

EXAMPLE

| 01/1 |

Backward Euler (BE) Implicit Integration

Use BE with $\Delta t = 0.1$ to find y_1 & y_2 for the following IVP:

$$\begin{cases} \dot{y} = -0.1y + \sin t \\ y(0) = 0 \end{cases} \Rightarrow f(y, t) = -0.1y + \sin t$$

BE: $y_1 = y_0 + \Delta t \cdot f(y_1, t_1)$

Then $y_1 = 0 + \Delta t \cdot (-0.1y_1 + \sin t_1)$

$$\Rightarrow y_1(1 + 0.1 \cdot \Delta t) = \sin t_1 \cdot \Delta t \Rightarrow y_1 = \frac{\sin(0.1) \cdot 0.1}{1 + (0.1)^2} = 9.88 \cdot 10^{-3}$$

Take now another integration time step:

BE: $y_2 = y_1 + \Delta t \cdot f(y_2, t_2)$

Then $y_2 = 9.88 \cdot 10^{-3} + 0.1(-0.1 \cdot y_2 + \sin(0.2))$

$$\Rightarrow y_2(1 + (0.1)^2) = 9.88 \cdot 10^{-3} + 0.1 \cdot \sin(0.2)$$

$$\Rightarrow y_2 = \frac{9.88 \cdot 10^{-3} + 0.1 \cdot \sin(0.2)}{1 + (0.1)^2} = 29.46 \cdot 10^{-3}$$

