ME451
Kinematics and Dynamics of Machine Systems

Start Position, Velocity, and Acc. Analysis 3.6
October 21, 2010

Before we get started…

- Last Time
  - Discussed driving constraints
    - A set of *ndof* independent drivers specified to “occupy” all the degrees of freedom
    - Driver constraints
      - Absolute: x, y, ϕ
      - Relative: distance, motion on a revolute joint, motion on a translational joint

- Today:
  - Excavator example, setting up driving constraints
  - Kinematic Analysis Wrap-up: Position, Velocity, and Acceleration Stages
  - Start wrecker-boom example

- Final Project proposal due in one week

- Midterm exam coming up on November 2
  - Review for exam: Monday, Nov 1, starting at 6 PM (precise room/time TBA)

Example:
Specifying Relative Distance Drivers

- Generalized coordinates: $\mathbf{q} = [\phi_1, x_2, y_2, \phi_2]^T$

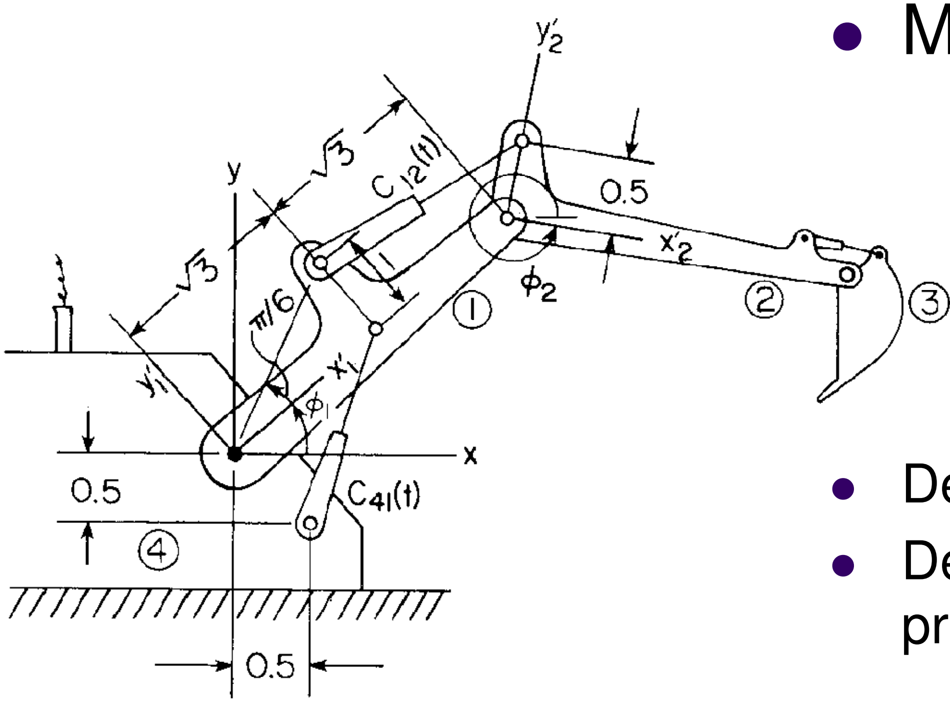

Figure 3.5.6 Excavator boom assembly with two distance drivers.

- Motions prescribed:

$$C_{41}(t) = \frac{1}{5}t + 1.8$$

$$C_{12}(t) = \frac{1}{10}t + 1.9$$

- Derive the constraints acting on system
- Derive linear system whose solution provides velocities $\dot{\mathbf{q}} = [\dot{\phi}_1, \dot{x}_2, \dot{y}_2, \dot{\phi}_2]^T$

Mechanism Analysis: Steps

- **Step A**: Identify *all* physical joints and drivers present in the system
- **Step B**: Identify the corresponding constraint equations $\Phi(\mathbf{q},t)$
- **Step C**: Solve for the Position as a function of time ($\Phi_{\mathbf{q}}$ is needed)
- **Step D**: Solve for the Velocities as a function of time (ν is needed)
- **Step E**: Solve for the Accelerations as a function of time (γ is needed)

Position, Velocity, and Acceleration Analysis (Section 3.6)

- The position analysis [Step C]:
  - It's the tougher of the three
  - Requires the solution of a system of nonlinear equations
  - What you are after is determining at each time the location and orientation of each component (body) of the mechanism

- The velocity analysis [Step D]:
  - Requires the solution of a linear system of equations
  - Relatively simple
  - Carried out <u>after</u> you are finished with the position analysis

- The acceleration analysis [Step E]:
  - Requires the solution of a linear system of equations
  - What is challenging is generating the RHS of acceleration equation, γ
  - Carried out <u>after</u> you are finished with the velocity analysis

Position Analysis

- Framework:
  - Somebody presents you with a mechanism and you select the set of *nc* generalized coordinates to position and orient each body of the mechanism:

$$\mathbf{q} = [x_1, y_1, \phi_1, x_2, y_2, \phi_2, \ldots]^T \in \mathbb{R}^{nc}$$

  - You inspect the mechanism and identify a set of *nk* kinematic constraints that must be satisfied by your coordinates:

$$\Phi^K(\mathbf{q}) = \mathbf{0}$$

  - Next, you identify the set of *nd* driver constraints that move the mechanism:

$$\Phi^D(\mathbf{q}, \mathbf{t}) = \mathbf{0}$$

NOTE: YOU MUST HAVE *nc* = *nk* + *nd*

Position Analysis

- We end up with this problem: given a time *t*, find that set of generalized coordinates **q** that satisfy the equations:

$$\boldsymbol{\Phi}(\mathbf{q},t) = \begin{bmatrix} \boldsymbol{\Phi}^K(\mathbf{q}) \\ \boldsymbol{\Phi}^D(\mathbf{q},t) \end{bmatrix} = \mathbf{0}$$

- What's the idea here?
  - Set time t=0, and find a solution **q** by solving above equations
  - Next, set time t=0.001 and find a solution **q** by solving above equations
  - Next, set time t=0.002 and find a solution **q** by solving above equations
  - Next, set time t=0.003 and find a solution **q** by solving above equations
  - …
  - Stop when you reach the end of the interval in which you are interested in the position

- What you do is find the time evolution on a **<u>time grid</u>** with step size Δt=0.001
  - You can then plot the solution as a function of time and get the time evolution of your mechanism

Position Analysis

- Two issues with the described methodology for finding the time evolution of the mechanism
  - The equations that we have to solve at each time *t* are nonlinear, so a first hurdle is being able to solve this nonlinear system
    - Deal with this issue later (next week, Newton-Raphson method)

  - The second issue comes when you start thinking about the solution that you've just got using a numerical algorithm
    - How do you know that what you got is a meaningful thing?
      - Remember, a nonlinear system can have an arbitrary number of solutions
    - Deal with this issue now

Position Analysis: Implicit Function Theorem

- Is the solution of our nonlinear system well behaved? A sufficient condition is provided by the **Implicit Function Theorem**

- In layman's words, this is what the theorem says:
  - Let's say that we are at some time t_k, and we just found the solution q_k and we question the quality of this solution
  - If the constraint Jacobian is nonsingular in this configuration, that is,

$$\det |\mathbf{\Phi_q}(\mathbf{q}_k, t_k)| \neq 0$$

  - … then, we can conclude that the solution is unique, and not only at t_k, but in a small interval δ about time t_k.
  - Additionally, in this small time interval, there is an explicit functional dependency of **q** on t, that is, there is a function $f(t)$ such that:

$$\mathbf{q}(t) = \mathbf{f}(t) \quad \text{for} \quad |t - t_k| < \delta$$

End Position Analysis

Begin Velocity Analysis

Velocity Analysis

- This is simple. What is the framework?

- You just found **q** at time *t*, that is, the location and orientation of each component of the mechanism at time t, and now you want to find the velocity of each component (body) of the mechanism

- Taking one time derivative of the constraints leads to the velocity equation:

$$\mathbf{\Phi}(\mathbf{q},t)=\mathbf{0} \quad \Rightarrow \quad \dot{\mathbf{\Phi}}(\mathbf{q},t)=\mathbf{0} \quad \Leftrightarrow \quad \mathbf{\Phi}_{\mathbf{q}}(\mathbf{q},t)\cdot\dot{\mathbf{q}}=\nu$$

- In layman's words, once you have $\mathbf{q}_{(k)}$ you can find $\dot{\mathbf{q}}_{(k)}$ at time t_k by solving the linear system

$$\mathbf{\Phi}_{\mathbf{q}}(\mathbf{q}_{(k)},t_k)\cdot\dot{\mathbf{q}}_{(k)}=\nu_{(k)}$$

Velocity Analysis

- Notation: please note the subscript is in parentheses

$$\mathbf{q}_{(k)}, \quad \dot{\mathbf{q}}_{(k)}$$

  - It indicates that that quantity is evaluated at t_k
  - If no parentheses, can be mistaken for the coordinates associated with body "k"

- Observations:
  - Note that as long as the constraint Jacobian is nonsingular, you can solve this linear system and recover the velocity $\dot{\mathbf{q}}_{(k)}$

  - The reason velocity analysis is easy is that, unlike for position analysis where you have to solve a nonlinear system, now you are dealing with a linear system, which is easy to solve

  - Note that the velocity analysis comes after the position analysis is completed, and you are in a new configuration of the mechanism in which you are about to find out its velocity

End Velocity Analysis

Begin Acceleration Analysis

Acceleration Analysis

- This is also fairly simple. What is the framework?
- You just found $\mathbf{q}_{(k)}$ and $\dot{\mathbf{q}}_{(k)}$ at time t_k, that is, where the mechanism is at time t_k, and what its velocity is
- You'd like to know the acceleration of each component of the model
- Taking two time derivatives of the constraints leads to the acceleration equation:

$$\mathbf{\Phi}(\mathbf{q}_{(k)}, t_k) = \mathbf{0} \quad \Rightarrow \quad \ddot{\mathbf{\Phi}}(\mathbf{q}_{(k)}, t_k) = \mathbf{0} \quad \Leftrightarrow \quad \mathbf{\Phi}_{\mathbf{q}}(\mathbf{q}_{(k)}, t_k) \cdot \ddot{\mathbf{q}}_{(k)} = \gamma_{(k)}$$

Acceleration Analysis

- In other words, you find the acceleration (second time derivative of **q** at time t_k) as the solution of a linear system:

$$\mathbf{\Phi}_{\mathbf{q}}(\mathbf{q}_{(k)}, t_k) \cdot \ddot{\mathbf{q}}_{(k)} = \gamma_{(k)}$$

- Observations:
  - The equation above illustrates why we have been interested in the expression of γ, the RHS of the acceleration equation:

$$\gamma = -(\mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}})_{\mathbf{q}}\dot{\mathbf{q}} - 2\mathbf{\Phi}_{\mathbf{q}}t\dot{\mathbf{q}} - \mathbf{\Phi}_{tt}$$

  - Note that you again want the constraint Jacobian to be nonsingular, since then you can solve the acceleration linear system and obtained the acceleration $\ddot{\mathbf{q}}_{(k)}$

SUMMARY OF CHAPTER 3

- We looked at the KINEMATICS of a mechanism
  - That is, we are interested in how this mechanism moves in response to a set of kinematic drives (motions) applied to it

- What one has to do:
  - **Step A**: Identify *all* physical *joints* and *drivers* present in the system
  - **Step B**: Identify the corresponding constraint equations $\Phi(\mathbf{q},t)$
  - **Step C**: Solve for the Position as a function of time ($\Phi_{\mathbf{q}}$ is needed)
  - **Step D**: Solve for the Velocities as a function of time (ν is needed)
  - **Step E**: Solve for the Accelerations as a function of time (γ is needed)