Example

Solve IVP using Backward Euler

$$\begin{cases} \dot{y} = -y^2 \\ y(0) = 2.4 \end{cases} \qquad h = \Delta t = 0.1$$

$$y_1 = y_0 + h\dot{y}_1 = y_0 + h(-y_1^2)$$

$$\Rightarrow \quad h y_1^2 + y_1 - y_0 = 0 \qquad (*)$$

You have to solve the above equation for y_1. This is a nonlinear equation, has actually two solutions:

$$0.1 \cdot y_1^2 + y_1 - 2.4 = 0.$$

$$y_1 = \frac{-1 \pm \sqrt{1 + 0.96}}{0.2} = \begin{cases} \dfrac{-1+1.4}{0.2} \Rightarrow y_1 = 2 \\ \dfrac{-1-1.4}{0.2} \Rightarrow y_1 = -12 \end{cases}$$

Which is the correct solution?

For this problem it is easy to see that as $h \to 0$, the correct solution is the one closer to $y_0 = 2$. To understand why, look at Eq. (*), take $h \to 0 \Rightarrow y_1 \to y_0$.

In general it might not be obvious which solution is the good one. In general, when you use Newton-Raphson to solve the algebraic problem if you start with a

wrong initial guess you will get the wrong solution. However, if h is very small and your guess for the Newton method is the value of the solution at the previous time step, then it is very likely you will get the correct solution.