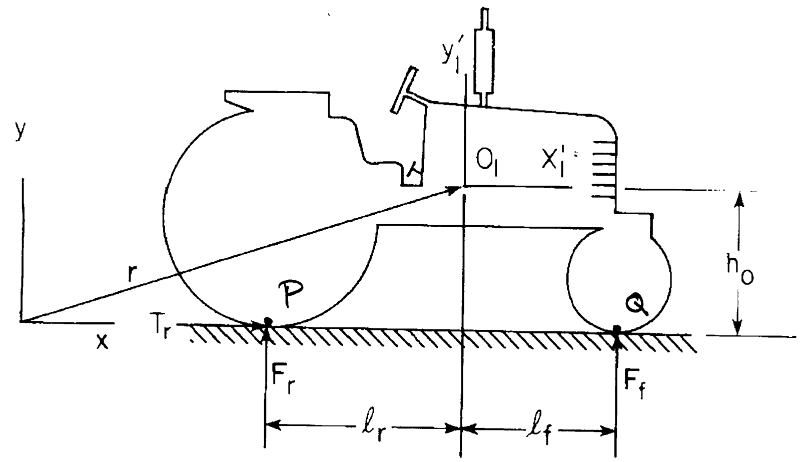
Example 6.1.1



Location of O_1:

$$r = \begin{bmatrix} x \\ y \end{bmatrix}$$

Orientation of local centroidal RF ϕ.

Small angle assumption $\sin\phi \approx \phi$ $\quad \cos\phi \approx 1$

$$r^P = r + A\begin{bmatrix} -\ell_r \\ -h_0 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 & -\phi \\ \phi & 1 \end{bmatrix}\begin{bmatrix} -\ell_r \\ -h_0 \end{bmatrix} = \begin{bmatrix} x - \ell_r + h_0\phi \\ y - \ell_r\phi - h_0 \end{bmatrix}$$

$$r^Q = r + A\begin{bmatrix} \ell_f \\ -h_0 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 & -\phi \\ \phi & 1 \end{bmatrix}\begin{bmatrix} \ell_f \\ -h_0 \end{bmatrix} = \begin{bmatrix} x + \ell_f + h_0\phi \\ y + \ell_f\phi - h_0 \end{bmatrix}$$

Deflection in rear tire: $d_r = y^P = y - \ell_r\phi - h_0$

Deflection in front tire $d_f = y^Q = y + \ell_f\phi - h_0$.

Assume that both deflections are negative. The force in front and rear tires are:

$$\begin{cases} F_f = -k \cdot d_f = k(h_0 - y - \ell_f\phi) \\ F_r = -k d_r = k(h_0 - y + \ell_r\phi) \end{cases}$$

The equations of motion assume the form:

$$\begin{cases} m\ddot{r} = F \\ J'\ddot{\phi} = n. \end{cases}$$

$$F = \begin{bmatrix} 0 \\ -mg \end{bmatrix} + \begin{bmatrix} 0 \\ F_f + F_t \end{bmatrix} + \begin{bmatrix} T_r \\ 0 \end{bmatrix}$$

$$n = T_r y + F_f l_f - F_r l_r$$

Therefore, the equations of motion are:

$$\begin{cases} m \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 \\ -mg \end{bmatrix} + \begin{bmatrix} 0 \\ k(h_0 - y - l_f \phi) + k(h_0 - y + l_r \cdot \phi) \end{bmatrix} + \begin{bmatrix} T_r \\ 0 \end{bmatrix} \\ J' \ddot{\phi} = T_r y + k l_f (h_0 - y - l_f \phi) - k l_r (h_0 - y + l_r \phi) \end{cases}$$

Note that the book has in the last equation $T_r h_0$ instead of $T_r y$, which is a mistake.