MATLAB Assignment

Due Date: December 07, 2010 @ 11:59 PM

December 7, 2010

<u>How to turn in your homework</u>: Place all your files in a directory called "lastName-Date"; zip that directory and email to TA.

Problem 1. Use Forward Euler to solve the Initial Value Problem below. Use an integration step-size of $h = 0.01$ and generate a numerical solution for the time interval $t \in [0, 10]$.

$$\begin{cases} \dot{y} &= \sin(y) \\ y(0) &= 1.0 \end{cases}$$

- Generate a plot (png or jpg) that displays the solution as a function of time for the given interval.
- Solve the problem using MATLAB's `ODE45` function and compare the MATLAB solution with your solution. Specifically, generate a second png or jpg that plots the difference between the two solutions $y_{you} - y_{MATLAB}$ as a function of time.

Problem 2. Use Backward Euler to solve the Initial Value Problem above. Provide a png or jpg plot of the solution, and the difference between Forward Euler (FE) and Backward Euler (BE) solutions $y_{FE} - y_{BE}$ as a function of time. Use the same integration step size as in Problem 1.

Problem 3. The goal of this problem is to perform an accuracy order analysis. Basically, what you want to do is to verify that the order of Forward Euler is 1 and that the order of the Runge-Kutta method provided below is 4. In this exercise use the following IVP:

$$\begin{cases} \dot{y} &= -0.1y\sin(y) \\ y(0) &= 0.1 \end{cases}$$

For the Forward Euler, start with a step size $h = 0.1$, then use $h = 0.01$, $h = 0.001$, and $h = 0.0001$. The time interval is $t \in [0, 1.5]$. You will have to generate a convergence plot, which is a log-log plot that on the x-axis plots the step size value and on the y-axis plots the numerical error. In the convergence analysis for RK4, since this is a more accurate formula, you can use larger step-sizes, so please use $h = 0.25$, $h = 0.3$, $h = 0.5$, and $h = 0.75$.

How do you obtain the numerical error? In theory, you should subtract $y_{exact}(1.5) - y_{approx}(1.5)$ to get the error at the end of the interval. However, you don't have $y_{exact}(t)$. To get a good value for this number you will have to use Runge-Kutta with a tiny step-size so that you get something that comes close to $y_{exact}(1.5)$. This solution obtained with the tiny step-size will be your "reference" solution, called $\hat{y}_{exact}(1.5)$. Then, when you use, for instance, Forward Euler with $h = 0.01$ you will plot this pair of points $(0.01$ and $y_{FE0.01} - \hat{y}_{exact}(1.5))$.

Recall that you should use a log-log plot. If everything works well, you should see a straight line with slope 1.0 for Forward Euler, and slope 4.0 for Runge-Kutta. These are the orders of the two methods: 1 and 4, respectively.

Finally, the Runge-Kutta RK4 approximation y_{n+1} of the actual value $y(t_{n+1})$ is computed as:

$$
\begin{aligned}
y_{n+1} &= y_n + \tfrac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) \\
t_{n+1} &= t_n + h
\end{aligned}
$$

where h is the integration step-size and k_1 through k_4 are obtained as

$$
\begin{aligned}
k_1 &= f(t_n, y_n) \\
k_2 &= f(t_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}hk_1) \\
k_3 &= f(t_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}hk_2) \\
k_4 &= f(t_n + h, y_n + hk_3)
\end{aligned}
$$

Problem 4. Use the second order BDF method provided in class to solve the Initial Value Problem in Problem 1. Provide a png or jpg plot of the solution. Use the same integration step size as in Problem 1.

Note: you will need to carry along the numerical approximations y_{n-1} and y_n in order to compute the numerical solution y_{n+1} at time t_{n+1}. Therefore, you can only apply the BDF method to get y_2, y_3, y_4, etc. This being the case, use the RK4 to get y_1, and then start using BDF for the rest of the problem. This is one small drawback of BDF, it needs to be primed to start.