ME451
Kinematics and Dynamics of Machine Systems

Relative Constraints, Composite Joints – 3.3

September 30, 2010

Before we get started…

- Last Time
  - Discussed the concept of joint (constraint)
    - Absolute constraints
    - Relative constraints
  - We looked at several absolute constraints
    - x, y, ϕ relative constraints
    - Absolute Distance Constraint

  - Recall the drill that you have to go through in relation to each joint in order to provide what it takes to carry out Kinematics Analysis
    - Five step procedure:
      - Identify and analyze the physical joint
      - Derive the constraint equations associated with the joint
      - Compute constraint Jacobian $\mathbf{\Phi_q}$
      - Get $\boldsymbol{\nu}$ (RHS of velocity equation)
      - Get $\boldsymbol{\gamma}$ (RHS of acceleration equation, this is challenging in some cases)

- Today
  - Covering revolute, translational, and composite joints

Absolute distance-constraint

- Step 1: the distance from a point P_i to an absolute (or global) reference frame stays constant, and equal to some known value C_4

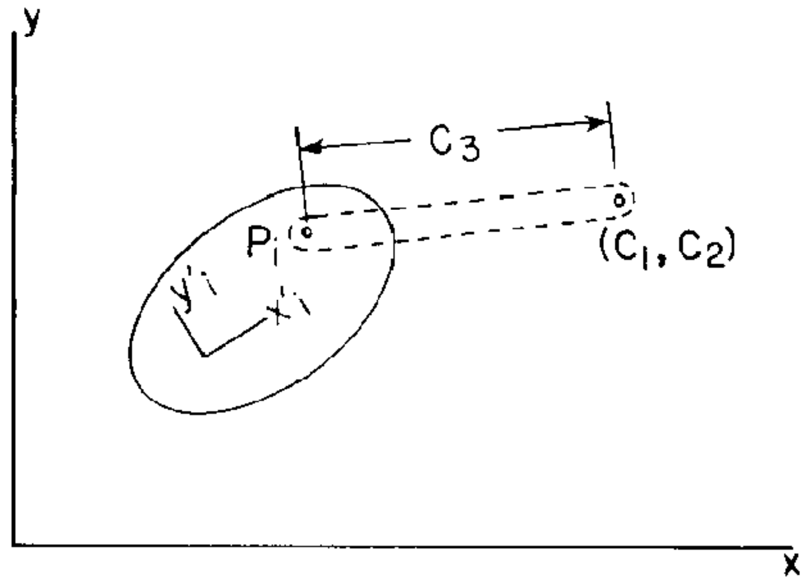


Fig. 3.2.1 Constraint imposes that distance between point P and point of coordinates (C_1, C_2) is C_3.

- Step 2: Identify $\Phi^{dx(i)}=0$
- Step 3: $\Phi^{dx(i)}_{\mathbf{q}} = ?$
- Step 4: $\nu^{dx(i)} = ?$
- Step 5: $\gamma^{dx(i)} = ?$

Example 3.2.1

- An example where you would have to use absolute constraints: the simple pendulum

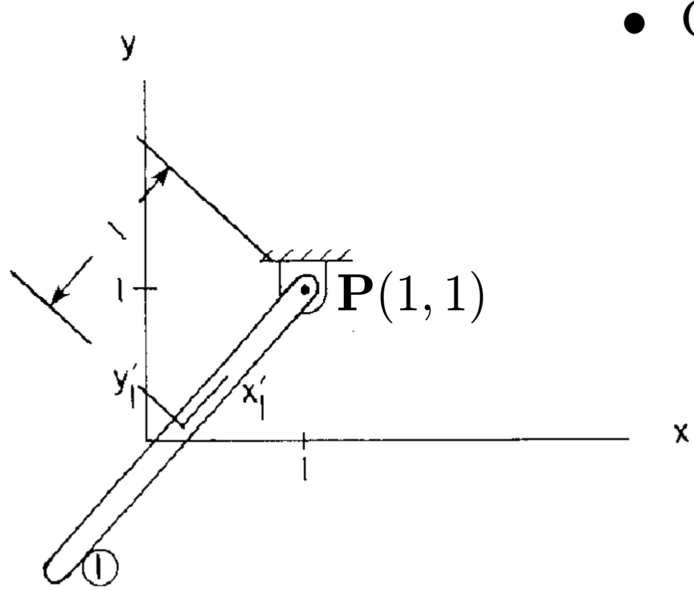


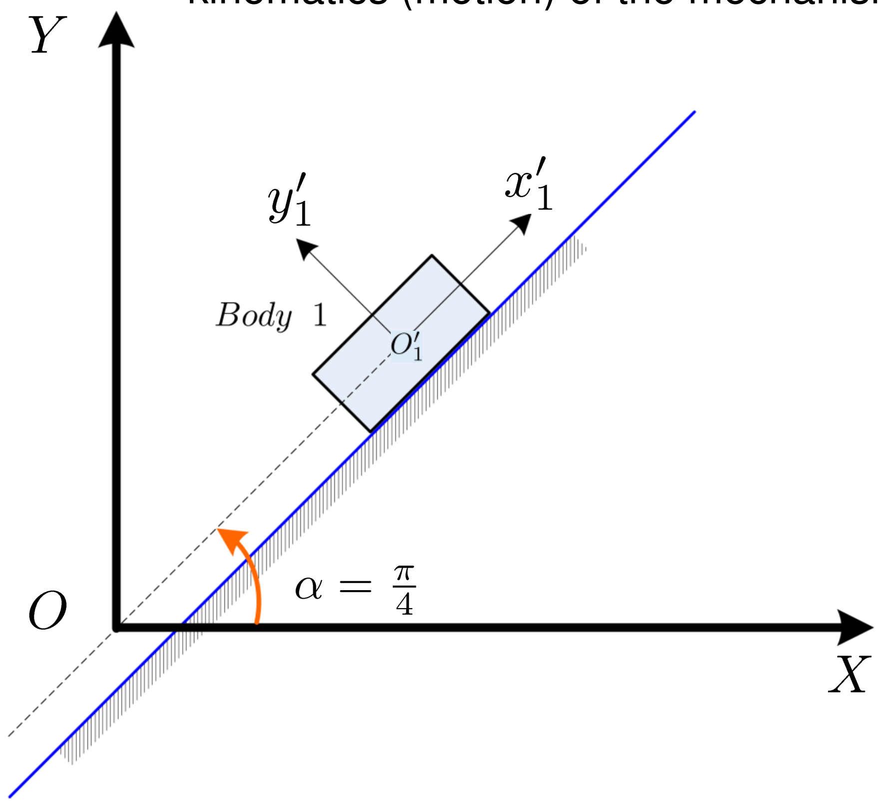
Figure 3.2.3 Simple pendulum with absolute constraints.

- Generalized coordinates used:

$$\mathbf{q}_1 = \begin{bmatrix} x_1 \\ y_1 \\ \phi_1 \end{bmatrix}$$

Example 3.1.3

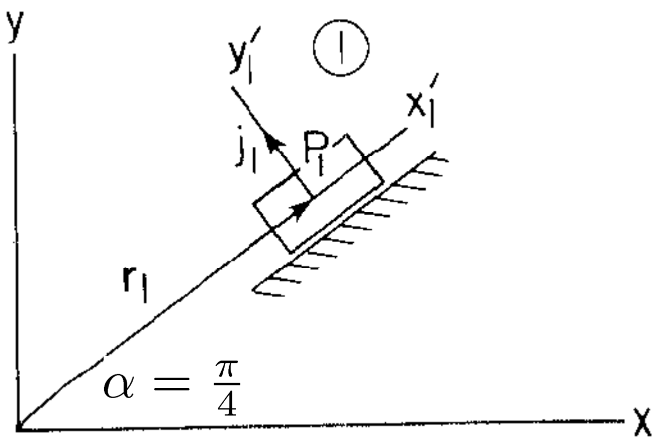
- A case when the algebraic constraints fail to imply (enforce) the actual kinematics of a mechanism: block moving down on incline, incline angle π/4
  - Use the following set of generalized coordinates: $\mathbf{q} = \begin{bmatrix} x_1 \\ y_1 \\ \varphi_1 \end{bmatrix}$
  - Motion prescribed like x_1=6-6t
  - Formulate constraints defining the kinematics (motion) of the mechanism

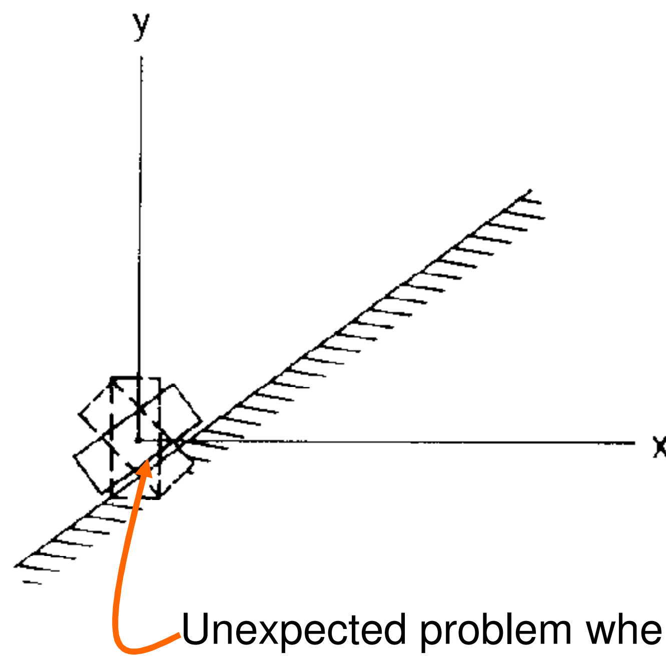


$$\mathbf{\Phi}(\mathbf{q}, t) = \begin{bmatrix} x_1 - y_1 \\ -x_1 \sin\varphi_1 + y_1 \cos\varphi_1 \\ x_1 - 6 + 6t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Example 3.1.3

- Note that when passing through the origin, the algebraic constraints fail to specify the actual kinematics of the mechanism





Unexpected problem when passing through origin…

Example 3.2.2

- An example where you would have to use an absolute angle constraint: slider along x-axis
  - Only Left ↔ Right motion is allowed

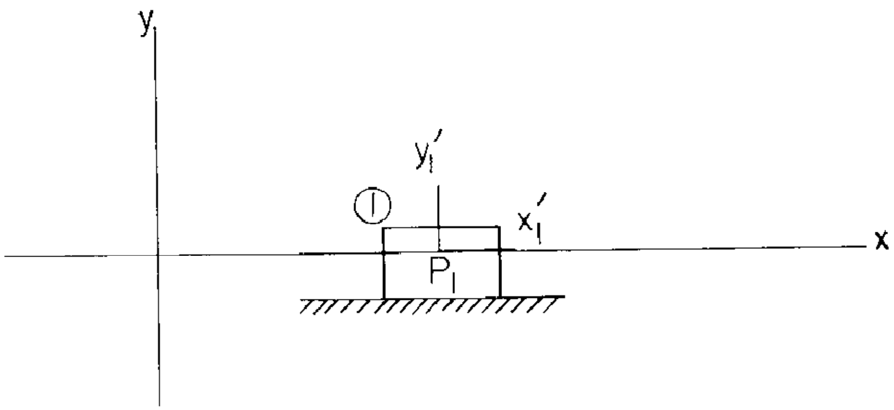


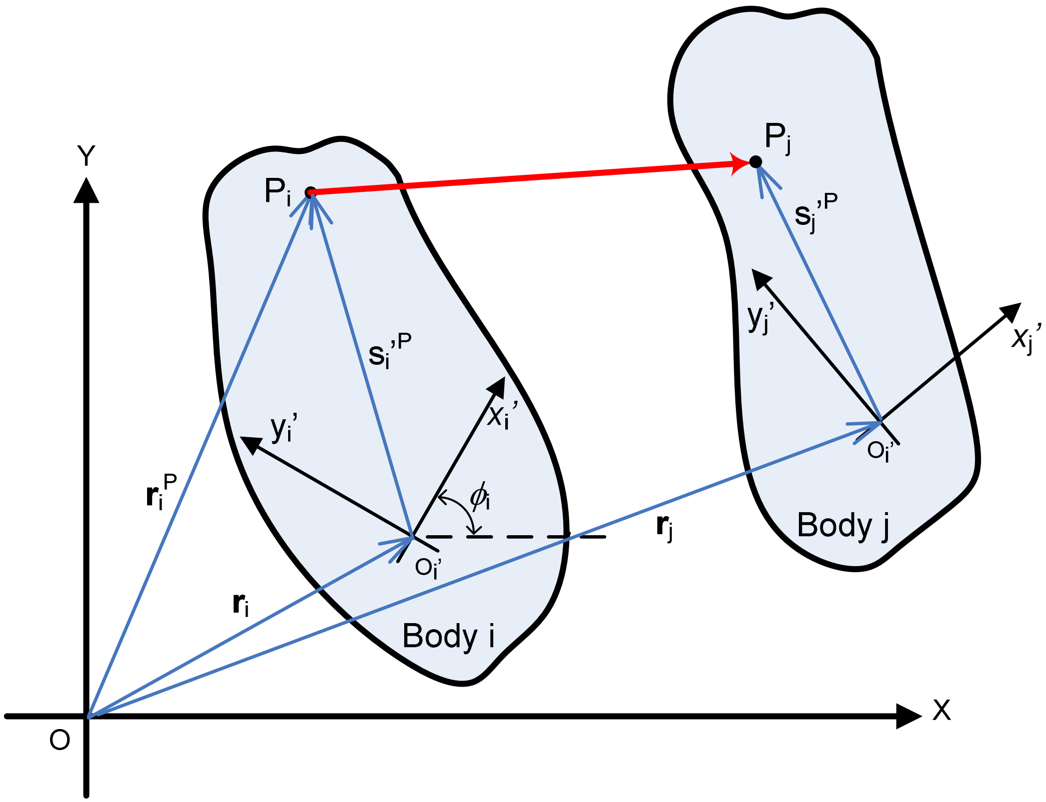
Figure 3.2.4 Slider along x axis.

Moving on to relative constraints (section 3.3)

Preliminary Discussion:

Relative Position, Velocity, and Acceleration of P_j with respect to P_i

- Something that we'll use a lot: the expression of the vector from P_i to P_j in terms of the generalized coordinates **q**



$$\mathbf{q} = \begin{bmatrix} x_i \\ y_i \\ \varphi_i \\ x_j \\ y_j \\ \varphi_j \end{bmatrix} = \begin{bmatrix} \mathbf{r}_i \\ \varphi_i \\ \mathbf{r}_j \\ \varphi_j \end{bmatrix} = \begin{bmatrix} \mathbf{q}_i \\ \mathbf{q}_j \end{bmatrix}$$

$$\overrightarrow{P_iP_j} = \mathbf{r}_j + \mathbf{A}_j \mathbf{s}'^{P_j} - \mathbf{r}_i - \mathbf{A}_i \mathbf{s}'^{P_i}$$

$$\frac{d(\overrightarrow{P_iP_j})}{dt} = \dot{\mathbf{r}}_j + \dot{\phi}_j \mathbf{B}_j \mathbf{s}'^{P_j} - \dot{\mathbf{r}}_i - \dot{\phi}_i \mathbf{B}_i \mathbf{s}'^{P_i}$$

$$\frac{d^2(\overrightarrow{P_iP_j})}{dt^2} = \ddot{\mathbf{r}}_j + \ddot{\phi}_j \mathbf{B}_j \mathbf{s}'^{P_j} - \dot{\phi}_j^2 \mathbf{A}_j \mathbf{s}'^{P_j} - \ddot{\mathbf{r}}_i - \ddot{\phi}_i \mathbf{B}_i \mathbf{s}'^{P_i} + \dot{\phi}_i^2 \mathbf{A}_i$$

Relative x-constraint

- Step 1: In layman's words, the difference between the x coordinates of point P_j and point P_i should stay constant and equal to some known value C_1

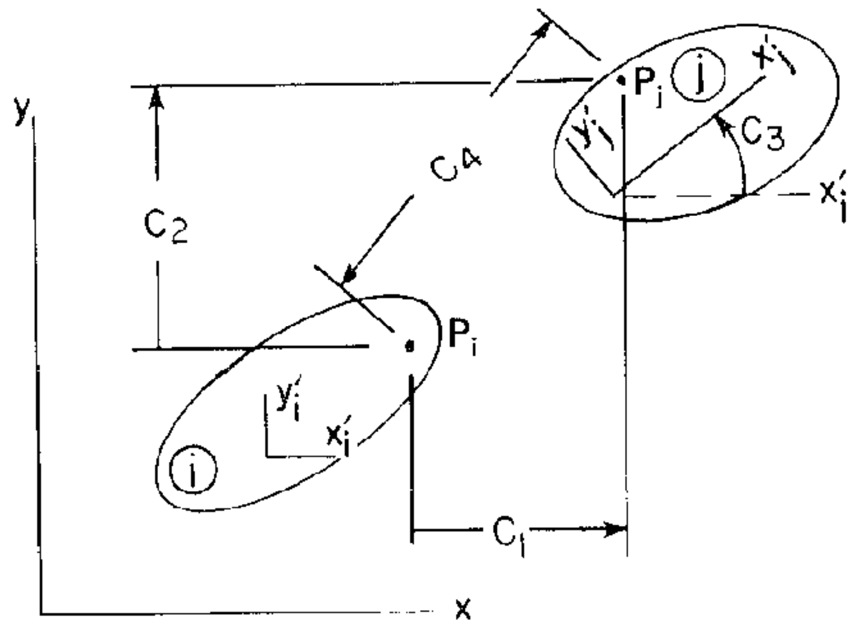


Figure 3.3.1 Simple constraints.

- Step 2: Identify $\Phi^{rx(i,j)}=0$
- Step 3: $\Phi^{rx(i,j)}_{\mathbf{q}} = ?$
- Step 4: $\nu^{rx(i,j)} = ?$
- Step 5 $\gamma^{rx(i,j)} = ?$

Relative y-constraint

- Step 1: The difference between the y coordinates of point P_j and point P_i should stay constant and equal to some known value C_2

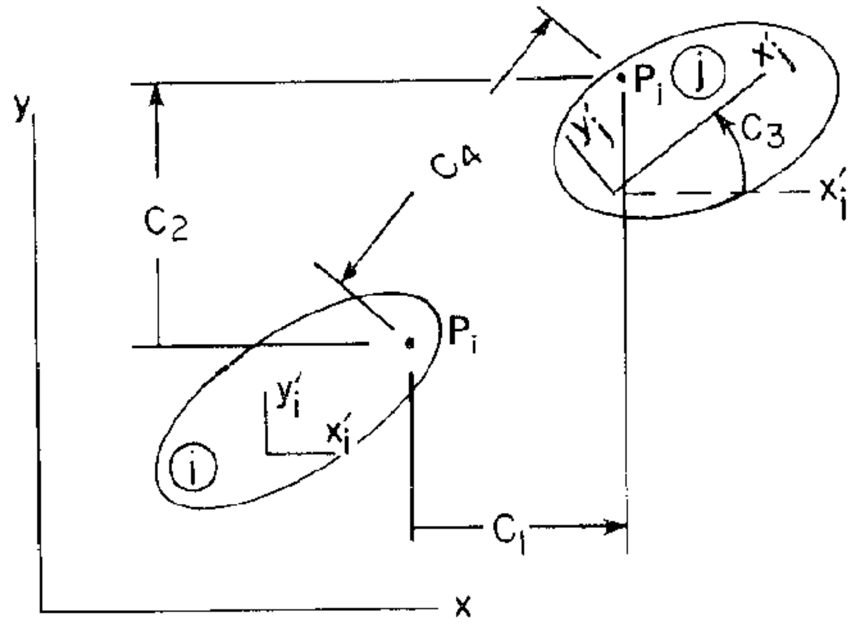


Figure 3.3.1 Simple constraints.

- Step 2: Identify $\Phi^{ry(i,j)}=0$
- Step 3: $\Phi^{ry(i,j)}_{\mathbf{q}} = ?$
- Step 4: $\nu^{ry(i,j)} = ?$
- Step 5: $\gamma^{ry(i,j)} = ?$

Relative angle-constraint

- Step 1: The difference between the orientation angles of the RFs associated with bodies i and j stays constant and equal to some known value C_3

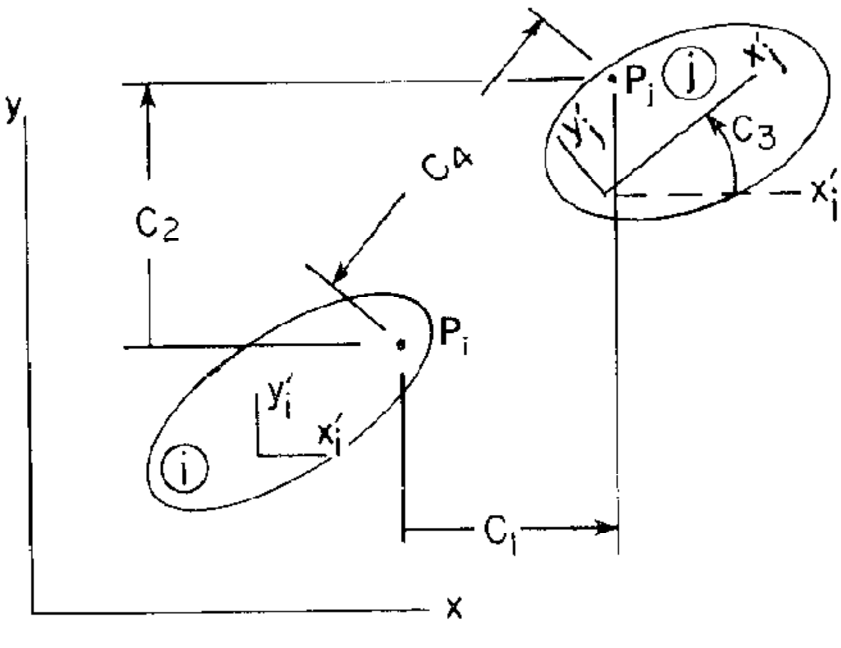


Figure 3.3.1 Simple constraints.

- Step 2: Identify $\Phi^{r\phi(i,j)}=0$
- Step 3: $\Phi^{r\phi(i,j)}_{\mathbf{q}} = ?$
- Step 4: $\nu^{r\phi(i,j)} = ?$
- Step 5: $\gamma^{r\phi(i,j)} = ?$

Relative distance-constraint

- Step 1: The distance between two points P_j and point P_i should stay constant and equal to some known value C_4

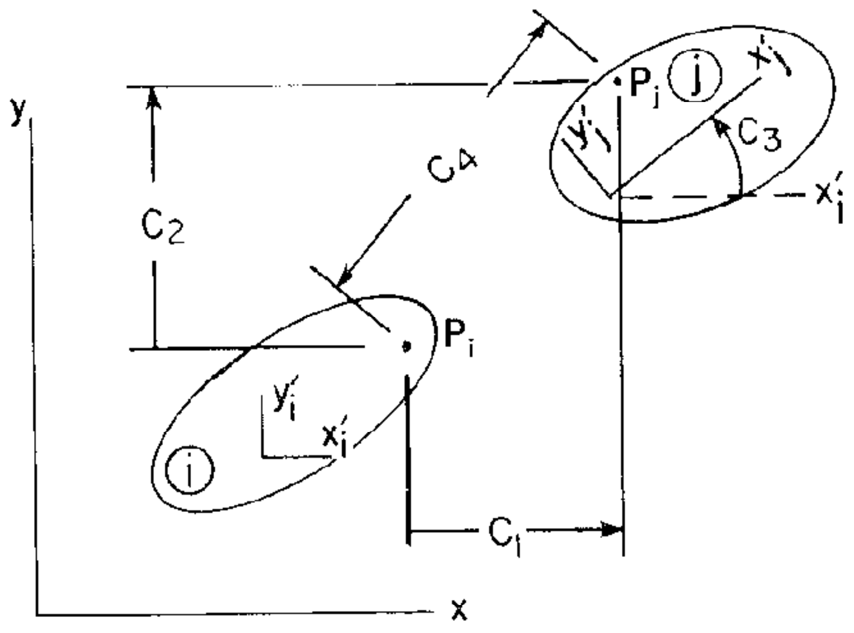


Figure 3.3.1 Simple constraints.

- Step 2: Identify $\Phi^{rd\,(i,j)}=0$
- Step 3: $\Phi^{rd\,(i,j)}_{\mathbf{q}} = ?$
- Step 4: $\nu^{rd\,(i,j)} = ?$
- Step 5: $\gamma^{rd\,(i,j)} = ?$