ME451
Kinematics and Dynamics of Machine Systems

Absolute Kinematic Constraints – 3.2

Relative Kinematic Constraints – 3.3

September 28, 2010

Before we get started…

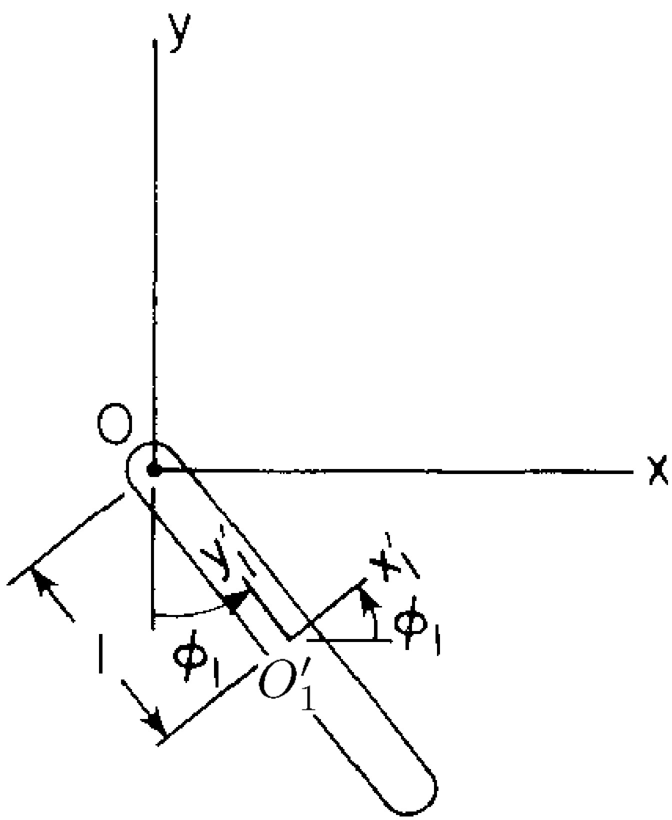
- HW (due in one week):
  - 2.6.1, 3.1.1, 3.1.2, 3.1.3
  - ADAMS component & MATLAB assignment will be emailed to you

- Last Time
  - Discussed stages of Kinematics Analysis
  - Boiler plate approach:
    - At each time step do
      - Position Analysis (system of nonlinear equations)
      - Velocity Analysis (system of linear equations, rhs denoted by ν)
      - Acceleration Analysis (system of linear equations, rhs denoted by γ)

- Today
  - Loose end: moving between two local reference frames
  - Start discussion about geometric constraints
      - Real life counterpart: joints between bodies

Example 3.1.1

- A motion $\phi_1 = 4t^2$ is applied to the pendulum
- Formulate the velocity analysis problem
- Formulate the acceleration analysis problem



Producing RHS of Acceleration Eq.
[Comments, in light of previous example]

- RHS was shown to be computed as

$$\ddot{\mathbf{\Phi}} = \frac{d^2}{dt^2}\mathbf{\Phi}(\mathbf{q},t) = \mathbf{0} \qquad \Rightarrow \qquad \mathbf{\Phi_q}\ddot{\mathbf{q}} = -(\mathbf{\Phi_q}\dot{\mathbf{q}})_\mathbf{q}\,\dot{\mathbf{q}} - 2\mathbf{\Phi}_{\mathbf{q}t}\,\dot{\mathbf{q}} - \mathbf{\Phi}_{tt} \qquad (1)$$

- Note that the RHS contains (is made up of) everything that does *not* depend on the generalized accelerations

- Implication:
  - When doing small examples in class, don't fell like you must compute the RHS using Equation (1) above
    - Equation (1) is always used in ADAMS, when shooting for a uniform approach to all problems

  - If you don't feel like using Equation (1), simply take two time derivatives of your constraints and move everything that does *not* depend on $\ddot{\mathbf{q}}$ to the RHS (to make up γ)

Loose Ends:
Moving between two Reference Frames with rotation matrices A_i and A_j, respectively

- Setup:
  - You expressed the location of a point in a RF with $\mathbf{A}_j$. (described by angle ϕ_j)
  - Now you want to express the location of the same point in a different RF with associated rotation matrix $\mathbf{A}_i$. (described by angle ϕ_i)
  - How do you do this?

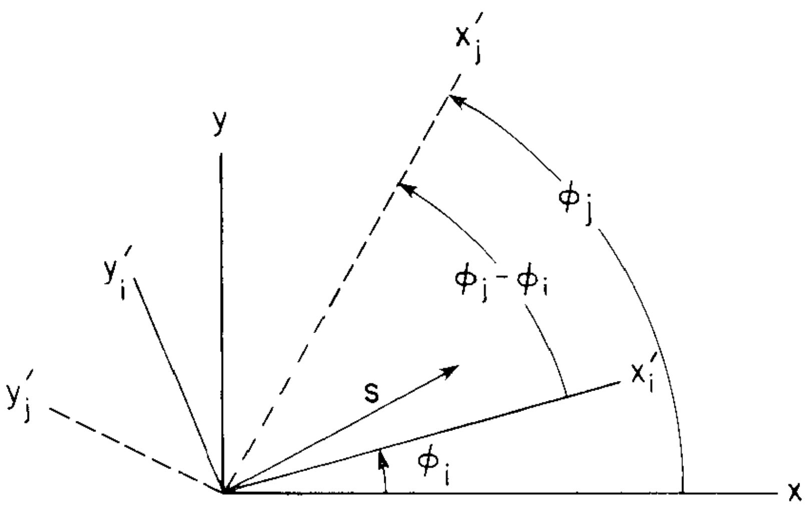


Figure 2.4.4 Three reference frames with coincident origins.

$$\mathbf{s}_i' = \mathbf{A}_i^T \mathbf{A}_j \mathbf{s}_j'$$

Loose Ends, Continued:
Notation (related to moving from A_j to A_i)

- Notation used: $$\mathbf{A}_{ij} \equiv \mathbf{A}_i^T \mathbf{A}_j = \begin{bmatrix} \cos\phi_i & \sin\phi_i \\ -\sin\phi_i & \cos\phi_i \end{bmatrix} \cdot \begin{bmatrix} \cos\phi_j & -\sin\phi_j \\ \sin\phi_j & \cos\phi_j \end{bmatrix}$$

$$\Downarrow$$

$$\mathbf{A}_{ij} = \begin{bmatrix} \cos(\phi_j - \phi_i) & -\sin(\phi_j - \phi_i) \\ \sin(\phi_j - \phi_i) & \cos(\phi_j - \phi_i) \end{bmatrix}$$

- Therefore,

$$\mathbf{s}'_i = \mathbf{A}_{ij}\mathbf{s}'_j$$

- Note that the order is important: it is "$\mathbf{A}_{ij}$" and not "$\mathbf{A}_{ji}$"
- $\mathbf{A}_{ij}$ gets multiplied from the right by a vector represented in the "j" Reference Frame (RF) and produces a vector represented in the "i" RF
- Note that when you see $\mathbf{A}_{ij}$ in fact you should have had $\mathbf{A}_{0j}$, where "0" is used to symbolize the global reference frame

Loose Ends, Final Slide

(regarding moving from A_j to A_i)

- For later reference, it is useful to recall that,

$$\mathbf{B} = \mathbf{A}\mathbf{R} = \mathbf{R}\mathbf{A} \qquad \text{where} \qquad \mathbf{R} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

- Therefore

$$\mathbf{B}_{ij} = \mathbf{R}\mathbf{A}_{ij} = \mathbf{A}_{ij}\mathbf{R} = \begin{bmatrix} -\sin(\phi_j - \phi_i) & -\cos(\phi_j - \phi_i) \\ \cos(\phi_j - \phi_i) & -\sin(\phi_j - \phi_i) \end{bmatrix}$$

[For the Rest of This Lecture:]
Focus on Geometric Constraints

- Learn how to write kinematic constraints that specify that the location and/or attitude of a body wrt the global (or absolute) RF is constrained in a certain way
  - Sometimes called <u>absolute</u> constraints

- Learn how to write kinematic constraints that couple the relative motion of two bodies
  - Sometimes called <u>relative</u> constraints

The Drill…

- Step 1: Identify a kinematic constraint (revolute, translational, relative distance, etc., i.e., the *physical* thing) acting between two components of a mechanism
- Step 2: Formulate the algebraic equations that capture that constraint, $\Phi(\mathbf{q})=\mathbf{0}$
  - This is called “modeling”
- Step 3: Compute the Jacobian (or the sensitivity matrix) $\Phi_{\mathbf{q}}$
- Step 4: Compute ν, the right side of the velocity equation
- Step 5: Compute γ, the right side of the acceleration equation (ugly…)

This is what we do almost exclusively in Chapter 3 (about two weeks)

Absolute Constraints

- Called "Absolute" since they express constraint between a body in a system and an absolute (ground) reference frame

- Types of Absolute Constraints
  - Absolute position constraints
  - Absolute orientation constraints
  - Absolute distance constraints

Absolute Constraints (Cntd.)

- Absolute position constraints
  - x-coordinate of P_i

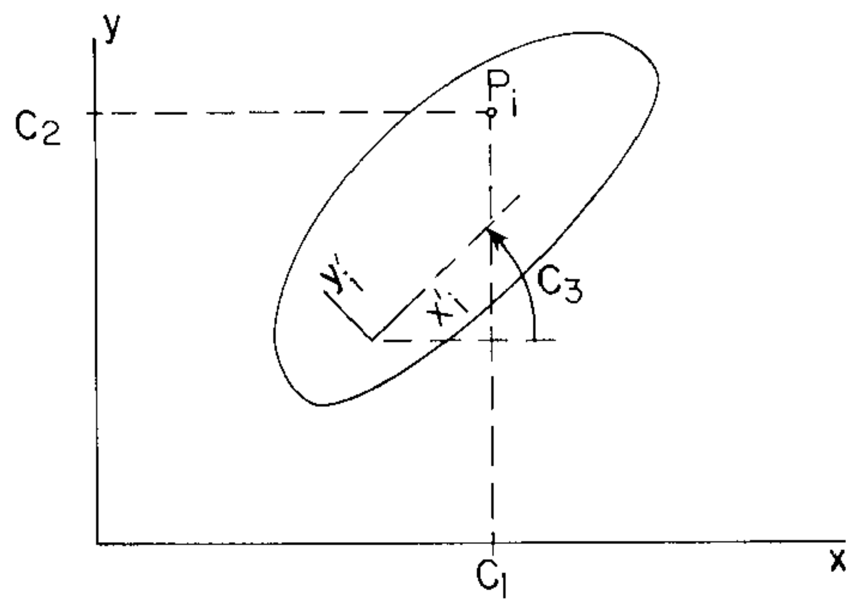
$$x^{P_i} - C_1 = 0$$

  - y-coordinate of P^i

$$y^{P_i} - C_2 = 0$$

- Absolute orientation constraint
  - Orientation ϕ of body

$$\phi_i - C_3 = 0$$



$$\mathbf{r}^{P_i} = \mathbf{r}_i + \mathbf{A}_i \mathbf{s}'^{P_i} \quad \Rightarrow \quad \mathbf{r}^{P_i} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} \cos\phi_i & -\sin\phi_i \\ \sin\phi_i & \cos\phi_i \end{bmatrix} \begin{bmatrix} x'^P_i \\ y'^P_i \end{bmatrix}$$

Absolute x-constraint

- Step 1: the absolute x component of the location of a point P_i in an absolute (or global) reference frame stays constant, and equal to some known value C_1

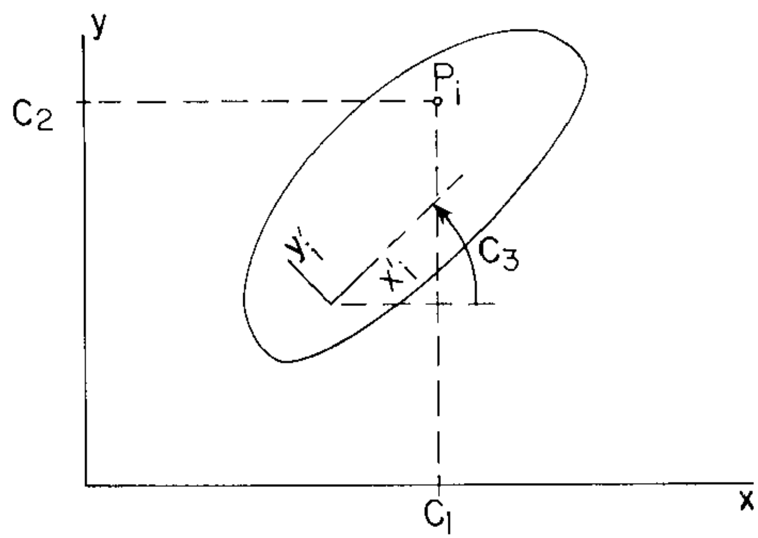


Figure 3.2.2 Constraints on absolute coordinates of point P_i and on angular orientation.

- Step 2: Identify $\Phi^{ax(i)}=0$
- Step 3: $\Phi^{ax(i)}_{\mathbf{q}} = ?$
- Step 4: $\nu^{ax(i)} = ?$
- Step 5: $\gamma^{ax(i)} = ?$

NOTE: The same approach is used to get the y- and angle-constraints