Example 3.5.3

$$q = \begin{bmatrix} \phi_1 \\ x_2 \\ y_2 \\ \phi_2 \end{bmatrix}$$

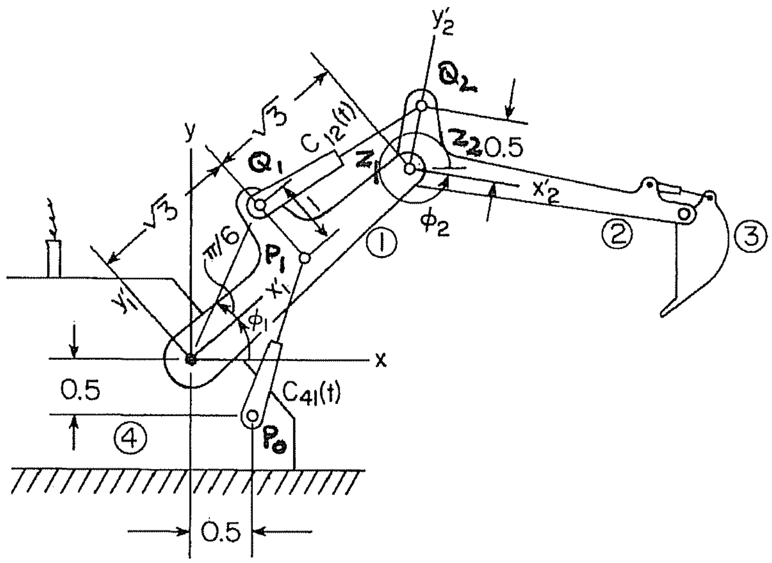

Figure 3.5.6 Excavator boom assembly with two distance drivers.

$$r^{P_0} = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}$$

$$r^{P_1} = A_1 \begin{bmatrix} \sqrt{3} \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{3}\, c\phi_1 \\ \sqrt{3}\, s\phi_1 \end{bmatrix}$$

$$r^{Q_1} = A_1 \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3}\, c\phi_1 & -s\phi_1 \\ \sqrt{3}\, s\phi_1 & +c\phi_1 \end{bmatrix}$$

$$r^{Q_2} = r_2 + A_2 \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} -0.5\, s\phi_2 \\ 0.5\, c\phi_2 \end{bmatrix} = \begin{bmatrix} x_2 - 0.5\, s\phi_2 \\ y_2 + 0.5\, c\phi_2 \end{bmatrix}$$

A set of two constraints coming out of the revolute joint between bodies 1 and 2:

$$r^{Z_1} = r^{Z_2}$$

$$r^{Z_1} = A_1 \begin{bmatrix} 2\sqrt{3} \\ 0 \end{bmatrix} = \begin{bmatrix} 2\sqrt{3}\, c\phi_1 \\ 2\sqrt{3}\, s\phi_1 \end{bmatrix} \qquad r^{Z_2} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

Therefore the constraint assumes the form:

$$r_2^z - r_1^z = \begin{bmatrix} x_2 - 2\sqrt{3}\, c\phi_1 \\ y_2 - 2\sqrt{3}\, s\phi_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

In addition, we have two driving distance constraints.

$$\left(r_1^P - r_0^P\right)^T \left(r_1^P - r_0^P\right) - C_{41}^2(t) = 0$$

$$\left(r_1^Q - r_2^Q\right)^T \left(r_1^Q - r_2^Q\right) - C_{12}^2(t) = 0.$$

We have a set of 4 constraint equations and four unknowns: ϕ_1, x_2, y_2, ϕ_2.

Notation used (u, v introduced to simplify derivation):

$$r_2^Q - r_1^Q = \begin{bmatrix} -x_2 + 0.5\, s\phi_2 + (\sqrt{3}\, c\phi_1 - s\phi_1) \\ -y_2 - 0.5\, c\phi_2 + (\sqrt{3}\, s\phi_1 + c\phi_1) \end{bmatrix} \equiv \begin{bmatrix} u \\ v \end{bmatrix}.$$

Then the constraints are expressed as:

$$\Phi(\phi_1, x_2, y_2, \phi_2, t) = \begin{bmatrix} x_2 - 2\sqrt{3}\, c\phi_1 \\ y_2 - 2\sqrt{3}\, s\phi_1 \\ (\sqrt{3}\, c\phi_1 - 0.5)^2 + (\sqrt{3}\, s\phi_1 + 0.5)^2 - C_{41}^2(t) \\ u^2 + v^2 - C_{12}^2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Jacobian of Φ is provided on next page. Also note that u and v can be expressed as:

$u = 2\cos(\phi_1 + \frac{\pi}{6}) - x_2 + 0.5 \cdot s\phi_2$ & $v = 2\sin(\phi_1 + \frac{\pi}{6}) - y_2 - 0.5\, c\phi_2$.

We'll use the chain rule for differentiation. To that end we need:

$$\frac{\partial u}{\partial \phi_1} = -2\, s(\phi_1 + \tfrac{\pi}{6}) \qquad \frac{\partial u}{\partial x_2} = -1 \qquad \frac{\partial u}{\partial y_2} = 0 \qquad \frac{\partial u}{\partial \phi_2} = 0.5\, c\phi_2$$

$$\frac{\partial v}{\partial \phi_1} = 2\, c(\phi_1 + \tfrac{\pi}{6}) \qquad \frac{\partial v}{\partial x_2} = 0 \qquad \frac{\partial v}{\partial y_2} = -1 \qquad \frac{\partial v}{\partial \phi_2} = 0.5\, s\phi_2$$

Then

$$\Phi_q = \begin{array}{c} \begin{array}{cccc} \text{col } \phi_1 & \text{col } x_2 & \text{col } y_2 & \text{col } \phi_2 \end{array} \\ \left[\begin{array}{c|c|c|c} 2\sqrt{3}\, s\phi_1 & 1 & 0 & 0 \\ -2\sqrt{3}\, c\phi_1 & 0 & 1 & 0 \\ 2(\sqrt{3}\, c\phi_1 - 0.5)(-\sqrt{3}\, s\phi_1) + 2(\sqrt{3}\, s\phi_1 + 0.5)\cdot\sqrt{3}\, c\phi_1 & 0 & 0 & 0 \\ 2u\left[-2\cdot s(\phi_1 + \tfrac{\pi}{6})\right] + 2v\left[2c(\phi_1 + \tfrac{\pi}{6})\right] & -2u & -2v & 2u\cdot 0.5\cdot c\phi_2 + 2v\cdot 0.5\cdot s\phi_2 \end{array}\right] \end{array}$$

Computation of ν:

$$\frac{\partial \Phi}{\partial t} = -\begin{bmatrix} 0 \\ 0 \\ -2\, c_{41}\cdot \frac{\partial c_{41}}{\partial t} \\ -2\, c_{12}\cdot \frac{\partial c_{12}}{\partial t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2(\frac{1}{5}t + 1.8)\cdot \frac{1}{5} \\ 2(\frac{1}{10}t + 1.9)\cdot\frac{1}{10} \end{bmatrix}$$

At each instance in time, t_k, one solves

$$\Phi(\phi_1, x_2, y_2, \phi_2, t_k) = 0$$

to get $(\phi_1, x_2, y_2, \phi_2)_k$, the unknowns at t_k.

Then solves $\Phi_q \cdot \dot{q} = \nu_k$, to get $\dot{q}$ at time t_k.

I didn't compute γ, the RHS of the acceleration equation since it's too messy. Had we had that, we would have solved

$$\Phi_q \cdot \ddot{q} = \gamma_k$$

to get the acceleration $\ddot{q} = \begin{bmatrix} \ddot{\phi}_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \\ \ddot{\phi}_2 \end{bmatrix}$ at time t_k.