MATLAB Assignment 9

Due Date: December 6, 2011

Turning in your homework: place all your files in a directory called "lastNameDate"; zip that directory and drop it in the mailbox at Learn@UW.

Problem 1 [pretty straightforward]. Use Forward Euler to solve the Initial Value Problem below. Use an integration step-size of $h = 0.01$ and generate a numerical solution for the time interval $t \in [0, 10]$.

$$\begin{cases} \dot{y} &= \sin(y) \\ y(0) &= 1.0 \end{cases}$$

- Generate a plot that displays the solution as a function of time for the given interval and upload it onto the forum.
- [Bonus] Solve the problem using MATLAB's `ODE45` function and compare the MATLAB solution with your solution. Specifically, generate and upload onto the forum a second png or jpg that plots the difference between the two solutions $y_{you} - y_{MATLAB}$ as a function of time.

Problem 2 [relatively straightforward]. Use Backward Euler to solve the Initial Value Problem above. Generate and upload onto the forum a plot of the solution, and the difference between Forward Euler (FE) and Backward Euler (BE) solutions $y_{FE} - y_{BE}$ as a function of time. Use the same integration step size as in Problem 1.

Problem 3 [relatively straightforward, but requires slightly more coding]. The goal of this problem is to perform an "order analysis". Basically, what you want to do is to verify that the order of Forward Euler is 1 and that the order of the Runge-Kutta method provided below is 4. In this exercise use the following IVP:

$$\begin{cases} \dot{y} &= -0.1y\sin(y) \\ y(0) &= 0.1 \end{cases}$$

For the Forward Euler, start with a step size $h = 0.1$, then use $h = 0.01$, $h = 0.001$, and $h = 0.0001$. The time interval is $t \in [0, 1.5]$. You will have to generate and upload to the forum a convergence plot, which is a log-log plot that on the x-axis plots the step size value and on the y-axis plots the numerical error. In the convergence analysis for RK4, since this is a more accurate formula, you can use larger step-sizes, so please use $h = 0.25$, $h = 0.3$, $h = 0.5$, and $h = 0.75$.

How do you obtain the numerical error? In theory, you should subtract $y_{exact}(1.5) - y_{approx}(1.5)$ to get the error at the end of the interval. However, you don't have $y_{exact}(t)$. To get a good value for this number you will have to use Runge-Kutta with a tiny step-size so that you get something that comes close to $y_{exact}(1.5)$. This solution obtained with the tiny step-size will be your "reference" solution, called $\hat{y}_{exact}(1.5)$. Then, when you use, for instance, Forward Euler with $h = 0.01$ you will plot this pair of points (0.01 and $y_{FE0.01} - \hat{y}_{exact}(1.5)$).

Recall that you should use a log-log plot. If everything works well, you should see a straight line with slope 1.0 for Forward Euler, and slope 4.0 for Runge-Kutta. These are the orders of the two methods: 1 and 4, respectively.

Finally, the Runge-Kutta RK4 approximation y_{n+1} of the actual value $y(t_{n+1})$ is computed as:

$$\begin{aligned} y_{n+1} &= y_n + \tfrac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\ t_{n+1} &= t_n + h \end{aligned}$$

where h is the integration step-size and k_1 through k_4 are obtained as

$$\begin{aligned} k_1 &= f(t_n, y_n) \\ k_2 &= f(t_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}hk_1) \\ k_3 &= f(t_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}hk_2) \\ k_4 &= f(t_n + h, y_n + hk_3) \end{aligned}$$

Problem 4 [not that straightforward]. Use the second order BDF method provided in class to solve the Initial Value Problem in Problem 1. Generate a plot of the solution and also upload it to the forum. Use the same integration step size as in Problem 1.
Note: you will need to carry along the numerical approximations y_{n-1} and y_n in order to compute the numerical solution y_{n+1} at time t_{n+1}. Therefore, you can only apply the BDF method to get y_2, y_3, y_4, etc. This being the case, use the RK4 to get y_1, and then start using BDF for the rest of the problem. This is one small drawback of BDF, it needs to be primed to start.

Problem 5 [difficult]. Consider the double pendulum in Fig.6.5.2 (page 233). Notice that a horizontal force is specified on the second body. Assume that this concentrated force always stays horizontal, and its magnitude is always 5 N. Although not shown, assume that a positive torque $n = 2.5\ N \cdot m$

is applied to body 1. Additionally, take $l_1 = l_2 = 1$, and $m_1 = m_2 = 2$ (all units are SI). The mass moment of inertia should be computed based on information in Table 6.1.1 (page 208). The bodies are moving in a gravitational field with $g = -9.81 m/s^2$. Use for this problem Cartesian generalized coordinates in conjunction with centroidal reference frames as shown in the textbook.

Use the doublePend.acf and doublePend.adm files that you generated for your previous MATLAB assignment to carry out Dynamics Analysis of this mechanism using your simEngine2D. To this end, you will have to use the Newmark integration formula discussed in class and solve the resulting nonlinear algebraic system obtained at each time step t_n upon discretization of the constrained equations of motion. For this problem,

(a) Generate and upload to the forum a plot of the time evolution of the x coordinate of body 2.

(b) Generate and upload to the forum a plot of the reaction force induced on body 2 by the presence of the revolute joint connecting bodies 1 and 2. When computing this force, take the point P in conjunction with which you report the reaction force to be the point where the joint is located on body 2.

(c) Just like above, generate and upload to the forum a plot of the reaction torque induced on body 2 by the presence of the revolute joint connecting bodies 1 and 2. When computing this reaction torque, take the point P in conjunction with which you report the reaction torque to be the point where the joint is located on body 2.