ME451
Kinematics and Dynamics of Machine Systems

Composite Joints – 3.3
Gears and Cam Followers – 3.4
October 7, 2010

Before we get started…

- Last time:
  - Revolute Joint
  - Translational Joint

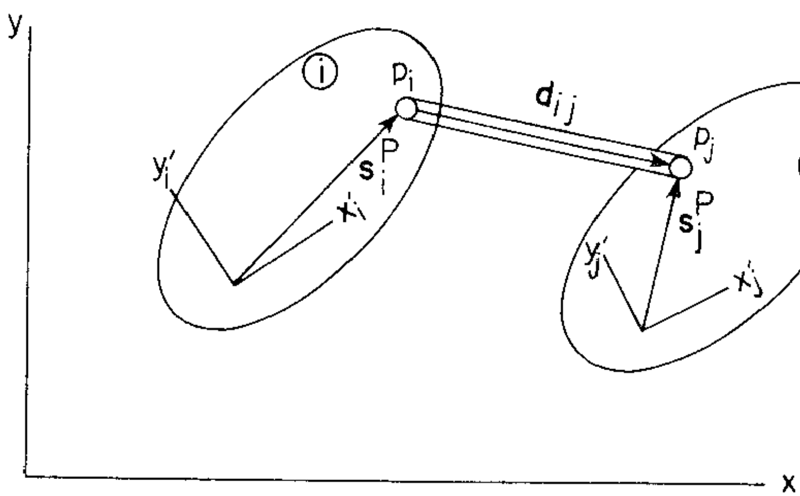
- Today: Move on and cover a couple of other constraints
  - Composite joints:
    - distance constraint
    - revolute-translational
  - Gears – I'll skip these
  - Cam-follower
  - Point follower

- Next week:
  - ADAMS Tutorial, part two. No lecture on Tu and Th
  - Focus is on ADAMS co-simulation and interfacing to MATLAB for mechatronics
  - No office hours, I'll be traveling the entire week

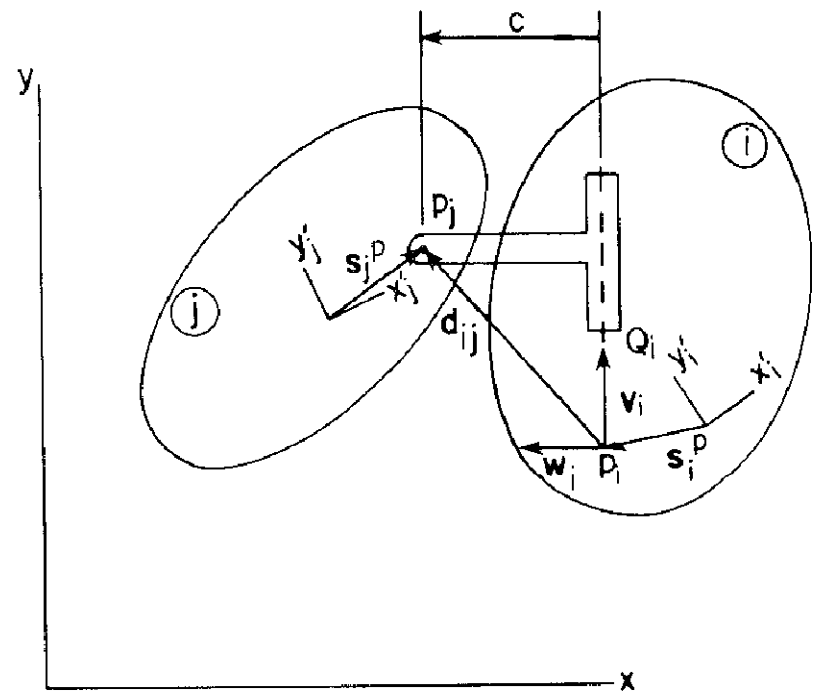
Composite Joints (CJ)

- Just a means to eliminate one intermediate body whose kinematics you are not interested in

- Revolute-Revolute CJ
  - Also called a coupler
  - Practically eliminates need of connecting rod
  - Given to you (joint attributes):
    - Location of points P_i and P_j
    - Distance d_{ij} of the massless rod



- Revolute-Translational CJ
  - Given to you (joint attributes):
    - Distance c
    - Point P_j (location of revolute joint)
    - Axis of translation $\mathbf{v}_i$'
    - $\mathbf{v}_i$' is assumed to be a unit vector



Composite Joints

- One follows exactly the same steps as for any joint:
  - Step 1: Physically, what type of motion does the joint allow?
  - Step 2: Constraint Equations $\Phi(\mathbf{q},t)$ = ?
  - Step 3: Constraint Jacobian $\Phi_{\mathbf{q}}$ = ?
  - Step 4: ν = ?
  - Step 5: γ = ?

Moving on to gears (section 3.4)

Gears

- Convex-convex gears
  - Gear teeth on the periphery of the gears cause the pitch circles shown to roll relative to each other, without slip
- First Goal: find the angle θ , that is, the angle of the carrier

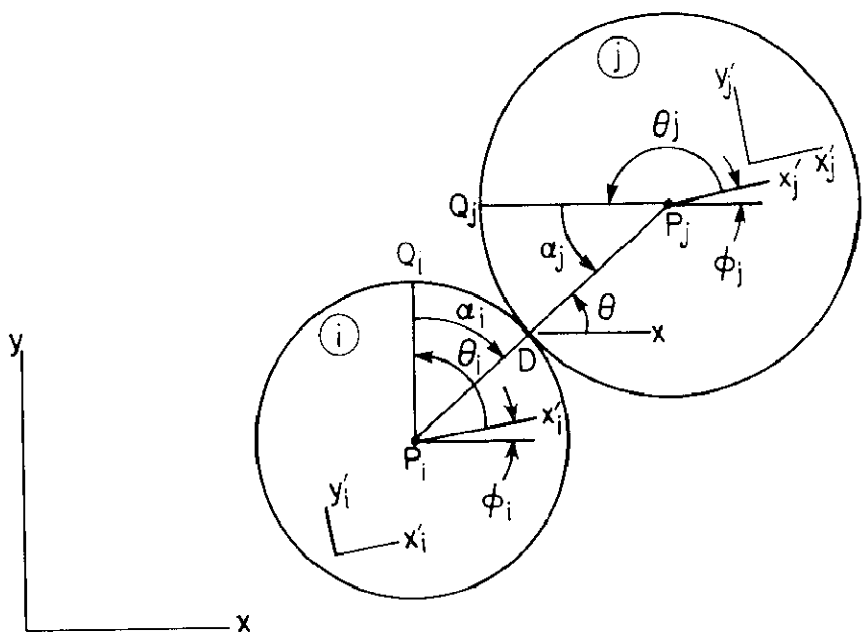


Figure 3.4.2 Geometry of gear set.

- What's known:
  - Angles θ_i and θ_j
  - The radii R_i and R_j
- You need to express θ as a function of these four quantities plus the orientation angles ϕ_i and ϕ_j
- Kinematically: P_iP_j should always be perpendicular to the contact plane

Gears - Discussion of Figure 3.4.2 (Geometry of gear set)

$O'_i x'_i y'_i$ represents the local reference frame attached to gear i. This reference frame is rotated by an angle ϕ_i with respect to the global reference frame. This angle ϕ_i depends on time, and changes as the attitude (orientation) of body i changes.

$O'_j x'_j y'_j$ represents the local reference frame attached to gear j. This reference frame is rotated by an angle ϕ_j with respect to the global reference frame. This angle ϕ_j depends on time, and changes as the attitude (orientation) of body j changes.

When the gears were assembled, the points Q_i and Q_j were the contact points between the two gears (that's where the gears came in contact for the first time, at time $t = 0$). If one gear is activated and it starts rotating, the second gear will follow and the two points Q_i and Q_j will separate and follow their destiny, away from the contact point which is denoted by D.

There is an angle α_i that indicates how far the original contact point, Q_i, is from the current contact point, D. By the same token, there is an angle α_j that indicates how far the original contact point, Q_j, is from the current contact point, D.

Since the local reference frame $O'_i x'_i y'_i$ is attached to gear i, and the point Q_i is also attached to gear i, the angle that positions Q_i in $O'_i x'_i y'_i$ stays constant at all times, and it's denoted by θ_i. For gear j a similar angle θ_j is defined.

Gears - Discussion of Figure 3.4.2 (Geometry of gear set)

The contact point D is always on the line that connects P_i and P_j, the centers of the two gears, respectively.

What we are after is determining the angle θ that implicitly defines the perpendicular on the plane of contact (the plane that goes through D and is tangent to the two gears at point D). Once this angle θ becomes available, the vector that is perpendicular on the tangent plane assumes the expression:

$$\mathbf{u}^{\perp} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}.$$

Using Figure 3.4.2, after performing some manipulations, the angle θ is expressed as

$$\theta = \frac{R_i(\phi_i + \theta_i) + R_j(\phi_j + \theta_j - \pi)}{R_i + R_j}. \tag{1}$$

The kinematic constraint associated with the gear set requires that the vectors $\vec{P_iP_j}$ and $\vec{\mathbf{u}}$ are parallel, or in other words,

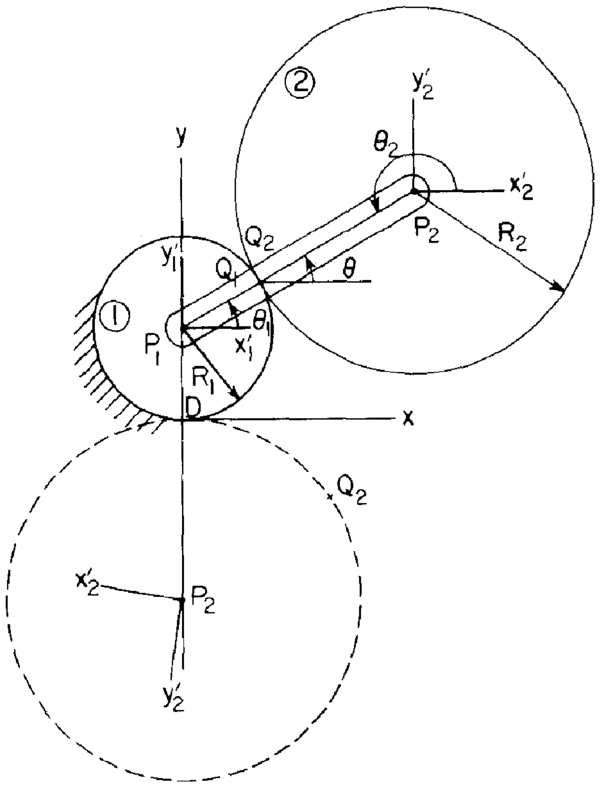
$$\Phi^{g(i,j)} = (\mathbf{r}_j^P - \mathbf{r}_i^P)^T \cdot \mathbf{u}^{\perp} = 0 \tag{2}$$

Note that the important thing is that this angle θ depends on the value of ϕ_i and ϕ_j, which in turn depend on the orientation of the two gears. What Eq. (2) is telling us is that ϕ_i and ϕ_j can not be arbitrarily changing. Rather, as they change in time, they should change in such a way so that the angle θ computed with Eq. (1) will satisfy the condition of Eq. (2).

Note: there are a couple of mistakes in the book, see Errata slide before

Example: 3.4.1

- Gear 1 is fixed to ground
- Given to you: $\phi_1 = 0$, $\theta_1 = \pi/6$, $\theta_2 = 7\pi/6$, $R_1 = 1$, $R_2 = 2$
- Find ϕ_2 as gear 2 falls to the position shown (carrier line P_1P_2 becomes vertical)



Gears (Convex-Concave)

- Convex-concave gears – we are not going to look into this class of gears
- The approach is the same, that is, expressing the angle θ that allows on to find the angle of the
- Next, a perpendicularity condition using **u** and P_iP_j is imposed (just like for convex-convex gears)

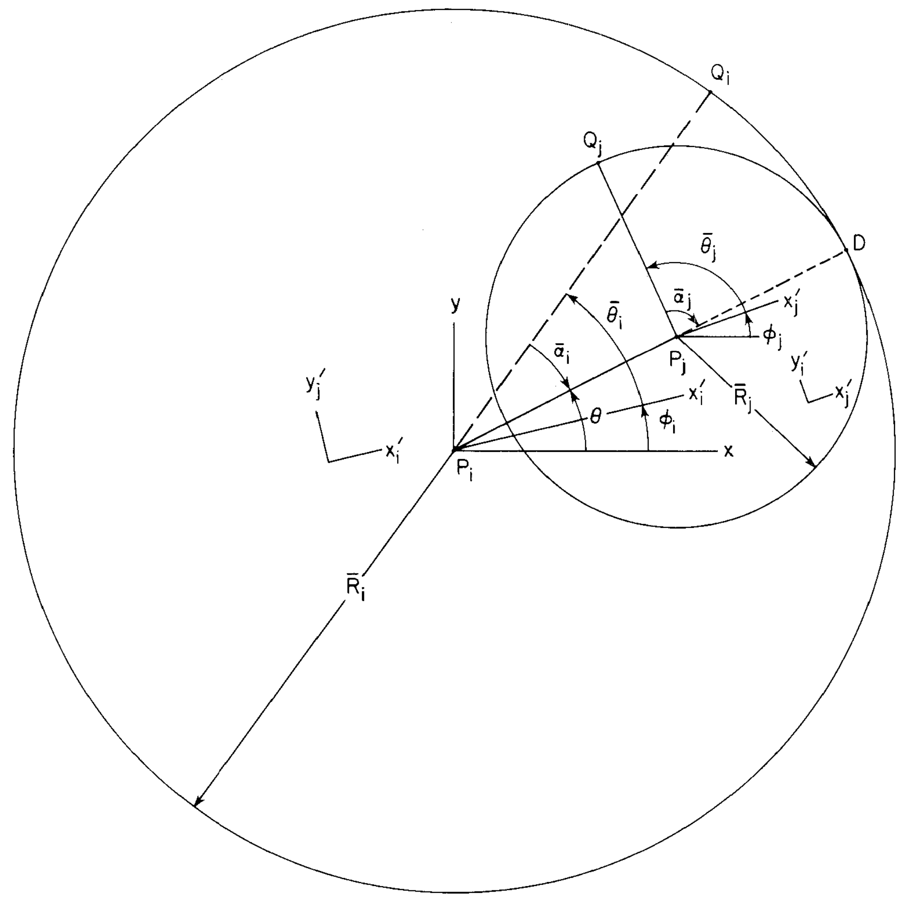
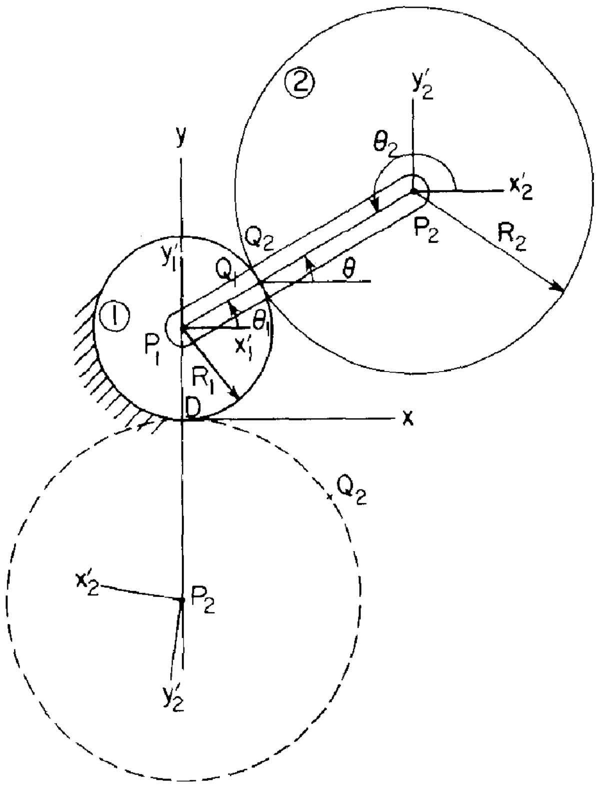


Figure 3.4.4 Concave–convex gear set.

Example: 3.4.1

- Gear 1 is fixed to ground
- Given to you: $\phi_1 = 0$, $\theta_1 = \pi/6$, $\theta_2 = 7\pi/6$, $R_1 = 1$, $R_2 = 2$
- Find ϕ_2 as gear 2 falls to the position shown (carrier line P_1P_2 becomes vertical)



Rack and Pinion Preamble

- Framework:
  - Two points P_i and Q_i on body i define the rack center line
  - Radius of pitch circle for pinion is R_j
  - There is no relative sliding between pitch circle and rack center line
  - Q_i and Q_j are the points where the rack and pinion were in contact at time t=0

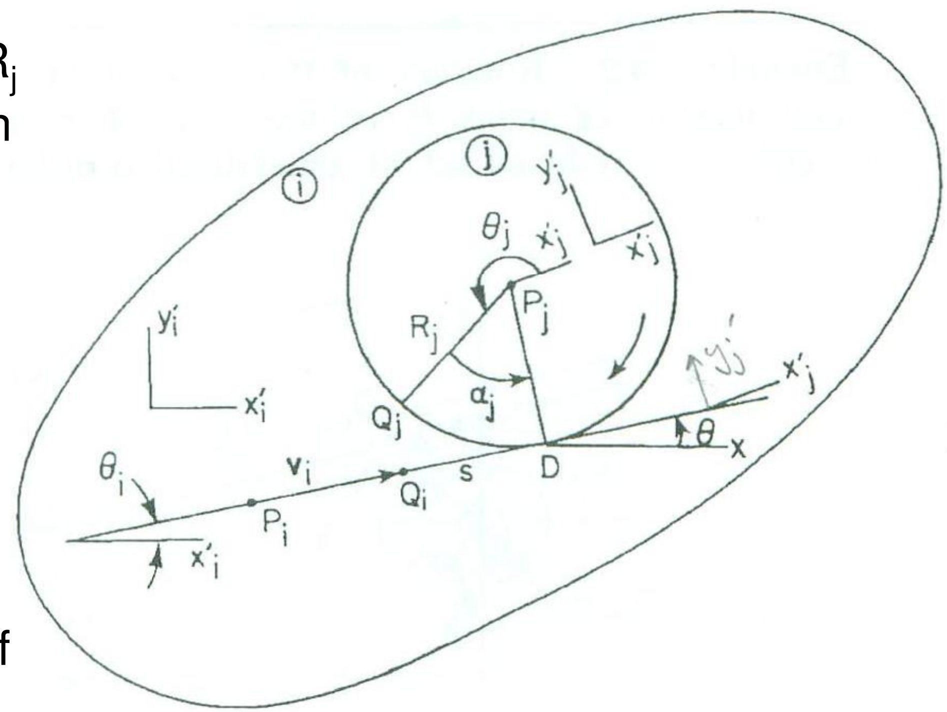


Figure 3.4.5 Rack and pinion.

- NOTE:
  - A rack-and-pinion type kinematic constraint is a limit case of a pair of convex-convex gears
    - Take the radius R_i to infinity, and the pitch line for gear i will become the rack center line

Rack and Pinion Kinematics

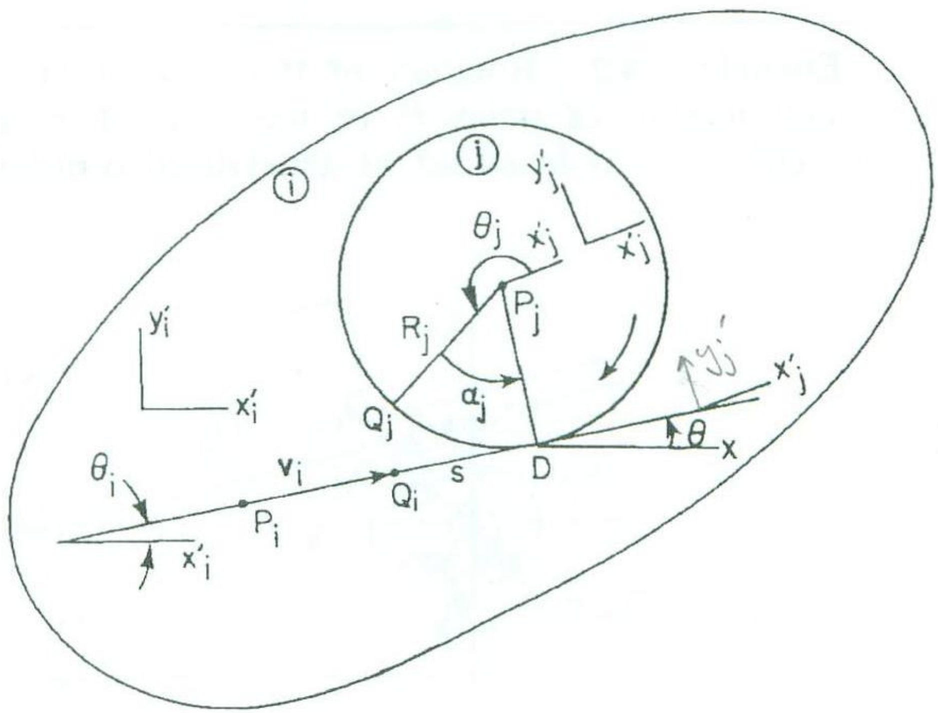


Figure 3.4.5 Rack and pinion.

- Kinematic constraints that define the relative motion:
  - At any time, the distance between the point P_j and the contact point D should stay constant (this is equal to the radius of the gear R_j)
  - The length of the segment Q_iD and the length of the arc Q_jD should be equal (no slip condition)
- Rack-and-pinion removes two DOFs of the relative motion between these two bodies

Rack and Pinion Pair

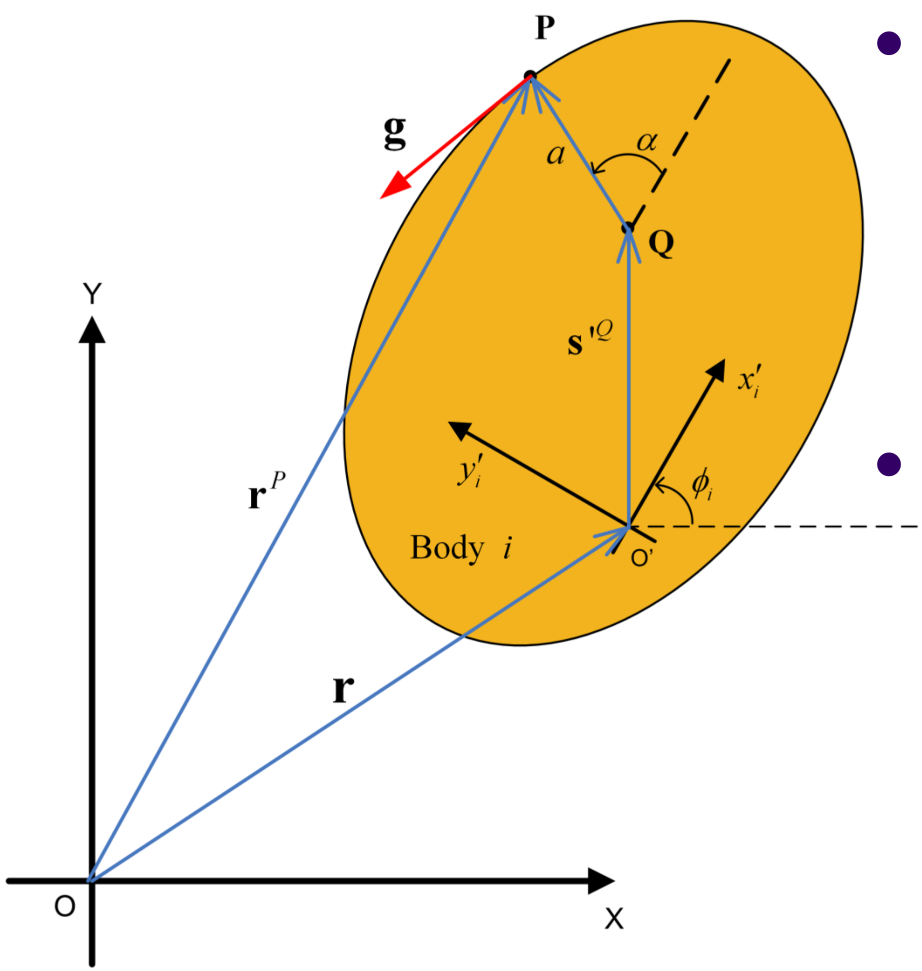
- Step 1: Understand the physical element
- Step 2: Constraint Equations $\Phi(\mathbf{q},t) = ?$
- Step 3: Constraint Jacobian $\Phi_{\mathbf{q}} = ?$
- Step 4: $\nu = ?$
- Step 5: $\gamma = ?$

End Gear Kinematics

Begin Cam-Follower Kinematics

Preamble: Boundary of a Convex Body

- Assumption: the bodies we are dealing with are convex
  - To any point on the boundary corresponds one value of the angle α (this is like the yaw angle, see figure below)



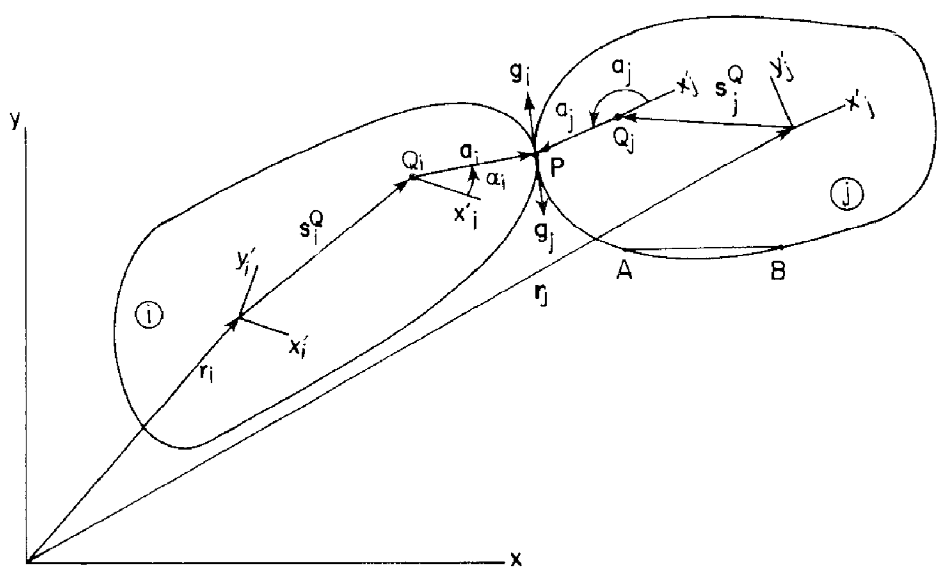
- The distance from the reference point Q to any point P on the convex boundary is a function of α:

$$||\mathbf{PQ}|| = \rho(\alpha)$$

- It all boils down to expressing **two** quantities as functions of α
  - The position of P, denoted by $\mathbf{r}^P$
  - The tangent at point P, denoted by **g**

Cam-Follower Pair

- Assumption: no chattering takes place
- The basic idea: two bodies are in contact, and at the contact point the two bodies share:
  - The contact point
  - The tangent to the boundaries



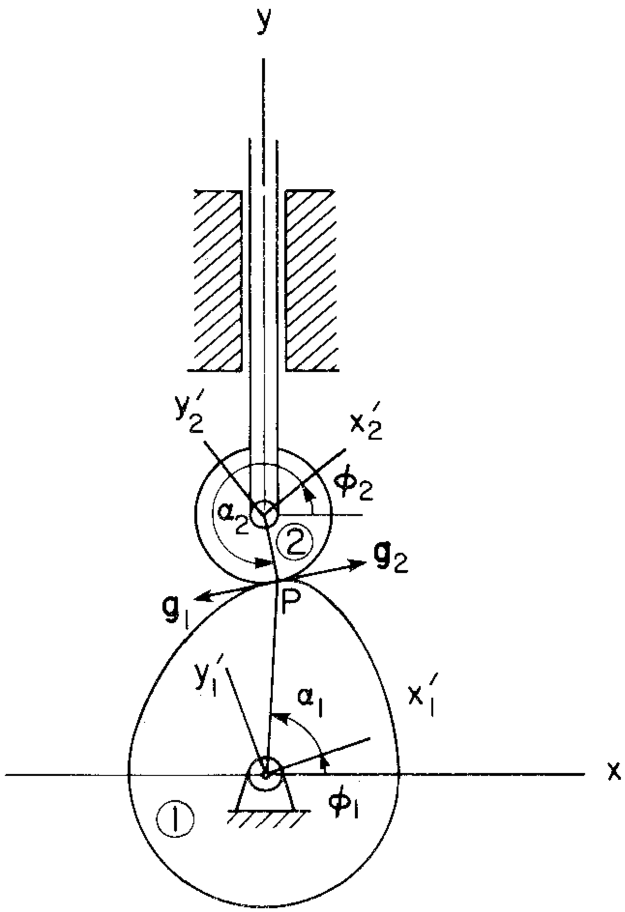
- Recall that a point is located by the angle α_i on body i, and α_j on body j.
- Therefore, when dealing with a cam-follower, in addition to the x,y,θ coordinates for <u>each</u> body one needs to rely on one additional generalized coordinate, namely the contact point angle α:
  - Body i: x_i, y_i, ϕ_i, α_i
  - Body j: x_j, y_j, ϕ_j, α_j

Cam-Follower Constraint

- Step 1: Understand the physical element
- Step 2: Constraint Equations $\Phi(\mathbf{q},t)$ = ?
- Step 3: Constraint Jacobian $\Phi_{\mathbf{q}}$ = ?
- Step 4: ν = ?
- Step 5: γ = ?

Example

- Determine the expression of the tangents $\mathbf{g}_1$ and $\mathbf{g}_2$



$$\rho_1(\alpha_1) = \begin{cases} -\frac{1}{4}\cos 3\alpha_1 + \frac{5}{4} & \text{if} \quad 0 \leq \alpha_1 < \frac{2\pi}{3} \\ 1 & \text{if} \quad \frac{2\pi}{3} \leq \alpha_1 \leq 2\pi \end{cases}$$

$$\rho_2(\alpha_2) = \frac{1}{4}$$

Cam flat-faced-follower Pair

- A particular case of the general cam-follower pair
  - Cam stays just like before
  - Flat follower
  - Typical application: internal combustion engine
  - Not covered in detail, HW touches on this case

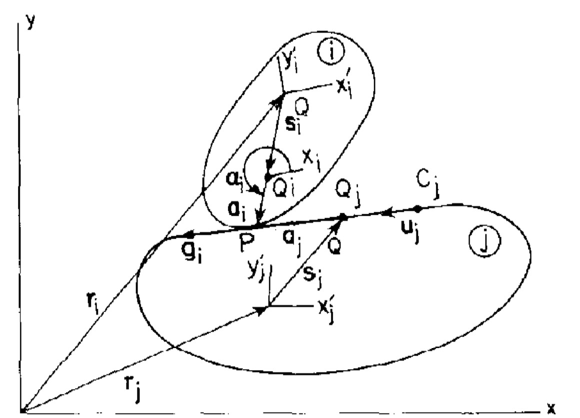


Figure 3.4.10 Cam–flat-faced follower pair.

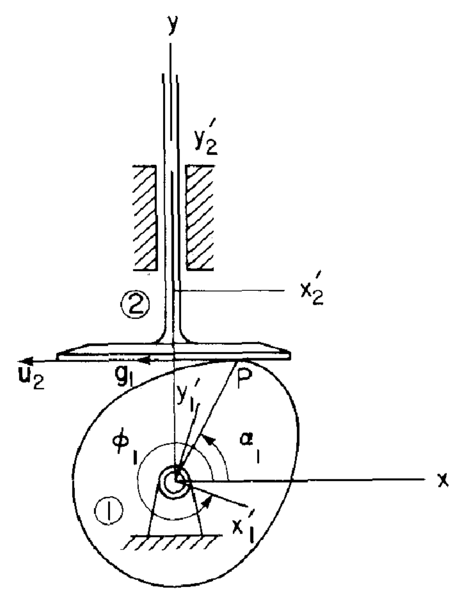


Figure 3.4.11 Cam–flat-faced follower in an internal combustion engine.

Point-Follower Pair

- Framework (Step 1):
  - Pin P is attached to body i and can move in slot attached to body j.
  - NOTE: the book forgot to mention what $\mathbf{g}_j$ is (pp.85, eq. 3.4.32)
    - It represents the tangent to the slot in which P is allowed to move

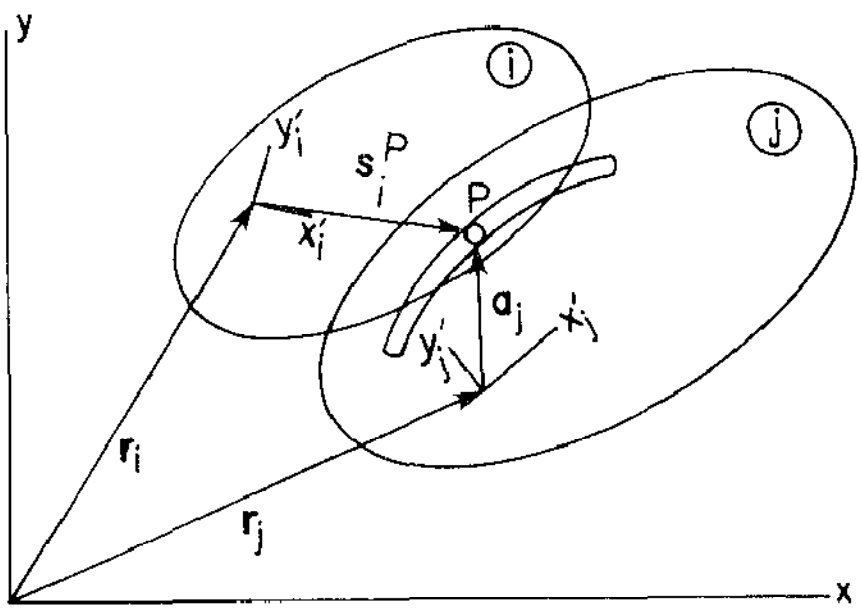


Figure 3.4.12 Point–follower pair.

- The location of point P in slot attached to body j is captured by angle α_j that parameterizes the slot.
- Therefore, when dealing with a point-follower we'll be dealing with the following set of generalized coordinates:
  - Body i: x_i, y_i, ϕ_i,
  - Body j: x_j, y_j, ϕ_j, α_j

Point-Follower Pair

- Step 2: Constraint Equations $\Phi(\mathbf{q},t) = ?$
- Step 3: Constraint Jacobian $\Phi_{\mathbf{q}} = ?$
- Step 4: $\nu = ?$
- Step 5: $\gamma = ?$