EXPRESSION OF THE TANGENT

$$r_i^P = r_i + A(\varphi_i)\left[\bar{s}_i^Q + \bar{\rho}_i(\alpha_i^P)\right]$$

Using Figure 3.4.8 :

$$\underbrace{\bar{\rho}_i(\alpha_i^P)}_{\text{vector}} = \underbrace{\rho_i}_{\text{scalar}}(\alpha_i^P) \cdot \begin{bmatrix} c\alpha_i \\ s\alpha_i \end{bmatrix} \equiv \rho_i(\alpha_i^P) \cdot e(\alpha_i)$$

$$e(\alpha) \equiv \begin{bmatrix} c\alpha \\ s\alpha \end{bmatrix}$$

$$\bar{g}_i(\alpha) \equiv \frac{\partial \bar{\rho}(\alpha)}{\partial \alpha} = \rho'(\alpha) \cdot e(\alpha) + \rho(\alpha) \cdot e'(\alpha)$$

$$= \begin{bmatrix} c\alpha & -s\alpha \\ s\alpha & c\alpha \end{bmatrix} \cdot \begin{bmatrix} \rho'(\alpha) \\ \rho(\alpha) \end{bmatrix} \equiv A(\alpha) \cdot \hat{\rho}(\alpha),$$

where $\hat{\rho}(\alpha) \equiv \begin{bmatrix} \rho'(\alpha) \\ \rho(\alpha) \end{bmatrix}$.

Then,

$$\boxed{\bar{g}_i^P = A(\alpha_i^P) \cdot \hat{\rho}_i(\alpha_i^P)}$$