Before we get started...

● Last Time
  ● Discussed the concept of joint (constraint)
    ● Absolute constraints
    ● Relative constraints
  ● We looked at several absolute constraints
    ● $x$, $y$, $\phi$ relative constraints
    ● Absolute Distance Constraint
  ● Recall the drill that you have to go through in relation to each joint in order to provide what it takes to carry out Kinematics Analysis
    ● Five step procedure:
      ▪ Identify and analyze the physical joint
      ▪ Derive the constraint equations associated with the joint
      ▪ Compute constraint Jacobian $\Phi_q$
      ▪ Get $\nu$ (RHS of velocity equation)
      ▪ Get $\gamma$ (RHS of acceleration equation, this is challenging in some cases)

● Today
  ● Covering revolute, translational, and composite joints
Absolute distance-constraint

- Step 1: the distance from a point $P_i$ to an absolute (or global) reference frame stays constant, and equal to some known value $C_4$

- Step 2: Identify $\Phi^{dx(i)}=0$

- Step 3: $\Phi^{dx(i)}_q = ?$

- Step 4: $\nu^{dx(i)} = ?$

- Step 5: $\gamma^{dx(i)} = ?$

Fig. 3.2.1 Constraint imposes that distance between point $P$ and point of coordinates $(C_1, C_2)$ is $C_3$. 
Example 3.2.1

- An example where you would have to use absolute constraints: the simple pendulum

- Generalized coordinates used:

\[ q_1 = \begin{bmatrix} x_1 \\ y_1 \\ \phi_1 \end{bmatrix} \]

**Figure 3.2.3** Simple pendulum with absolute constraints.
Example 3.1.3

- A case when the algebraic constraints fail to imply (enforce) the actual kinematics of a mechanism: block moving down on incline, incline angle $\pi/4$
  - Use the following set of generalized coordinates: $q = \begin{bmatrix} x_1 \\ y_1 \\ \varphi_1 \end{bmatrix}$
  - Motion prescribed like $x_1 = 6 - 6t$
  - Formulate constraints defining the kinematics (motion) of the mechanism

\[
\Phi(q, t) = \begin{bmatrix}
x_1 - y_1 \\
-x_1 \sin \varphi_1 + y_1 \cos \varphi_1 \\
x_1 - 6 + 6t
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]
Example 3.1.3

- Note that when passing through the origin, the algebraic constraints fail to specify the actual kinematics of the mechanism.
Example 3.2.2

- An example where you would have to use an absolute angle constraint: slider along x-axis
  - Only Left ↔ Right motion is allowed

Figure 3.2.4  Slider along x axis.
Moving on to relative constraints (section 3.3)
Preliminary Discussion:
Relative Position, Velocity, and Acceleration of \( P_j \) with respect to \( P_i \)

- Something that we’ll use a lot: the expression of the vector from \( P_i \) to \( P_j \) in terms of the generalized coordinates \( \mathbf{q} \)

\[
\mathbf{q} = \begin{bmatrix}
    x_i \\
    y_i \\
    \varphi_i \\
    x_j \\
    y_j \\
    \varphi_j
\end{bmatrix} = \begin{bmatrix}
    \mathbf{r}_i \\
    \mathbf{r}_j \\
    \mathbf{q}_i \\
    \mathbf{q}_j
\end{bmatrix}
\]

\[
\overrightarrow{P_iP_j} = \mathbf{r}_j + A_j s'^P - \mathbf{r}_i - A_i s'^{P_i}
\]

\[
\frac{d}{dt} \left( \overrightarrow{P_iP_j} \right) = \dot{\mathbf{r}}_j + \dot{\varphi}_j \mathbf{B}_j s'^P - \dot{\mathbf{r}}_i - \dot{\varphi}_i \mathbf{B}_i s'^{P_i}
\]

\[
\frac{d^2}{dt^2} \left( \overrightarrow{P_iP_j} \right) = \ddot{\mathbf{r}}_j + \ddot{\varphi}_j \mathbf{B}_j s'^P - \ddot{\mathbf{r}}_i - \ddot{\varphi}_i \mathbf{B}_i s'^{P_i} + \dddot{\varphi}_i \mathbf{A}_i
\]
Relative x-constraint

- Step 1: In layman’s words, the difference between the x coordinates of point $P_j$ and point $P_i$ should stay constant and equal to some known value $C_1$

- Step 2: Identify $\Phi^{rx(i,j)} = 0$

- Step 3: $\Phi^{rx(i,j)}_q = ?$

- Step 4: $\nu^{rx(i,j)} = ?$

- Step 5 $\gamma^{rx(i,j)} = ?$

*Figure 3.3.1* Simple constraints.
Relative y-constraint

- Step 1: The difference between the y coordinates of point $P_j$ and point $P_i$ should stay constant and equal to some known value $C_2$

- Step 2: Identify $\Phi_{ry(i,j)} = 0$

- Step 3: $\Phi_{ry(i,j)}^q = ?$

- Step 4: $\nu_{ry(i,j)} = ?$

- Step 5: $\gamma_{ry(i,j)} = ?$

*Figure 3.3.1 Simple constraints.*
Relative angle-constraint

- Step 1: The difference between the orientation angles of the RFs associated with bodies $i$ and $j$ stays constant and equal to some known value $C_3$

- Step 2: Identify $\Phi_{r\phi(i,j)} = 0$

- Step 3: $\Phi_{r\phi(i,j)} q = ?$

- Step 4: $\nu_{r\phi(i,j)} = ?$

- Step 5: $\gamma_{r\phi(i,j)} = ?$

Figure 3.3.1 Simple constraints.
Relative distance-constraint

- Step 1: The distance between two points $P_j$ and point $P_i$ should stay constant and equal to some known value $C_4$

- Step 2: Identify $\Phi^{rd}_{(i,j)} = 0$

- Step 3: $\Phi^{rd}_{(i,j)} q = ?$

- Step 4: $\nu^{rd}_{(i,j)} = ?$

- Step 5: $\gamma^{rd}_{(i,j)} = ?$

Figure 3.3.1 Simple constraints.