}{\partial x_2} & \frac{\partial \mathbf{\Phi}}{\partial x_3} \end{bmatrix} = ?$$

What is the dimension of the result $\mathbf{\Phi}_{\mathbf{x}}$?