ME451 Kinematics and Dynamics of Machine Systems

Singular Configurations of Mechanisms 3.7

Dynamics of Planar Systems: Chapter 6

October 27, 2011

"Getting information off the Internet is like taking a drink from a fire hydrant."
Mitchell Kapor

Before we get started…

- Last Time
  - Discussed Newton-Raphson method to solve nonlinear algebraic equations
  - Discussed the three stages of the Kinematics Analysis:
    - Position Analysis
    - Velocity Analysis
    - Acceleration Analysis
  - Mentioned why the Implicit Function Theorem is your friend
- Today:
  - Discuss “Singular Configurations of Mechanisms” (Section 3.7)
  - Start the “Dynamics Analysis” part of the course (Chapter 6)
- HW (due on November 3 at 11:59 PM): emailed to you this weekend
  - ADAMS
  - MATLAB
- Quick Remarks:
  - Exam Review on Nov. 2, starting at 6PM in room 1153ME
  - Note the room is the one next door

Singular Configurations

- What are “singular configurations”?
  - Abnormal situations that should be avoided since they indicate either a malfunction of the mechanism (poor design), or a bad model associated with an otherwise well designed mechanism
  - Singular configurations come in two flavors:
    - Physical Singularities (PS): reflect bad design decisions
    - Modeling Singularities (MS): reflect bad modeling decisions
  - Singular configurations do not represent the norm, but you must be aware of their existence
    - A PS is particularly bad and can lead to dangerous situations

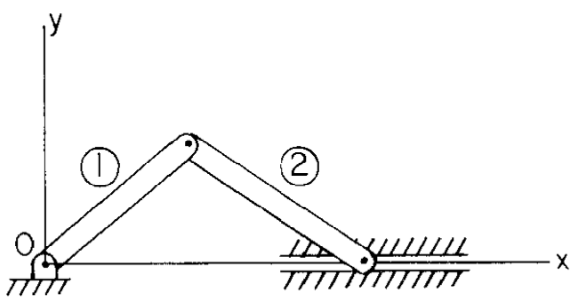
Singular Configurations

- In a *singular configuration*, one of three things can happen:
  - PS1: Your mechanism locks-up
  - PS2: Your mechanism hits a bifurcation
  - MS1: Your mechanism has redundant constraints
- The important question:
  - How can we characterize a singular configuration in a formal way such that we are able to diagnose it?
- Next: example of what happens in a singular configuration

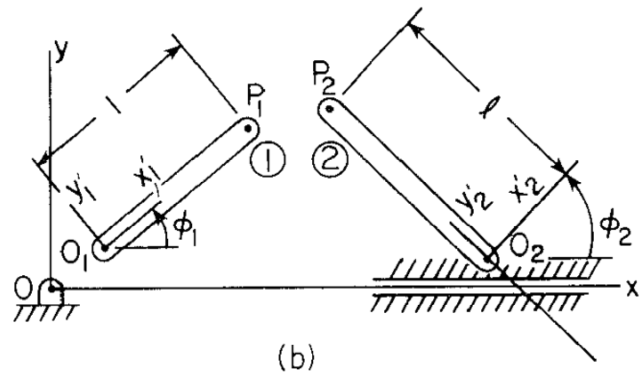
Mechanism Lock-Up: PS1
(Example 3.7.5, draws on 3.1.2)

- Investigate what happens to this mechanism when length $l = 0.5$

$$\mathbf{q} = \begin{bmatrix} x_2 \\ \phi_1 \\ \phi_2 \end{bmatrix}$$



(a)



(b)

$$\boldsymbol{\Phi}(\mathbf{q}, t) = \begin{bmatrix} -x_2 + \cos\phi_1 + l\sin\phi_2 \\ \sin\phi_1 - l\cos\phi_2 \\ \phi_1 - \frac{\pi}{12}t \end{bmatrix} = \mathbf{0}$$

$$\boldsymbol{\Phi}_{\mathbf{q}}(\mathbf{q}) = \begin{bmatrix} -1 & -\sin\phi_1 & l\cos\phi_2 \\ 0 & \cos\phi_1 & l\sin\phi_2 \\ 0 & 1 & 0 \end{bmatrix}$$

- Can you ever get in trouble?
- Yes, check what happens when t=2
  - Mechanism hits a lock-up configuration
  - When t=2: $x_2 = \frac{\sqrt{3}}{2}$

    $\phi_1 = \frac{\pi}{6}$

    $\phi_2 = 0$

Mechanism Lock-Up

- Definition of **lock-up** configuration:
  - At t=2, the mechanism cannot proceed anymore from configuration

$$\mathbf{q}^\star = [\sqrt{3}/2, \quad \pi/6, \quad 0]^T$$

  - Symptoms of "lock-up":
    - Jacobian in that configuration is singular (indeed, check the Jacobian on previous slide):

$$\det |\mathbf{\Phi_q}(\mathbf{q}^\star, 2)| = 0$$

    - The rank of the *velocity augmented constraint Jacobian* is higher than the rank of the constraint Jacobian

Velocity augmented constraint Jacobian

$$\widehat{\mathbf{J}}_{vel} = [\mathbf{\Phi_q} \quad \nu] \qquad rank(\widehat{\mathbf{J}}_{vel}) > rank(\mathbf{\Phi_q})$$

    - The velocities and accelerations assume huge values (in fact, going to ∞)
      - That is, you're sure not to miss it…

Mechanism Lock-Up (Cntd.)

- Investigate rank of augmented Jacobian

$$rank(\widehat{\mathbf{J}}_{vel}) = 3 \quad > \quad 2 = rank(\mathbf{\Phi_q})$$

- Carry out velocity analysis

time = 1.85	vel =	[-0.71392649808689	0.26179938779915	-1.27150008402231]	
time = 1.90	vel =	[-0.85975114686538	0.26179938779915	-1.54001421905491]	
time = 1.95	vel =	[-1.18022664998825	0.26179938779915	-2.15362292657357]	
time = 2.00	vel = 1.0e+007*[	-1.52152519881098	0.00000002617994	-3.04305037144201]	

Mechanism moves faster than speed of light…

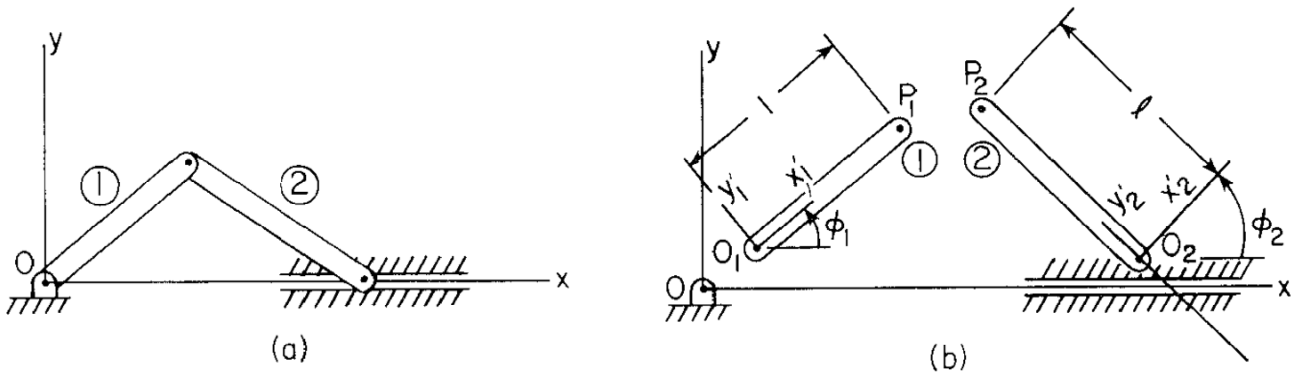
- Carry out acceleration analysis

time = 1.80	acc =	[-1.47292585680960	0	-2.53780315286818]
time = 1.85	acc =	[-2.19722185658353	0	-3.95600397951865]
time = 1.90	acc =	[-3.92446925376964	0	-7.35587287703508]
time = 1.95	acc =	[-10.83795211380501	0	-21.05152842858363]
time = 2.00	acc = 1.0e+022*[	-3.10719260152581	0	-6.21438520305161]

Bifurcation: PS2 (Example 3.7.5, draws on 3.1.2)

- Investigate what happens to this mechanism when length $l = 1$

$$\mathbf{q} = \begin{bmatrix} x_2 \\ \phi_1 \\ \phi_2 \end{bmatrix}$$



$$\boldsymbol{\Phi}(\mathbf{q}, t) = \begin{bmatrix} -x_2 + \cos\phi_1 + l\sin\phi_2 \\ \sin\phi_1 - l\cos\phi_2 \\ \phi_1 - \frac{\pi}{12}t \end{bmatrix} = \mathbf{0}$$

$$\boldsymbol{\Phi}_{\mathbf{q}}(\mathbf{q}) = \begin{bmatrix} -1 & -\sin\phi_1 & l\cos\phi_2 \\ 0 & \cos\phi_1 & l\sin\phi_2 \\ 0 & 1 & 0 \end{bmatrix}$$

- Can you ever get in trouble?
- Yes, check what happens when t=6
  - Mechanism hits a bifurcation
  - When t=6: $x_2 = 0$

    $\phi_1 = \frac{\pi}{2}$

    $\phi_2 = 0$

Bifurcation (Cntd.)

- Definition of **bifurcation** configuration:
  - The mechanism can proceed in more than one way
  - Symptoms of "bifurcation":
    - Jacobian in that configuration is singular
    - The rank of the *velocity and acceleration augmented constraint Jacobians* is equal to the rank of the constraint Jacobian

Acceleration augmented constraint Jacobian

$$\widehat{\mathbf{J}}_{acc} = [\mathbf{\Phi_q} \quad \gamma] \qquad\qquad rank(\widehat{\mathbf{J}}_{acc}) = rank(\widehat{\mathbf{J}}_{vel}) = rank(\mathbf{\Phi_q})$$

- The velocities and accelerations do not assume huge values
  - That's why it's tough to spot a bifurcation (unlike a lock-up), often times you cruise through it without knowing it…

Bifurcation, Scenario 1: Time Step is 0.06 [s]

- Investigate rank of augmented Jacobians

$$rank(\widehat{\mathbf{J}}_{acc}) = rank(\widehat{\mathbf{J}}_{vel}) = rank(\mathbf{\Phi_q}) = 2$$

Bifurcation Time: T=6

- Carry out velocity analysis

time =	5.80	vel = [	-0.52288120167379	0.26179938779915	-0.26179938779915]
time =	5.86	vel = [	-0.52324712340312	0.26179938779915	-0.26179938779915]
time =	5.92	vel = [	-0.52348394173427	0.26179938779915	-0.26179938779916]
time =	5.98	vel = [	-0.52359159823540	0.26179938779915	-0.26179938779871]
time =	6.04	vel = [	0.00000000000002	0.26179938779915	0.26179938779917]
time =	6.10	vel = [	-0.00000000000001	0.26179938779915	0.26179938779914]
time =	6.16	vel = [	0.00000000000000	0.26179938779915	0.26179938779915]

NOTE: Stepped over bifurcation configuration and hardly noticed

- Carry out acceleration analysis

time = 5.80	acc =	[	-0.00717409977873	0	0.00000000000003]
time = 5.86	acc =	[	-0.00502304039889	0	0.00000000000003]
time = 5.92	acc =	[	-0.00287074165928	0	-0.00000000000025]
time = 5.98	acc =	1.0e-003 *[	-0.71773456266700	0	0.00000004392366]
time = 6.04	acc =	1.0e-012 *[	-0.99659190644620	0	-0.99659201484942]
time = 6.10	acc =	1.0e-012 *[	0.22531501249805	0	0.22531502713580]
time = 6.16	acc =	1.0e-013 *[	-0.43745210091874	0	-0.43745418339683]

Stepping over bifurcation...

Bifurcation, Scenario 2: Time Step is 0.05 [s]

- Carry out velocity analysis

time = 5.85 vel = [-0.52319509991791 0.26179938779915 -0.26179938779915]
time = 5.90 vel = [-0.52341935137507 0.26179938779915 -0.26179938779914]
time = 5.95 vel = [-0.52355391762090 0.26179938779915 -0.26179938779910]
time = 6.00 vel = [NaN NaN -Inf]
Warning: Matrix is singular to working precision.
> In function bifurcation at line 14
time = 6.05 vel = [-0.00000000000005 0.26179938779915 0.26179938779910]
time = 6.10 vel = [-0.00000000000001 0.26179938779915 0.26179938779914]
time = 6.15 vel = [-0.00000000000001 0.26179938779915 0.26179938779914]

Bifurcation Time: T=6

- Carry out acceleration analysis

time = 5.85 acc = [-0.00538165069997 0 0.00000000000005]
time = 5.90 acc = [-0.00358827950303 0 0.00000000000011]
time = 5.95 acc = [-0.00179429347120 0 0.00000000000185]
time = 6.00 acc = [NaN NaN NaN]
Warning: Matrix is singular to working precision.
> In function bifurcation at line 19
time = 6.05000000000000 acc = 1.0e-011 *[0.21214905163374 0 0.21214905572961]
time = 6.10000000000000 acc = 1.0e-012 *[0.22531501249805 0 0.22531502713580]
time = 6.15000000000000 acc = 1.0e-012 *[0.10145027567015 0 0.10145042771686]
time = 6.20000000000000 acc = 1.0e-013 *[0.49056387941139 0 0.49055963218247]

NOTE: On previous slide we were “lucky”. Here, by chance, we chose a step-size that happen to hit the bifurcation

Singular Configurations

- In the end, what is the pattern that emerges?

- The important remark:
  - The only case when you run into problems is when the constraint Jacobian becomes singular:

$$\det|\mathbf{\Phi}_{\mathbf{q}}(\mathbf{q}^{\star}, t^{\star})| = 0$$

- Otherwise, the Implicit Function Theorem (IFT) gives you the answer:
  - If the constraint Jacobian is nonsingular, IFT says that you cannot be in a singular configuration. And that's that.

Singularities: Closing Remarks

- Remember that you seldom see singularities

- To summarize, if the constraint Jacobian is singular,
  - You can be in a lock-up configuration (you won't miss this, PS1)
  - You might face a bifurcation situation (very hard to spot, PS2)
  - You might have redundant constraints (we didn't say anything about this, MS1)

- Singularity analysis is a tough topic. Textbook gives a broader perspective, although not necessarily deeper

End of Kinematics

Beginning of Dynamics (Chapter 6)

Purpose of Chapter 6

- At the end of this chapter you should understand what “dynamics” means and how you should go about carrying out a dynamics analysis
- We’ll learn a couple of things:
  - How to formulate and then solve the equations that govern the time evolution of a system of bodies in planar motion
    - These equations are differential equations and they are called equations of motion
    - As many bodies as you wish, connected by any joints we’ve learned about…
  - How to compute the reaction forces in any joint connecting any two bodies in the mechanism
  - Understand how to properly handle the applied (external) forces to correctly use them in formulating the equations of motion

The Idea, in a Nutshell…

- First part of the class: **Kinematics**
  - You have as many constraints (kinematic and driving) as generalized coordinates: NDOF=0
  - No spare degrees of freedom left
  - Position, velocity, acceleration found as the solution of algebraic problems (both nonlinear and linear)
  - We do not care whatsoever about forces applied to the system, we are told what the motions are and that's enough for the purpose of kinematics

- Second part of the class: **Dynamics**
  - You only have a few constraints imposed on the system
  - You have extra degrees of freedom: NDOF>0
  - The system evolves in time as a result of external forces applied on it
  - We very much care about forces applied and inertia properties of the components of the mechanism (mass, mass moment of inertia)

Some clarifications

- Dynamics **<u>key</u>** question: how can I get the acceleration of each body of the mechanism?
  - Why is acceleration so relevant? If you know the acceleration you can integrate it twice to get velocity and position information for each body
  - How is the acceleration of a body "*i* "measured in the first place?
    - You attach a reference frame on body "*i* " and measure the acceleration of the body reference frame with respect to the global reference frame:

$$\ddot{\mathbf{q}}_i = \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \\ \ddot{\phi}_i \end{bmatrix}$$

- The answer to the key question: To get the acceleration of each body, you first need to formulate the equations of motion
  - Remember **F=ma**?
    - Actually, the proper way to state this is **ma=F**, which is the "equation of motion", that is, what we are after here

Equations of motion of ONE planar RIGID body

- Framework:
  - We are dealing with rigid bodies
  - For this lecture, we'll consider only one body
    - We'll extend to more bodies in two weeks…

- What are we after?
  - Proving that for one body with a **reference frame attached at its center of mass location** the equations of motion are:

$$
\begin{aligned}
m\ddot{\mathbf{r}} - \mathbf{F} &= \mathbf{0} \qquad \text{Equations of Motion governing translation} \\
J'\ddot{\phi} - n &= 0 \qquad \text{Equation of Motion governing rotation}
\end{aligned}
$$

r is the position of the body local reference frame

ϕ is the orientation of the body local reference frame

Equations of Motion (EOM)
Some clarifications…

- Centroidal reference frame of a body
  - A reference frame located right at the center of mass of that body
  - How is this special? It's special since a certain integral vanishes...

$$\int_m \mathbf{s}'^P \, d\mathrm{m}(P) = 0$$

- What is J' ?
  - Mass moment of inertia

$$J' = \int_m \left[\mathbf{s}'^P\right]^T \mathbf{s}'^P \, d\mathrm{m}(P)$$

NOTE: Textbook uses misleading notation

$$\mathbf{s}'^{PT} \Leftrightarrow \left[\mathbf{s}'^P\right]^T$$