

Example

1/1

Solve IVP using Backward Euler

$$\begin{cases} \dot{y} = \sin(y) \\ y(0) = 0 \end{cases} \quad \Delta t = h = 0.1$$

$$y_1 = y_0 + h \dot{y}_1 = y_0 + h \sin(y_1) = 0 + 0.1 \cdot \sin(y_1)$$

Then, the solution of the IVP at time $t_1 = 0.1$ is the solution of the following nonlinear algebraic problem:

$$y_1 - 0.1 \sin(y_1) = 0$$

This problem is solved using Newton-Raphson:

$$\Psi(y_1) = y_1 - 0.1 \sin(y_1)$$

$$y_1^{(k+1)} = y_1^{(k)} - \left[\frac{\Psi(y_1^{(k)})}{y_1} \right]^{-1} \cdot \Psi(y_1^{(k)})$$

Conclusion:

Whenever $f(t, y)$ assume a complicated form, you will have to use Newton-Raphson to produce the solution of the algebraic problem

