ME451
Kinematics and Dynamics of Machine Systems

Basic Concepts in Planar Kinematics - 3.1
Absolute Kinematic Constraints – 3.2
Relative Kinematic Constraints – 3.3

September 29, 2011

"There is no reason for any individual to have a computer in their home."
Ken Olson, president and founder, Digital Equipment Corporation, 1977.

Before we get started…

- Last time:
  - Computing the velocity and acceleration of a point attached to a moving rigid body
  - Absolute vs. relative generalized coordinates
  - Start Chapter 3: Kinematics Analysis

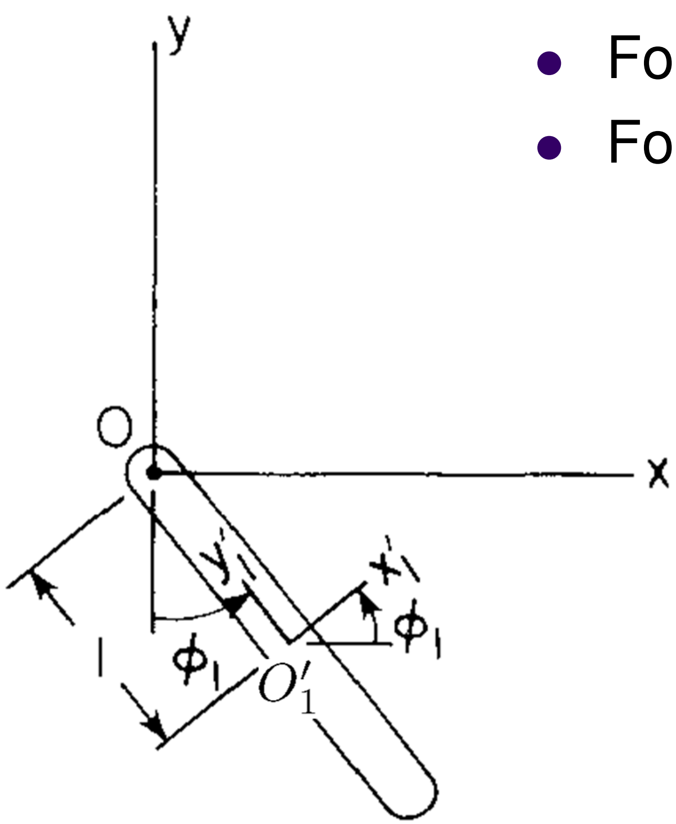
- Today:
  - Wrap up high level discussion of Kinematics Analysis after introducing the concept of Jacobian
  - Start discussion on how to formulate Kinematic constraints associated with a mechanism
    - Absolute kinematic constraints
    - Relative kinematic constraints

- Assignment 4 due one week from today:
  - Problems 2.6.1, 3.1.1, 3.1.2, 3.1.3
  - ADAMS and MATLAB components emailed to you

- Assignment 3 due today
  - Problems due in class
  - MATLAB and ADAMS part due at 23:59 PM

Example 3.1.1

- A motion $\phi_1 = 4t^2$ is applied to the pendulum
- Use Cartesian generalized coordinates
- Formulate the velocity analysis problem
- Formulate the acceleration analysis problem



Kinematic Analysis Stages

- Position Analysis Stage
  - Challenging
- Velocity Analysis Stage
  - Simple
- Acceleration Analysis Stage
  - OK

- To take care of all these stages, ONE step is critical:
  - Write down the constraint equations associated with the joints present in your mechanism
- Once you have the constraints, the rest is boilerplate

Once you have the constraints…
(Going beyond the critical step)

- The three stages of Kinematics Analysis: position analysis, velocity analysis, and acceleration analysis they each follow *very* similar recipes for finding for each body of the mechanism its position, velocity and acceleration, respectively

- ALL STAGES RELY ON THE CONCEPT OF JACOBIAN MATRIX:
  - $\mathbf{\Phi_q}$ – the partial derivative of the constraints wrt the generalized coordinates

- ALL STAGES REQUIRE THE SOLUTION OF A SYSTEM OF EQUATIONS

$$\mathbf{\Phi_q}\,\mathbf{x} = \mathbf{b}$$

- WHAT IS *DIFFERENT* BETWEEN THE THREE STAGES IS THE EXPRESSION OF THE RIGHT-SIDE OF THE LINEAR EQUATION, "**b**"

The Details…

- As we pointed out, it all boils down to this:
  - Step 1: Before anything, write down the constraint equations associated with your model
  - Step 2: For each stage, construct $\mathbf{\Phi}_q$ and the specific **b** , then solve for **x**

- So how do you get the <u>position</u> configuration of the mechanism?
  - Kinematic Analysis key observation: The number of constraints (kinematic and driving) should be equal to the number of generalized coordinates
    - This is, NDOF=0, a prerequisite for Kinematic Analysis

$$\mathbf{\Phi}(\mathbf{q},t) = \begin{bmatrix} \mathbf{\Phi}^K(\mathbf{q}) \\ \mathbf{\Phi}^D(\mathbf{q},t) \end{bmatrix}_{nc\times 1} = \mathbf{0}$$

$$\mathbf{q} \in \mathbb{R}^{nc} \qquad \mathbf{\Phi} : \mathbb{R}^{nc+1} \longrightarrow \mathbb{R}^{nc}$$

IMPORTANT: This is a nonlinear systems with *nc* equations and *nc* unknowns that you must solve to find **q**

Getting the Velocity and Acceleration of the Mechanism

- Previous slide taught us how to find the positions **q**
  - At each time step t_k, generalized coordinates $\mathbf{q}_k$ are the solution of a nonlinear system

- Take one time derivative of constraints Φ(q,t) to obtain the **velocity equation**:

$$\frac{d}{dt}\mathbf{\Phi}(\mathbf{q},t) = \mathbf{0} \qquad \Rightarrow \qquad \mathbf{\Phi_q}\dot{\mathbf{q}} = -\mathbf{\Phi}_t$$

- Take yet one more time derivative to obtain the **acceleration equation:**

$$\ddot{\mathbf{\Phi}} = \frac{d^2}{dt^2}\mathbf{\Phi}(\mathbf{q},t) = \mathbf{0} \qquad \Rightarrow \qquad \mathbf{\Phi_q}\ddot{\mathbf{q}} = -(\mathbf{\Phi_q}\dot{\mathbf{q}})_{\mathbf{q}}\,\dot{\mathbf{q}} - 2\mathbf{\Phi}_{\mathbf{q}t}\,\dot{\mathbf{q}} - \mathbf{\Phi}_{tt}$$

- NOTE: Getting right-hand side of acceleration equation is tedious

Producing RHS of Acceleration Eq.

[In light of previous example]

- RHS was shown to be computed as

$$\ddot{\mathbf{\Phi}} = \frac{d^2}{dt^2}\mathbf{\Phi}(\mathbf{q}, t) = \mathbf{0} \qquad \Rightarrow \qquad \mathbf{\Phi_q}\ddot{\mathbf{q}} = -(\mathbf{\Phi_q}\dot{\mathbf{q}})_{\mathbf{q}}\,\dot{\mathbf{q}} - 2\mathbf{\Phi}_{\mathbf{q}t}\,\dot{\mathbf{q}} - \mathbf{\Phi}_{tt}$$

- Note that the RHS contains (is made up of) everything that does *not* depend on the generalized accelerations

- Implication:
  - When doing small examples in class, don't bother to compute the RHS using expression above
    - This is done only in ADAMS, when you shoot for a uniform approach to all problems
  - Simply take two time derivatives of your simple constraints and move everything that does *not* depend on acceleration to the RHS

[What comes next:]

Focus on Geometric Constraints

- Learn how to write kinematic constraints that specify that the location and/or attitude of a body wrt the global (or absolute) RF is constrained in a certain way
  - Sometimes called <u>absolute</u> constraints

- Learn how to write kinematic constraints that couple the relative motion of two bodies
  - Sometimes called <u>relative</u> constraints

The Drill…

[related to assignment]

- Step 1: Identify a kinematic constraint (revolute, translational, relative distance, etc., i.e., the *physical* thing) acting between two components of a mechanism
- Step 2: Formulate the algebraic equations that capture that constraint, $\Phi(\mathbf{q})=\mathbf{0}$
  - This is called "modeling"
- Step 3: Compute the Jacobian (or the sensitivity matrix) $\Phi_{\mathbf{q}}$
- Step 4: Compute ν, the right side of the velocity equation
- Step 5: Compute γ, the right side of the acceleration equation (ugly…)

This is what we do almost exclusively in Chapter 3 (about two weeks)

Absolute Constraints

- Called “Absolute” since they express constraint between a body in a system and an absolute (ground) reference frame

- Types of Absolute Constraints
  - Absolute position constraints
  - Absolute orientation constraints
  - Absolute distance constraints

Absolute Constraints (Cntd.)

- Absolute position constraints
  - x-coordinate of P_i

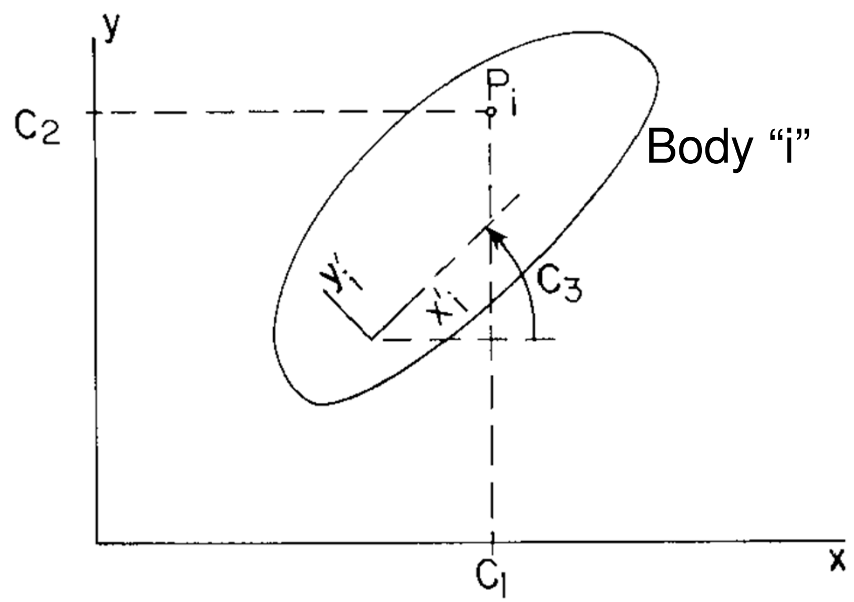
$$x^{P_i} - C_1 = 0$$

  - y-coordinate of P^i

$$y^{P_i} - C_2 = 0$$

- Absolute orientation constraint
  - Orientation ϕ of body

$$\phi_i - C_3 = 0$$



$$\mathbf{r}^{P_i} = \mathbf{r}_i + \mathbf{A}_i \mathbf{s}'^{P_i} \quad \Rightarrow \quad \mathbf{r}^{P_i} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} \cos\phi_i & -\sin\phi_i \\ \sin\phi_i & \cos\phi_i \end{bmatrix} \begin{bmatrix} x'^{P}_i \\ y'^{P}_i \end{bmatrix}$$

Absolute x-constraint

- Step 1: the absolute x component of the location of a point P_i in an absolute (or global) reference frame stays constant, and equal to some known value C_1

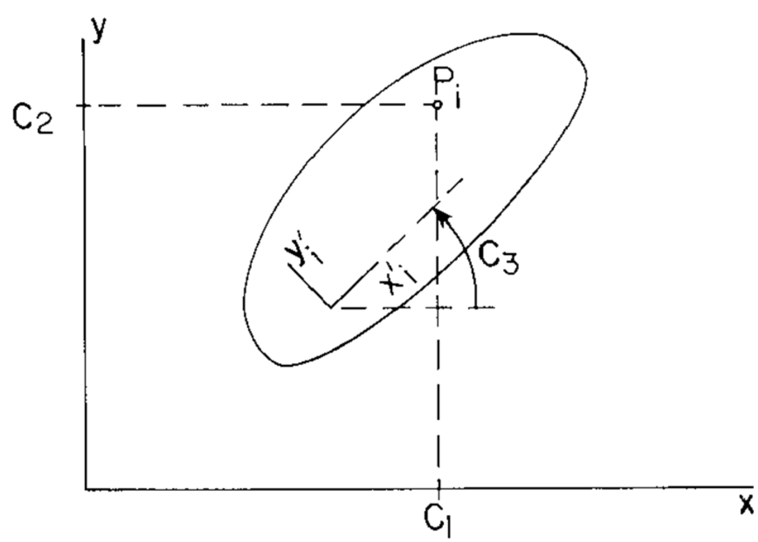


Figure 3.2.2 Constraints on absolute coordinates of point P_i and on angular orientation.

- Step 2: Identify $\Phi^{ax(i)}=0$
- Step 3: $\Phi^{ax(i)}_{\mathbf{q}} = ?$
- Step 4: $\nu^{ax(i)} = ?$
- Step 5: $\gamma^{ax(i)} = ?$

NOTE: The same approach is used to get the y- and angle-constraints

Absolute distance-constraint

- Step 1: the distance from a point P_i to an absolute (or global) reference frame stays constant, and equal to some known value C_4

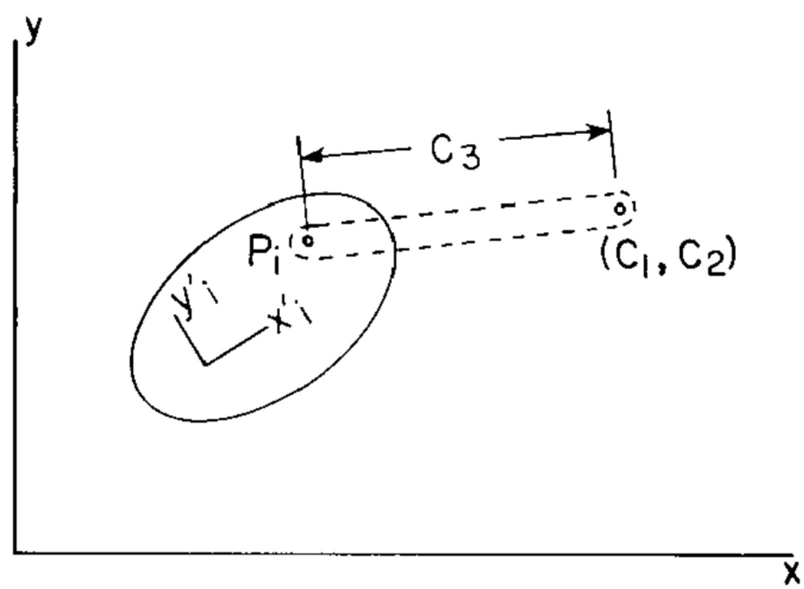


Figure 3.2.1 Constraint that distance from point P to (C_1, C_2) equals C_3.

- Step 2: Identify $\Phi^{dx(i)}=0$
- Step 3: $\Phi^{dx(i)}_{\mathbf{q}} = ?$
- Step 4: $\nu^{dx(i)} = ?$
- Step 5: $\gamma^{dx(i)} = ?$