ME451
Kinematics and Dynamics of Machine Systems

Singular Configurations of Mechanisms 3.7
October 26, 2010

Before we get started…

- Last Time
  - Discussed the three stages of the Kinematics Analysis:
    - Position Analysis
    - Velocity Analysis
    - Acceleration Analysis
  - Mentioned why the Implicit Function Theorem is your friend

- Today:
  - Cover an example: wrecker-boom
  - Start discussion on "Singular Configurations of Mechanisms" (Section 3.7)

- HW due next Tu (Nov. 2): 3.5.1, 3.5.4, 3.5.5, 3.5.6, ADAMS, MATLAB
  - 3.5.5: note that the angle ϕ_2 is not displayed correctly
  - 3.5.6: get rid of $\mathbf{v}_i$, take it unit vector
  - ADAMS problem: emailed by TA and due on November 2
  - MATLAB: due on November 9 , pdf posted online

- Quick Remarks:
  - Exam on Nov. 2 & Exam Review on Nov 1, 6-8PM, 1153ME
  - Forum to become operational before the end of the week
    - Post simEngine2D questions there, I'll answer your questions therein
    - When you have a problem, check the forum first. If no discussion on your topic, post it

Strategies for Kinematics Analysis

- You can embrace one of two strategies to carry out Kinematics Analysis
  - They are different based on the number of generalized coordinates used to carry out the analysis

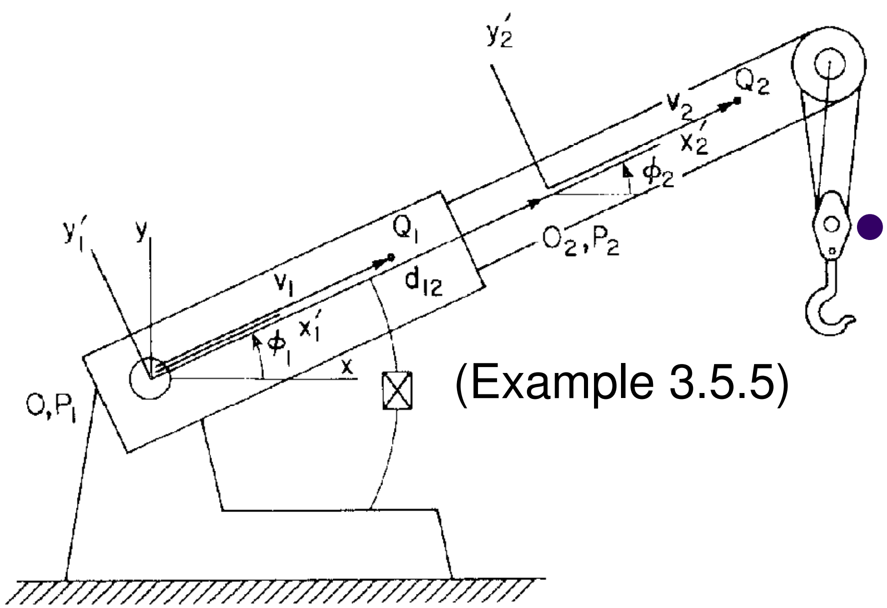
- Strategy 1: use a reduced set of generalized coordinates

- Strategy 2: use Cartesian generalized coordinates

The Reduced Set Strategy

- **<u>Reduced set</u>** of generalized coordinates:

$$\mathbf{q} = [\phi_1, x_2, y_2, \phi_2]^T$$



(Example 3.5.5)

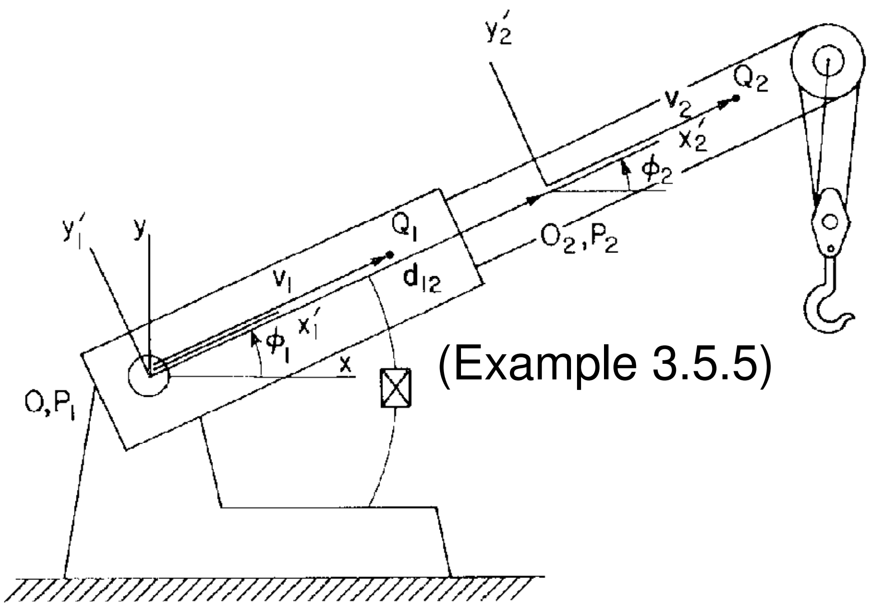
Figure 3.5.10 Wrecker boom with a translational-distance driver.

- Advantages
  - Few generalized coordinates lead to few kinematic constraints (that is, number of equations in $\mathbf{\Phi}(\mathbf{q},t)=\mathbf{0}$ is small)
  - For small mechanisms (3-4 bodies), easy to solve with pencil and paper
- Disadvantages
  - This strategy is not general (systematic), but rather is applied on a case by case situation
    - You are back to a ME240 situation, where each problem comes with its own solution
  - Not trivial to use for large systems, especially when you are dealing with 3D mechanisms

The Cartesian Set Strategy

- **Cartesian** generalized coordinates:

$$\mathbf{q} = [x_1, y_1, \phi_1, x_2, y_2, \phi_2]^T$$



(Example 3.5.5)

Figure 3.5.10 Wrecker boom with a translational-distance driver.

- Advantages
  - This strategy is general (systematic), it always works (used in ADAMS as well)
  - It is rather automatic, little thinking involved, simply following a recipe
  - It relies on a very limited number of building blocks (provided in book) to implement a systematic approach that allows for analysis of **any** mechanism no matter how large it is

- Disadvantages
  - Larger set of generalized coordinates leads to larger number of kinematic constraints (that is, number of equations in $\Phi(\mathbf{q},t)=\mathbf{0}$ is large)
  - For simple systems might be an overkill (the mosquito and the cannonball)

The Reduced Set Strategy: Example

- For the wrecker boom mechanism, use a reduced set of generalized coordinates and carry out the steps required by Kinematic Analysis

$$\mathbf{q} = [\phi_1, x_2, y_2, \phi_2]^T$$

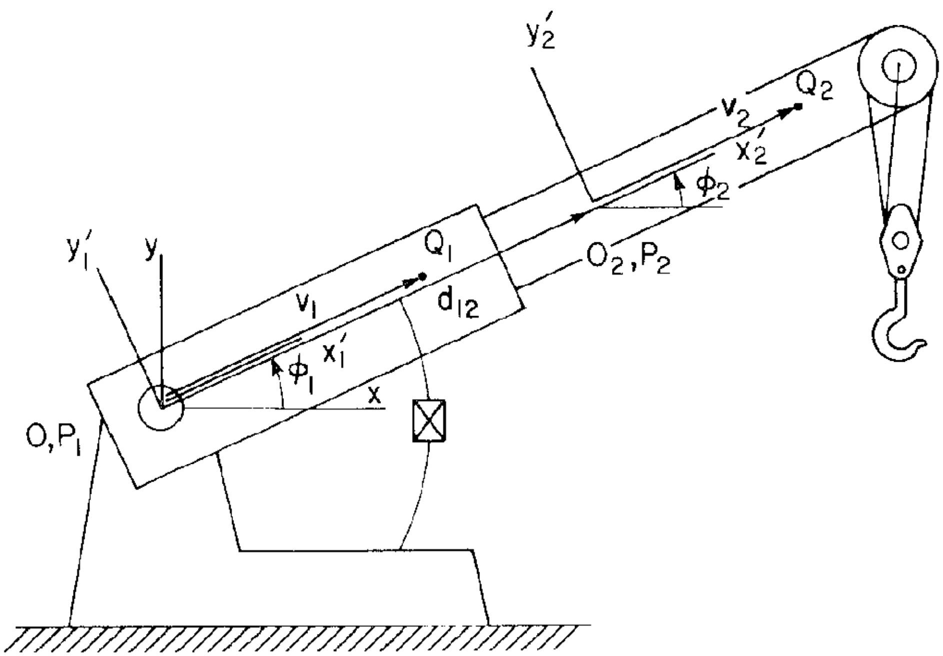


Figure 3.5.10 Wrecker boom with a translational-distance driver.

- Two motions prescribed to control the motion of the boom:

$$\mathbf{v}_i^T \mathbf{d}_{ij} - 0.1t = 0$$

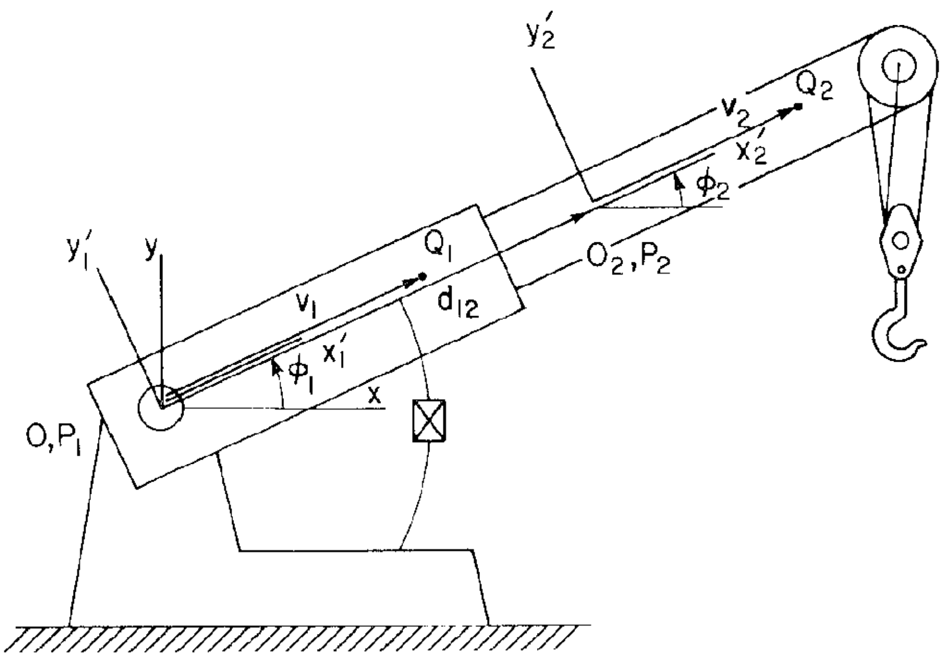
$$\phi_1 - 0.025t = 0$$

Example
The Reduced Set Strategy

The Cartesian Set Strategy: Example

- For the wrecker boom mechanism, use Cartesian coordinates and carry out the steps required by Kinematic Analysis

$$\mathbf{q} = [x_1, y_1, \phi_1, x_2, y_2, \phi_2]^T$$



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$$\mathbf{v}_i^T \mathbf{d}_{ij} - 0.1t = 0$$

$$\phi_1 - 0.025t = 0$$

Example

The Cartesian Set Strategy

simEngine2D

- Important note: use the wrecker-boom example discussed when you debug your simEngine2D

- simEngine2D requires you to think about how to stack the constraints together and how to use the MATLAB functions you have defined in your HW to assemble in matrix-vector form the following *four* quantities:

$$1: \ \mathbf{\Phi}(\mathbf{q}, t) \qquad 2: \ \mathbf{\Phi_q} \qquad 3: \ \nu \qquad 4: \ \gamma$$

- How is simEngine2D going to work?
  - It parses an input file for a model description (the topic of today's MATLAB assignment), generates the four quantities above, solves the required equations, and finally generates plots and an animation that describe the motion of the mechanism

END: Example

BEGIN: Singular Configurations of Mechanisms
Section 3.7

Singular Configurations

- What are “singular configurations”?
  - Abnormal situations that should be avoided since they indicate either a malfunction of the mechanism (poor design), or a bad model associated with an otherwise well designed mechanism
  - Singular configurations come in two flavors:
    - Physical Singularities (PS): reflect bad design decisions
    - Modeling Singularities (MS): reflect bad modeling decisions
  - Singular configurations do not represent the norm, but you must be aware of their existence
    - A PS is particularly bad and can lead to dangerous situations

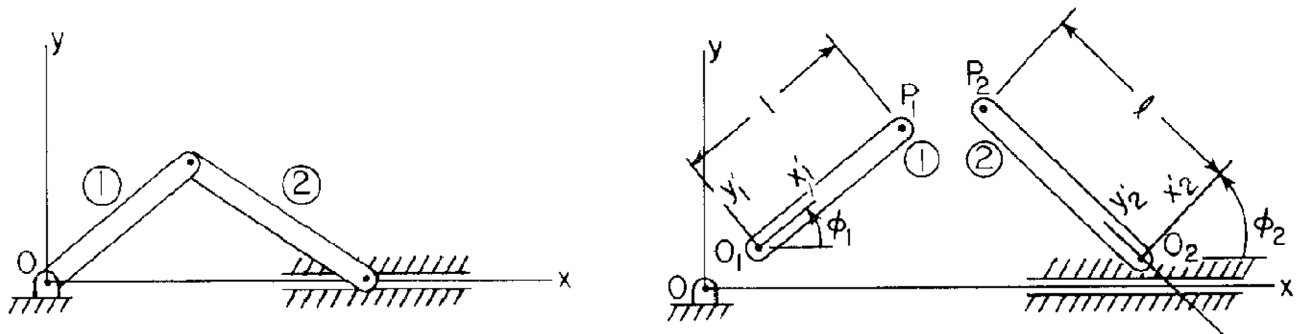
Singular Configurations

- In a *singular configuration*, one of three things can happen:
  - PS1: Your mechanism locks-up
  - PS2: Your mechanism hits a bifurcation
  - MS1: Your mechanism has redundant constraints
- The important question:
  - How can we characterize a singular configuration in a formal way such that we are able to diagnose it?
  - Look at two examples next to see what happens in a singular configuration

Mechanism Lock-Up: PS1 (Example 3.7.5, draws on 3.1.2)

- Investigate what happens to this mechanism when length $l = 0.5$

$$\mathbf{q} = \begin{bmatrix} x_2 \\ \phi_1 \\ \phi_2 \end{bmatrix}$$



(a) (b)

$$\mathbf{\Phi}(\mathbf{q}, t) = \begin{bmatrix} -x_2 + \cos\phi_1 + l\sin\phi_2 \\ \sin\phi_1 - l\cos\phi_2 \\ \phi_1 - \frac{\pi}{12}t \end{bmatrix} = \mathbf{0}$$

$$\mathbf{\Phi_q}(\mathbf{q}) = \begin{bmatrix} -1 & -\sin\phi_1 & l\cos\phi_2 \\ 0 & \cos\phi_1 & l\sin\phi_2 \\ 0 & 1 & 0 \end{bmatrix}$$

- Can you ever get in trouble?
- Yes, check what happens when t=2
  - Mechanism hits a lock-up configuration
  - When t=2: $x_2 = \frac{\sqrt{3}}{2}$

    $\phi_1 = \frac{\pi}{6}$

    $\phi_2 = 0$

Mechanism Lock-Up

- Definition of **lock-up** configuration:
  - The mechanism cannot proceed anymore
  - Symptoms of “lock-up”:
    - Jacobian in that configuration is singular
    - The rank of the *velocity augmented constraint Jacobian* is higher than the rank of the constraint Jacobian

Velocity augmented constraint Jacobian

$$\widehat{\mathbf{J}}_{vel} = [\mathbf{\Phi_q} \quad \nu] \qquad rank(\widehat{\mathbf{J}}_{vel}) > rank(\mathbf{\Phi_q})$$

- The velocities and accelerations assume huge values (in fact, going to infinity)
  - That is, you’re sure not to miss it…

Mechanism Lock-Up (Cntd.)

- Investigate rank of augmented Jacobian

$$rank(\widehat{\mathbf{J}}_{vel}) = 3 \quad > \quad 2 = rank(\mathbf{\Phi_q})$$

- Carry out velocity analysis

time = 1.85	vel =	[-0.71392649808689 0.26179938779915 -1.27150008402231]
time = 1.90	vel =	[-0.85975114686538 0.26179938779915 -1.54001421905491]
time = 1.95	vel =	[-1.18022664998825 0.26179938779915 -2.15362292657357]
time = 2.00	vel =	1.0e+007*[-1.52152519881098 0.00000002617994 -3.04305037144201]

Mechanism moves faster than speed of light…

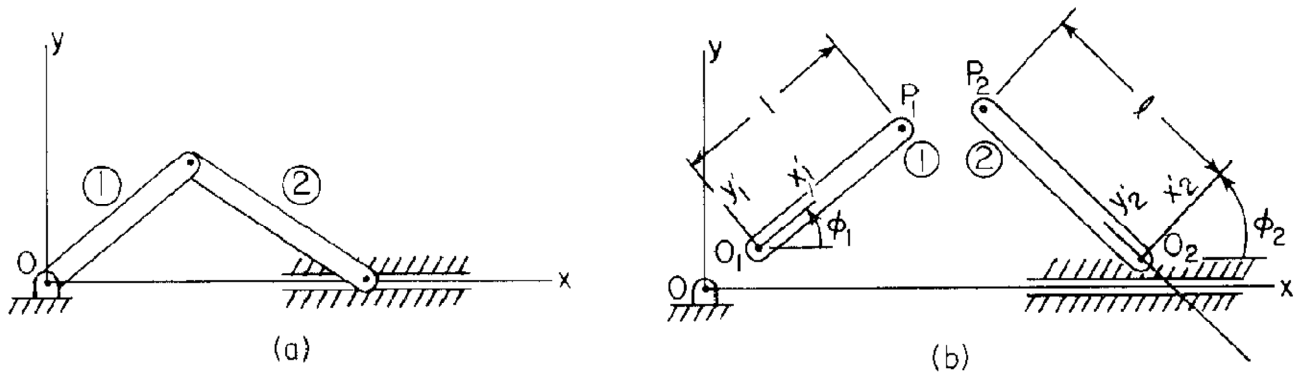
- Carry out acceleration analysis

time = 1.80	acc =	[-1.47292585680960	0	-2.53780315286818]
time = 1.85	acc =	[-2.19722185658353	0	-3.95600397951865]
time = 1.90	acc =	[-3.92446925376964	0	-7.35587287703508]
time = 1.95	acc =	[-10.83795211380501	0	-21.05152842858363]
time = 2.00	acc = 1.0e+022*[	-3.10719260152581	0	-6.21438520305161]

Bifurcation: PS2
(Example 3.7.5, draws on 3.1.2)

- Investigate what happens to this mechanism when length $l = 1$

$$\mathbf{q} = \begin{bmatrix} x_2 \\ \phi_1 \\ \phi_2 \end{bmatrix}$$



$$\mathbf{\Phi}(\mathbf{q}, t) = \begin{bmatrix} -x_2 + \cos\phi_1 + l\sin\phi_2 \\ \sin\phi_1 - l\cos\phi_2 \\ \phi_1 - \frac{\pi}{12}t \end{bmatrix} = \mathbf{0}$$

$$\mathbf{\Phi_q}(\mathbf{q}) = \begin{bmatrix} -1 & -\sin\phi_1 & l\cos\phi_2 \\ 0 & \cos\phi_1 & l\sin\phi_2 \\ 0 & 1 & 0 \end{bmatrix}$$

- Can you ever get in trouble?
- Yes, check what happens when t=6
  - Mechanism hits a bifurcation
  - When t=6: $x_2 = 0$

    $\phi_1 = \frac{\pi}{2}$

    $\phi_2 = 0$

Bifurcation (Cntd.)

- Definition of **bifurcation** configuration:
  - The mechanism can proceed in more than one way
  - Symptoms of “bifurcation”:
    - Jacobian in that configuration is singular
    - The rank of the *velocity and acceleration augmented constraint Jacobians* is equal to the rank of the constraint Jacobian

Acceleration augmented constraint Jacobian

$$\widehat{\mathbf{J}}_{acc} = [\mathbf{\Phi_q} \quad \gamma] \qquad rank(\widehat{\mathbf{J}}_{acc}) = rank(\widehat{\mathbf{J}}_{vel}) = rank(\mathbf{\Phi_q})$$

    - The velocities and accelerations <u>do not</u> assume huge values
      - That’s why it’s tough to spot a bifurcation (unlike a lock-up), often times you cruise through it without knowing it…

Bifurcation, Scenario 1: Time Step is 0.06 [s]

- Investigate rank of augmented Jacobians

$$rank(\widehat{\mathbf{J}}_{acc}) = rank(\widehat{\mathbf{J}}_{vel}) = rank(\mathbf{\Phi_q}) = 2$$

Bifurcation Time: T=6

- Carry out velocity analysis

time = 5.80	vel = [-0.52288120167379 0.26179938779915 -0.26179938779915]	
time = 5.86	vel = [-0.52324712340312 0.26179938779915 -0.26179938779915]	
time = 5.92	vel = [-0.52348394173427 0.26179938779915 -0.26179938779916]	
time = 5.98	vel = [-0.52359159823540 0.26179938779915 -0.26179938779871]	NOTE: Stepped over bifurcation configuration and hardly noticed
time = 6.04	vel = [0.00000000000002 0.26179938779915 0.26179938779917]	
time = 6.10	vel = [-0.00000000000001 0.26179938779915 0.26179938779914]	
time = 6.16	vel = [0.00000000000000 0.26179938779915 0.26179938779915]	

- Carry out acceleration analysis

time = 5.80	acc = [-0.00717409977873 0 0.00000000000003]	
time = 5.86	acc = [-0.00502304039889 0 0.00000000000003]	
time = 5.92	acc = [-0.00287074165928 0 -0.00000000000025]	
time = 5.98	acc = 1.0e-003 *[-0.71773456266700 0 0.00000004392366]	Stepping over bifurcation...
time = 6.04	acc = 1.0e-012 *[-0.99659190644620 0 -0.99659201484942]	
time = 6.10	acc = 1.0e-012 *[0.22531501249805 0 0.22531502713580]	
time = 6.16	acc = 1.0e-013 *[-0.43745210091874 0 -0.43745418339683]	

Bifurcation, Scenario 2: Time Step is 0.05 [s]

- Carry out velocity analysis

time = 5.85 vel = [-0.52319509991791 0.26179938779915 -0.26179938779915]
time = 5.90 vel = [-0.52341935137507 0.26179938779915 -0.26179938779914]
time = 5.95 vel = [-0.52355391762090 0.26179938779915 -0.26179938779910]
time = 6.00 vel = [NaN NaN -Inf]
Warning: Matrix is singular to working precision.
> In function bifurcation at line 14
time = 6.05 vel = [-0.00000000000005 0.26179938779915 0.26179938779910]
time = 6.10 vel = [-0.00000000000001 0.26179938779915 0.26179938779914]
time = 6.15 vel = [-0.00000000000001 0.26179938779915 0.26179938779914]

Bifurcation Time: T=6

- Carry out acceleration analysis

time = 5.85 acc = [-0.00538165069997 0 0.00000000000005]
time = 5.90 acc = [-0.00358827950303 0 0.00000000000011]
time = 5.95 acc = [-0.00179429347120 0 0.00000000000185]
time = 6.00 acc = [NaN NaN NaN]
Warning: Matrix is singular to working precision.
> In function bifurcation at line 19
time = 6.05000000000000 acc = 1.0e-011 *[0.21214905163374 0 0.21214905572961]
time = 6.10000000000000 acc = 1.0e-012 *[0.22531501249805 0 0.22531502713580]
time = 6.15000000000000 acc = 1.0e-012 *[0.10145027567015 0 0.10145042771686]
time = 6.20000000000000 acc = 1.0e-013 *[0.49056387941139 0 0.49055963218247]

NOTE: On previous slide we were “lucky”. Here, by chance, we chose a step-size that happen to hit the bifurcation

Singular Configurations

- In the end, what is the pattern that emerges?

- The important remark:
  - The only case when you run into problems is when the constraint Jacobian becomes singular:

$$\det\left|\mathbf{\Phi}_{\mathbf{q}}(\mathbf{q}^{\star}, t^{\star})\right| = 0$$

- Otherwise, the Implicit Function Theorem (IFT) gives you the answer:
  - If the constraint Jacobian is nonsingular, IFT says that you cannot be in a singular configuration. And that's that.

Singularities: Closing Remarks

- Remember that you seldom see singularities

- To summarize, if the constraint Jacobian is singular,
  - You can be in a lock-up configuration (you won't miss this, PS1)
  - You might face a bifurcation situation (very hard to spot, PS2)
  - You might have redundant constraints (we didn't say anything about this, MS1)

- Singularity analysis is a tough topic. Textbook gives a broader perspective, although not necessarily deeper