ME451
Kinematics and Dynamics of Machine Systems

Dynamics of Planar Systems

December 6, 2011

Equilibrium Analysis & Inverse Dynamics Analysis

ME451 Wrap Up

"My personal philosophy is not to undertake a project unless it is manifestly important and nearly impossible."

Edwin Land (Polaroid camera inventor)

Before we get started…

- Last Time:
  - Finished the discussion on how to solve the Differential Algebraic Equations that govern the time evolution (dynamics) of a mechanism
    - Covered Newmark's method and discussed about the quasi-Newton approach that will help with the solution of the nonlinear system obtained after the discretization of the constrained equations of motion

- Today
  - Discuss the final exam
  - Inverse Dynamics Analysis
  - Equilibrium Analysis
  - We hug each other and stuff

- No more assignments in ME451 (MATLAB or textbook-based)

- Final Exam
  - Tu, Dec. 17 at 2:45 PM, Room: 1255ME

Final Exam Related

- For simEngine2D, please make sure you support the following modeling elements:
  - Bodies
  - Abs constraints, revolute & translational joint, distance constraints
  - Point forces, Translational-Spring-Damper Forces, Rotational-Spring-Damper Torques
  - Computation of reaction forces in joints

- Note: only the last 15 points (out of 100) of the final exam are tied to simEngine2D. In other words, if you don't even have a simEngine2D and answer all the other questions right you'll end up with a score of 85.

- Do this prior to December 17:
  - Stop by 1255ME and check that your simEngine2D works as expected in conjunction with the version of MATLAB available on the computers in that room

Final Exam, Rules of Engagement

- Identify the set of constraints present in the mechanism
- Determine a set of consistent Initial Conditions for the mechanism
- Define an acf and adm pair of files associated with the type of analysis that you are supposed to carry out and the model that you were given.
- Run simulations (Kinematics and/or Dynamics) using your simEngine2D.
- Generate a set of plots that show the time evolution of an attribute of the model (the motion of a point, the value of a reaction force as a function of time, etc.)
- Use the Learn@UW drop-box to provide a zipped directory that contains your code, adm/acf files, and png plots of your results. The naming convention for this directory should be "LastnameFinalME451.zip". For instance, "NegrutFinalME451.zip".

Final Exam, Comments

- Running your simEngine2D code should also report the amount of time it took for completing a simulation. This information should be included in the zipped directory in a text file "readme.txt".

- I will not insist on having simEngine2D that you use during the exam be implemented exclusively by you. However, in good faith, you will have to indicate in the text file "readme.txt" the percentage of your contribution to the simEngine2D code that you are using in the final exam. I will then understand that the remaining percent came from code written by other ME451 colleague[s]. This is absolutely fine (it won't impact your final's score), but should be acknowledged.

- If you contributed more than 66% to your simEngine2D, you qualify for entering the race for the fastest solver. Winning that race translates into an automatic A-grade in the course.

- One other automatic A grade *might* be assigned for the most general, flexible, and neatly organized simEngine2D code.

Miscellaneous Tidbits…

- No more ME451 lectures from now till the final exam
  - Make-up class of last Th to count as lecture for this Th
  - Last week: work on your take-home midterm and on the Final Project
- I will travel so I won’t be able to meet face-to-face with you. Post questions on the forum and I’ll do my best to answer them as soon as possible
- Your homework with the lowest score will be dropped when computing the final grade

[New Topic]

Inverse Dynamics: The idea

- First of all, what does dynamics analysis mean?
  - You apply some forces/torques on a mechanical system and look at how the configuration of the mechanism changes in time
  - How it moves also depends on the ICs associated with that mechanical system

- In *inverse* dynamics, the situation is quite the opposite:
  - You specify driving constraints on the mechanical system and you are interested in finding out the set of forces/torques that were actually applied to the mechanical system to lead to this motion
  - Note that you need a zero degree of freedom system after the application of the driving constraints

- When is *inverse* dynamics useful?
  - It's useful in controls. For instance in controlling the motion of a robot: you know how you want this robot to move, but you need to figure out what joint torques you should apply to make it move the way it should

Inverse Dynamics: The Math

- When can one talk about Inverse Dynamics?
  - Given a mechanical system, a prerequisite for Inverse Dynamics is that the number of degrees of freedom associated with the system is **zero**
    - You have as many generalized coordinates as constraints (THIS IS KEY)
  - This effectively makes the problem a Kinematics problem
  - The two stages of the Inverse Dynamics analysis
    - First solve for accelerations (recall the acceleration equation):

$$\mathbf{\Phi_q}\ddot{\mathbf{q}} = \gamma$$

    - Next you solve for the Lagrange multipliers and then the reaction forces:

$$\mathbf{\Phi_q}^T\lambda = \mathbf{Q}^A - \mathbf{M}\ddot{\mathbf{q}}$$

Inverse Dynamics: Closure

- Are we done once we computed the reaction forces?
  - Yes, because among the forces you computed, you get all the forces/torques that are necessary to impose the driving constraints Φ^D that you imposed on the system

$$\mathbf{F}^D = -\left[\mathbf{\Phi}^D_{\mathbf{r}_i}\right]^T \lambda$$

$$T^D = [(\mathbf{s'}^P_i)^T \mathbf{B}^T_i \left[\mathbf{\Phi}^D_{\mathbf{r}_i}\right]^T - \left[\mathbf{\Phi}^D_{\phi_i}\right]^T]\lambda$$

Here constraint Φ^D acts between body i and some other body. Reaction forces are computed as "felt" by body i

  - This gives you the forces/torques that you need to apply to get the prescribed motion

End Inverse Dynamics
Beginning Equilibrium Analysis

[New Topic]

Equilibrium Analysis: The Idea

- A mechanical system is in equilibrium if the following conditions hold:

$$\dot{\mathbf{q}} = \mathbf{0} \quad \& \quad \ddot{\mathbf{q}} = \mathbf{0}$$

- Equivalently, the system is at rest, with zero acceleration

- So what does it take to be in this state of equilibrium?
  - You need to be in a certain configuration **q**
  - The reaction forces, that is, Lagrange Multipliers, should assume certain values
  - What does "certain" mean?

Equilibrium Analysis: The Math

- Equations of Motion:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^T \lambda = \mathbf{Q}^A \quad \Rightarrow \quad \boxed{\mathbf{\Phi}_{\mathbf{q}}^T \lambda = \mathbf{Q}^A}$$

- Position Constraint Equations:

$$\boxed{\mathbf{\Phi}(\mathbf{q}, t) = \mathbf{0}}$$

- Velocity Constraint Equations:

$$\mathbf{\Phi}_{\mathbf{q}} \cdot \dot{\mathbf{q}} = -\mathbf{\Phi}_t \quad \Rightarrow \quad \boxed{\mathbf{0} = -\mathbf{\Phi}_t}$$

- Acceleration Constraint Equations:

$$\mathbf{\Phi}_{\mathbf{q}}\ddot{\mathbf{q}} = -(\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}})_{\mathbf{q}}\dot{\mathbf{q}} - 2\mathbf{\Phi}_{\mathbf{q}t}\dot{\mathbf{q}} - \mathbf{\Phi}_{tt} \quad \Rightarrow \quad \boxed{\mathbf{0} = -\mathbf{\Phi}_{tt}}$$

[Cntd.]

Equilibrium Analysis: The Math

- To conclude, one needs a configuration **q** and the Lagrange multipliers λ should be such that

$$
\begin{aligned}
\mathbf{\Phi}_{\mathbf{q}}^T \lambda &= \mathbf{Q}^A \\
\mathbf{\Phi}(\mathbf{q}, t) &= \mathbf{0} \\
\mathbf{\Phi}_t(\mathbf{q}, t) &= \mathbf{0}
\end{aligned}
$$

- How can you go about finding such a configuration?
  - Approach 1 (dumb, but powerful)
    - Add damping in a system and watch it move till it stops
  - Approach 2 (OK, but you need a good starting point)
    - Simply solve the nonlinear system to find **q** and $\mathbf{Q}^A$

$$
\begin{aligned}
\mathbf{\Phi}_{\mathbf{q}}^T \lambda &= \mathbf{Q}^A \\
\mathbf{\Phi}(\mathbf{q}) &= \mathbf{0}
\end{aligned}
$$

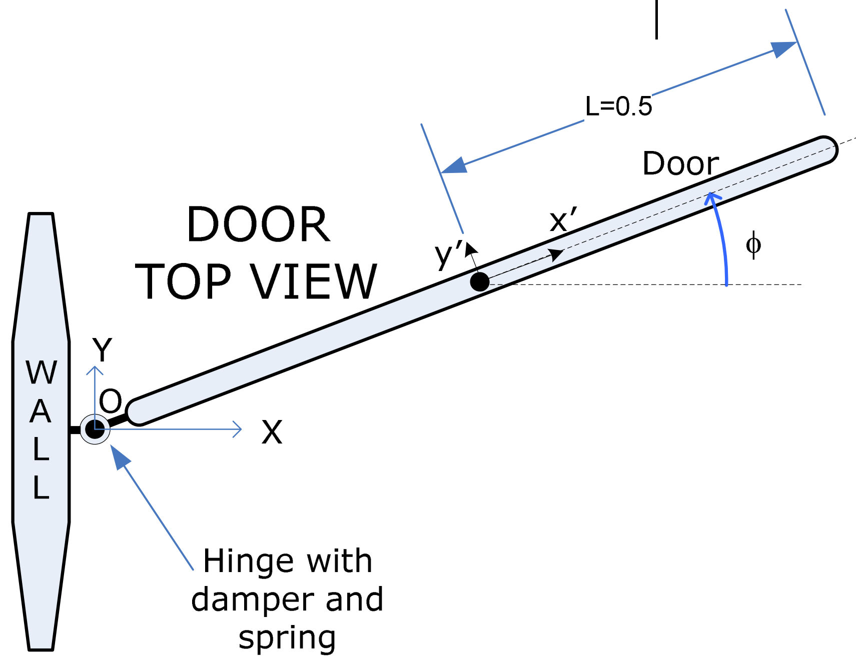
  - Approach 3 (not that common)
    - Cast it as an optimization problem
    - Works for conservative systems only

[AO]

Example: Inverse Dynamics

- Door Mass m = 30
- Mass Moment of Inertia J' = 2.5
- Spring/damping coefficients:

  K = 8 C = 1
- All units are SI.



- Zero Tension Angle of the spring:

$$\phi_{free} = 0$$

- Compute torque that electrical motor applies to *open* handicapped door
  - Apply motion for two seconds to open the door like

$$\Phi^D(t) = \phi - \frac{\pi}{2}\sin(\frac{\pi}{4}t)$$

[AO]

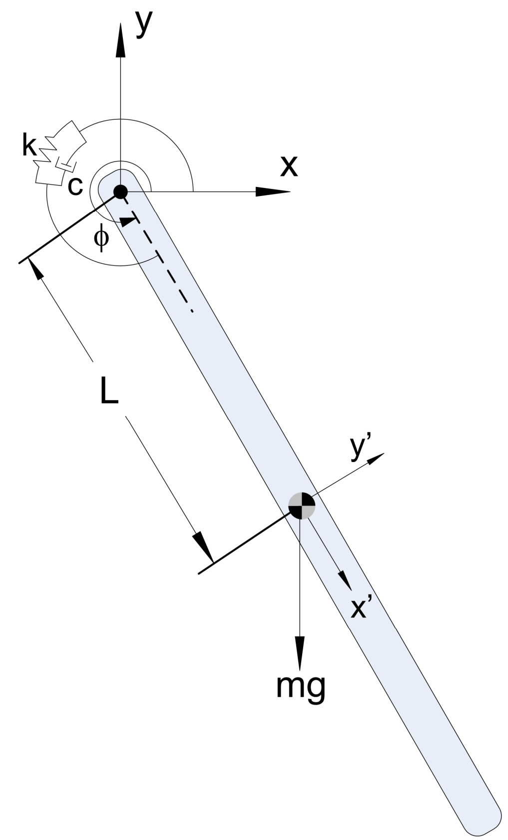
Example: Equilibrium Analysis

- Find the equilibrium configuration of the pendulum below
  - Pendulum connected to ground through a revolute joint and rotational spring-damper element

- Free angle of the spring:

$$\phi_{free} = 0$$

- Spring constant: k=25
- Mass m = 10
- Length L=1
- All units are SI.



Summary of ME451

- Pick a set of Cartesian generalized coordinates q.
- You can then formulate the following sets of equations

- Equations of Motion: $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^T \lambda = \mathbf{Q}^A$

- Position Constraint Equations: $\mathbf{\Phi}(\mathbf{q}, t) = \mathbf{0}$

- Velocity Constraint Equations: $\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}} = \nu$

- Acceleration Constraint Equations: $\mathbf{\Phi}_{\mathbf{q}}\ddot{\mathbf{q}} = \gamma$

What was ME451 good for?

- Understand how to pose a Kinematics/Dynamics problem
- Understand how to solve a Kinematics/Dynamics problem
- Improve your MATLAB programming skills

End ME451