PROOF :

$$\boxed{(\vec{a}+\vec{b})\,\vec{c} = \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}}\quad , \text{ for } \vec{c} \neq \underset{\uparrow \text{ zero vector}}{\vec{0}}$$

First note that the dot products above lead to a scalar value. If $\vec{c}$ is not a unit vector, then

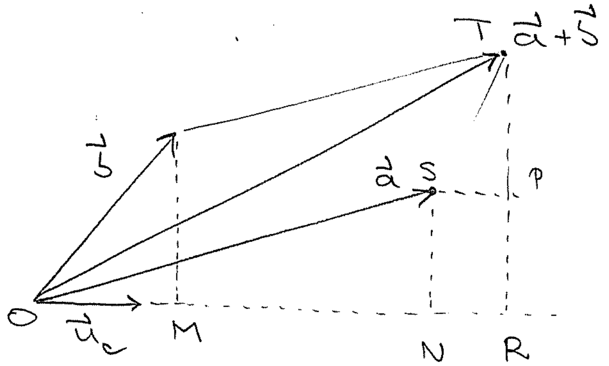
$$\vec{c} = \|\vec{c}\|\cdot \vec{u}_c ,$$

where $\vec{u}_c$ is a unit vector oriented like $\vec{c}$.

Then, we need to prove that

$$(\vec{a}+\vec{b})\,\|\vec{c}\|\cdot \vec{u}_c = \vec{a}\cdot\|\vec{c}\|\,\vec{u}_c + \vec{b}\cdot\|\vec{c}\|\cdot\vec{u}_c$$

$$\Leftrightarrow \quad (\vec{a}+\vec{b})\cdot\vec{u}_c = \vec{a}\,\vec{u}_c + \vec{b}\,\vec{u}_c \quad , \text{ with } \|\vec{u}_c\| = 1$$



$\vec{a}\,\vec{u}_c = ON$

$\vec{b}\cdot\vec{u}_c = OM = SP = NR$

Then,

$\vec{a}\,\vec{u}_c + \vec{b}\,\vec{u}_c = ON + NR = OR.$

At the same time,

$(\vec{a}+\vec{b})\,\vec{u}_c = OR,$

from where we have that

$$(\vec{a}+\vec{b})\,\vec{u}_c = \vec{a}\,\vec{u}_c + \vec{b}\cdot\vec{u}_c$$